Complex Objectives in Simple Stochastic Games

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Stochastic Games

Graphs where nodes belong to \( \text{Max} \square, \text{Min} \diamond \) or are \textit{randomized} \( \bigcirc \). Random nodes come with a probability distr. over the successors.
Stochastic Games

Graphs where nodes belong to $Max \Box$, $Min \lozenge$ or are randomized $\bigcirc$. Random nodes come with a probability distr. over the successors.

Plays
are infinite paths $\rho \in V^\omega$ in the graph.
Stochastic Games

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are sets of plays (considered good for Max).
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Graphs where nodes belong to Max □, Min ◇ or are randomized ○. Random nodes come with a probability distr. over the successors.

Plays
are infinite paths $\rho \in V^\omega$ in the graph.

Objectives
are sets of plays (considered good for Max).

Player Strategies
resolve choice of successor for nodes controlled by Max/Min. E.g., $\sigma : V^*V_\Box \rightarrow \mathcal{D}(V)$
Measuring Objectives

Fixing an initial state, strategies \( \sigma : V^* V_\square \to \mathcal{D}(V) \) and \( \tau : V^* V_\lozenge \to \mathcal{D}(V) \) induces a unique probability measure \( \mathbb{P}^{\sigma, \tau} : \mathcal{O} \to [0, 1] \).
Measuring Objectives

Fixing an initial state, strategies $\sigma : V^* V_{\square} \to D(V)$ and $\tau : V^* V_{\Diamond} \to D(V)$ induces a unique probability measure $P_{\sigma, \tau} : O \to [0, 1]$.

How?

- A cylinder is a set $wV^\omega$ of runs that share the prefix $w \in V^*$.
- Define a measure $P_{\sigma, \tau}$ on cylinders by $P_{\sigma, \tau}(sV^\omega) = \mu(s)$ and

\[
P_{\sigma, \tau}(wstV^\omega) = \begin{cases} 
P_{\sigma, \tau}(wsV^\omega) \cdot \sigma(ws)(t) & \text{for } s \in V_{\Diamond} \\
\mu(s) & \text{for } s \in V_{\square} \\
\mu(s) & \text{for } s \in V_{\Diamond} 
\end{cases}
\]

- Caratheodory’s theorem guarantees a unique extension of $P_{\sigma, \tau}$ to $O \overset{\text{def}}{=} \text{the Borel sigma-algebra generated by all cylinders.}$
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P^{\sigma, \tau}(wsV^\omega) \cdot \delta(ws)(t) & \text{for } s \in V^\Diamond
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- Caratheodory’s theorem guarantees a unique extension of $\mathbb{P}^{\sigma,\tau}_s$ to $\mathcal{O} \overset{\text{def}}{=} \text{the Borel sigma-algebra generated by all cylinders}$.

Max/Min choose their strategies to maximize/minimize $\mathbb{P}^{\sigma,\tau}(\text{Obj})$. 
Objectives – Values and Optimality

Martins Theorem

\[ \sup_{\sigma} \inf_{\tau} P^{\sigma,\tau}(Obj) = \inf_{\tau} \sup_{\sigma} P^{\sigma,\tau}(Obj) \]

in particular for finite arenas and Borel Objectives Obj as before. We call this quantity the value of Obj.
Objectives – Values and Optimality

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in particular for finite arenas and Borel Objectives $\text{Obj}$ as before. We call this quantity the value of $\text{Obj}$.

Immediate Consequence
For every $\varepsilon > 0$ there is some strategy $\sigma_\varepsilon$ that achieves, against all strategies $\tau$, that

$$P^{\sigma_\varepsilon,\tau}(\text{Obj}) \geq \text{value}(\text{Obj}) - \varepsilon.$$
Objectives – Values and Optimality

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in particular for finite arenas and Borel Objectives \(Obj\) as before. We call this quantity the value of \(Obj\).

Immediate Consequence

For every \(\varepsilon > 0\) there is some strategy \(\sigma_\varepsilon\) that achieves, against all strategies \(\tau\), that

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\mathbb{P}^{\sigma_\varepsilon,\tau}(Obj) \geq \text{value}(Obj) - \varepsilon.
\]

Such a strategy is called \(\varepsilon\)-optimal.
Objectives – Values and Optimality

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$$\sup_{\sigma} \inf_{\tau} P^{\sigma,\tau}(Obj) = \inf_{\tau} \sup_{\sigma} P^{\sigma,\tau}(Obj)$$

in particular for finite arenas and Borel Objectives $Obj$ as before. We call this quantity the *value* of $Obj$.

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For every $\varepsilon > 0$ there is some strategy $\sigma_\varepsilon$ that achieves, against all strategies $\tau$, that

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Such a strategy is called $\varepsilon$-optimal. . . . and simply *optimal* if the above holds for $\varepsilon = 0$.  

Objectives – Values and Optimality

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in particular for finite arenas and Borel Objectives \( Obj \) as before.
We call this quantity the value of \( Obj \).

Immediate Consequence
For every \( \varepsilon > 0 \) there is some strategy \( \sigma_{\varepsilon} \) that achieves, against all strategies \( \tau \), that

\[ \mathbb{P}^{\sigma_{\varepsilon},\tau}(Obj) \geq \text{value}(Obj) - \varepsilon. \]

Such a strategy is called \( \varepsilon \)-optimal. . . and simply optimal if the above holds for \( \varepsilon = 0 \).

How complex do strategies need to be?
Interesting Questions

Given an objective Obj,

1. exists a strategy $\sigma$ with $\lim_{\tau} P^{\sigma,\tau}(Obj) = 1$?
Interesting Questions

Given an objective $Obj$,

1. exists a strategy $\sigma$ with $\lim_\tau \mathbb{P}^{\sigma,\tau}(Obj) > 0$?
Interesting Questions

Given an objective $Obj$,

1. exists a strategy $\sigma$ with $\lim_{\tau} \mathbb{P}^{\sigma,\tau}(Obj) > \frac{1}{2}$?
Interesting Questions

Given an objective \( Obj \),

1. exists a strategy \( \sigma \) with \( \lim_{\tau} P^{\sigma,\tau}(Obj) \geq c \)?
2. do optimal strategies exist?
Interesting Questions

Given an objective $Obj$,

1. exists a strategy $\sigma$ with $\lim_{\tau} \mathbb{P}^{\sigma,\tau}(Obj) \geq c$ ?
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3. compute (approximate) values
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5. synthesize (optimal/a.s./good) strategies
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4. what can be achieved by a given type of strategy?
5. synthesize (optimal/a.s./good) strategies
6. . . .
Interesting Objectives

- Reachability
  A given (set of) node(s) is visited.

- Parity
  Maximal colour visited infinitely often is even.

- Energy($k$)
  For all length $n$ prefixes, $k + \sum_{i=0}^{n} \text{cost}(e_i) \geq 0$

- Mean Payoff
  $\lim_{n \to \infty} \frac{\sum_{i=0}^{n} \text{cost}(e_i)}{n} > 0$
Interesting Objectives

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Combined Objectives

Some combinations are compatible. E.g.,
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- ENERGY implies a Mean Payoff of $\geq 0$. 

- $\text{B"{u}chi}_1 \cup \text{B"{u}chi}_2$. 

- $\text{PARITY} \cap \text{MP} \geq 0$.
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- ENERGY implies a Mean Payoff of $\geq 0$.
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Combined Objectives

Some combinations are compatible. E.g.,

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Combined Objectives

Some combinations are compatible. E.g.,
- ENERGY implies a Mean Payoff of \( \geq 0 \).
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Some are conflicting.
- PARITY \( \cap MP_{\geq 0} \)
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- PARITY \( \cap MP_{\geq 0} \)
- PARITY \( \cap \) ENERGY
Part 2
MDPs with Energy-Parity Objectives

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Abstract—Energy-parity objectives combine ω-regular with quantitative objectives of reward MDPs. The controller needs to avoid to run out of energy while satisfying a parity objective.

We refute the common belief that, if an energy-parity objective holds almost-surely, then this can be realised by some finite memory strategy. We provide a surprisingly simple counterexample that only uses coBüchi conditions.

We introduce the new class of bounded (energy) storage objectives that, when combined with parity objectives, preserve the finite memory property. Based on these, we show that almost-sure and limit-sure energy-parity objectives, as well as almost-sure and limit-sure storage parity objectives, are in NP ∩ coNP and can be solved in pseudo-polynomial time for energy-parity MDPs.

I. INTRODUCTION

Context. Markov decision processes (MDPs) are a standard model for dynamic systems that exhibit both stochastic and controlled behaviour [1]. Such a process starts in an initial state and makes a sequence of transitions between states. Depending on the type of the current state, either the controller gets to choose an enabled transition (or a distribution over transitions), or the next transition is chosen randomly according to a predefined distribution. By fixing a strategy for the controller, restricted case of energy-Büchi objectives, finite memory optimal strategies exist, and that almost-sure satisfiability is in NP ∩ coNP and can be solved in pseudo-polynomial time.

They then describe a direct reduction from almost-sure energy-parity to almost-sure energy-Büchi. This reduction claimed that the winning strategy could be chosen among a certain subclass of strategies, that we call colour-committing. Such a strategy eventually commits to a particular winning even colour, where this colour must be seen infinitely often almost-surely and no smaller colour must ever been seen after committing.

However, this reduction from almost-sure energy-parity to almost-sure energy-Büchi in [2] (Sec. 3) contains a subtle error (which also appears in the survey in [3] (Theorem 4)). In fact, we show that strategies for almost-sure energy-parity may require infinite memory.

Our contributions can be summarised as follows.

1) We provide a simple counterexample that shows that, even for almost-sure energy-coBüchi objectives, the winning strategy requires infinite memory and cannot be chosen among the colour-committing strategies.

2) We introduce an energy storage objective, which requires
Almost-sure Problems for finite MDPs

- **ENERGY**
  - Determined
  - $\mathcal{NP} \cap \text{coNP}$
  - Pseudo P
- **Positive Mean-Payoff**
  - Determined
  - P-time
- **PARITY**
  - Determined
  - P-time
  - $\text{PosMP} \cap \text{PAR}$
  - Determined
  - P-time

Chatterjee/Doyen, MFCS'11
Almost-sure Problems for finite MDPs

ENERGY

- FD determined
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**Positive Mean-Payoff**
- MD determined
- P-time

**EN \(\cap\) PAR**
- reduces to \(EN \cap \text{B"uchi}\)
- reduces to EN games
- FD determined

**PARITY**
- MD determined
- P-time

\(EN \cap PAR\)

Chatterjee/Doyen, MFCS’11
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$\text{EN} \cap \text{PAR}$ reduces to $\text{EN} \cap \text{B"uchi}$
$\times$ reduces to $\text{EN}$ games
$\times$ FD determined

Chatterjee/Doyen, MFCS’11
What’s the Problem?

Aim: Satisfy ENERGY and avoid STAY in \{A, B, C\} and lose ENERGY ⏯ prefer D over B and lose PARITY

No FM-strategy wins (a.s.) ⏯ eventually commits to red or green ⏯ maintain bounded distance to D An (a.s.) winning strategy ⏯ "move to D only if energy level is 0" ⏯ works because P green (always > 0) > 1/2
What’s the Problem?

Aim: Satisfy ENERGY and avoid $D$

A (a.s.) winning strategy
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- stay in \{A, B, C\} and lose ENERGY
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- stay in $\{A, B, C\}$ and lose ENERGY
- prefer $D$ over $B$ and lose PARITY
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- pseudo $\text{P}$

PARITY
- MD determined
- P-time

Positive Mean-Payoff
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$\text{PosMP} \cap \text{PAR}$
- reduces to $\text{EN} \cap \mathbb{B}" uchi$
- reduces to $\text{EN}$ games
- FD determined
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- NP $\cap$ coNP
- pseudo P

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- P-time

**EN $\cap$ PAR**
- needs $\infty$ memory

**Positive Mean-Payoff**
- MD determined
- P-time

**PosMP $\cap$ PAR**
- MD determined
- P-time
The Storage Objective

A path satisfies the $k$-Storage condition $\text{ST}(k)$ if

$$k + \sum_{i=n}^{m} \text{cost}(e_i) \geq 0$$

for all indices $n < m$. 

CAUTION

(a.s.) ENERGY $\iff$ (a.s.) STORAGE

but

(a.s.) ENERGY $\cap$ PARITY $\not\iff$ (a.s.) STORAGE $\cap$ PARITY
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($\text{a.s.}$ $\text{ENERGY} \iff \text{a.s.} \text{STORAGE}$ but $\text{a.s.} \text{ENERGY} \cap \text{PARITY} \nsymbol{\not\iff} \text{a.s.} \text{STORAGE} \cap \text{PARITY}$)
The Storage Objective

A path satisfies the $k$-Storage condition $\text{ST}(k)$ if

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The STORAGE objective asks for ENERGY and $\text{ST}(k)$ for some $k$. 
The Storage Objective

A path satisfies the $k$-Storage condition $\text{ST}(k)$ if

$$k + \sum_{i=n}^{m} \text{cost}(e_i) \geq 0$$

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The STORAGE objective asks for ENERGY and $\text{ST}(k)$ for some $k$. (it holds a.s. if $\exists \sigma \exists k. \mathbb{P}^{\sigma}(\text{ST}(k)) = 1 \wedge \mathbb{P}^{\sigma}(\text{ENERGY}) = 1$)
The Storage Objective

A path satisfies the $k$-Storage condition $ST(k)$ if

$$k + \sum_{i=n}^{m} \text{cost}(e_i) \geq 0$$

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The STORAGE objective asks for ENERGY and $ST(k)$ for some $k$. (it holds a.s. if $\exists \sigma \ \exists k. \ \mathbb{P}^{\sigma}(ST(k)) = 1 \land \mathbb{P}^{\sigma}(ENERGY) = 1$)

CAUTION

(a.s.) ENERGY $\iff$ (a.s.) STORAGE

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(a.s.) ENERGY $\cap$ PARITY $\niff$ (a.s.) STORAGE $\cap$ PARITY
Almost-sure Problems for finite MDPs

**ENERGY**
- FD determined
- NP ∩ coNP
- pseudo P

**PARITY**
- MD determined
- P-time

**EN ∩ PAR**
- needs ∞ memory

**Positive Mean-Payoff**
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**ST ∩ PAR**
- FD determined
- reduces to ST ∩ Büchi
- reduces to STORAGE

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Computing Winning Sets

ENERGY \cap PARITY holds almost-surely iff

1. ST \cap PARITY holds a.s., or
Computing Winning Sets

ENERGY ∩ PARITY holds almost-surely iff

1. ST ∩ PARITY holds a.s., or
2. There are strategies σ, σ' such that
   2.1 σ witnesses a.s. ST ∩ PosMP
   2.2 σ stays in the winning set of σ'
   2.3 σ' witnesses a.s. PAR ∩ PosMP
   2.4 σ' stays in the winning set of σ
Computing Winning Sets

\( \text{ENERGY} \cap \text{PARITY} \) holds almost-surely iff

1. \( \text{ST} \cap \text{PARITY} \) holds a.s., or
2. There are strategies \( \sigma, \sigma' \) such that
   2.1 \( \sigma \) witnesses a.s. \( \text{ST} \cap \text{PosMP} \)
   2.2 \( \sigma \) stays in the winning set of \( \sigma' \)
   2.3 \( \sigma' \) witnesses a.s. \( \text{PAR} \cap \text{PosMP} \)
   2.4 \( \sigma' \) stays in the winning set of \( \sigma \)

Bailout: arbitrary increase
Computing Winning Sets

ENERGY \cap PARITY holds almost-surely iff

1. ST \cap PARITY holds a.s., or
2. There are strategies σ, σ′ such that
   2.1 σ witnesses a.s. ST \cap PosMP
   2.2 σ stays in the winning set of σ′
   2.3 σ′ witnesses a.s. PAR \cap PosMP
   2.4 σ′ stays in the winning set of σ

δ > 0 chance of success
Computing Winning Sets

ENERGY \cap \text{PARITY} holds almost-surely iff

1. ST \cap \text{PARITY} holds a.s., or
2. There are strategies \( \sigma, \sigma' \) such that
   2.1 \( \sigma \) witnesses a.s. \( \text{ST} \cap \text{PosMP} \)
   2.2 \( \sigma \) stays in the winning set of \( \sigma' \)
   2.3 \( \sigma' \) witnesses a.s. \( \text{PAR} \cap \text{PosMP} \)
   2.4 \( \sigma' \) stays in the winning set of \( \sigma \)

\( \text{switch and try again} \)
The Limit-sure Variant

$$\sup_{\sigma} P^\sigma(\text{Obj}) = 1?$$

Here, EN $\cap$ Büchi($C$) holds limit-surely but not almost-surely in $A$. 
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(equivalent: for all $\varepsilon > 0$ exists a strategy $\sigma$ with $\mathbb{P}^\sigma(Obj) > 1 - \varepsilon$)
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(equivalent: for all \( \varepsilon > 0 \) exists a strategy \( \sigma \) with \( \mathbb{P}^{\sigma}(\text{Obj}) > 1 - \varepsilon \))

Here, \( \text{EN} \cap \text{B"uchi}(C) \) holds limit-surely but not almost-surely in \( A \).
Computing Winning Sets

ENERGY \cap PARITY holds almost-surely iff

1. ST \cap PARITY holds a.s., or

2. There are strategies \( \sigma, \sigma' \) such that
   2.1 \( \sigma \) witnesses a.s. ST \( \cap \) PosMP
   2.2 \( \sigma \) stays in the winning set of \( \sigma' \)
   2.3 \( \sigma' \) witnesses a.s. PAR \( \cap \) PosMP
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ENERGY \cap PARITY objectives for finite MDPs

- Almost-sure optimal strategies need infinite memory.
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What next?
- generalize to full stochastic games
- multiple dimensions of energy
- properly classify infinite-memory strategies
- build practical solvers for 2-player Energy Games
Summary & Outlook

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thank you.