Point Distribution Models

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Point Distribution Models

Given:

Set of objects from the same class

Set of point positions $\{\vec{x}_i\}$ for each object instance

Assume:

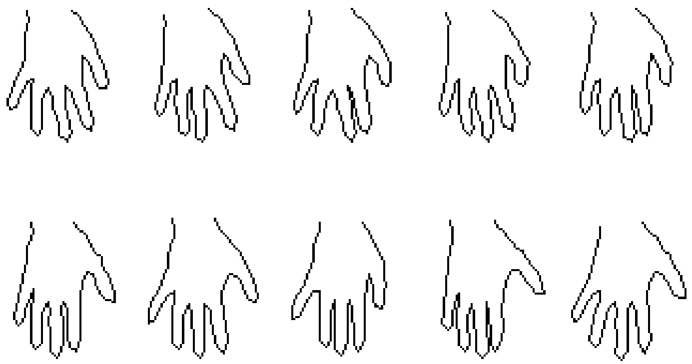
Point positions have a systematic structural variation plus a Gaussian noise point distribution

Thus, point position variations are correlated

Goals:
Construct a model that captures structural as well as statistical position variation.
Use model for recognition

Data Example

A family of objects with shape variations



How to represent?

Point Distribution Models - PDMs

Given a set of N observations, each with P boundary points $\{(r_{ik}, c_{ik})\}, k = 1..P, i = 1..N$ in corresponding positions.

Key Trick: rewrite $\{(r_{ik}, c_{ik})\}$ as a new $2P$ vector $\vec{x}_i = (r_{i1}, c_{i1}, r_{i2}, c_{i2}, ..., r_{ip}, c_{ip})'$

Gives N vectors $\{\vec{x}_i\}$ of dimension $2P$

PDMs II

If shape variations are random, then components of $\{\vec{x}_i\}$ will be uncorrelated.

If there is a systematic variation, then components will be correlated.

Use PCA to find correlated variations.

PDM II: The Structural Model

PCA over the set $\{\vec{x}_i\}$ gives a set of $2P$ axes such that

$$\vec{x}_i = \vec{m} + \sum_{j=1}^{2P} w_{ij}\vec{a}_j$$

$2P$ axes gives complete representation for $\{\vec{x}_i\}$.

Approximate shapes using a subset M of the most significant axes (based on the eigenvalue size from PCA):

$$\vec{x}_i \doteq \vec{m} + \sum_{j=1}^{M} w_{ij}\vec{a}_j \quad (1)$$

PDM II: The Structural Model

Represent $\vec{x}_i$ using $\vec{w}_i = (w_{i1}, ..., w_{iM})'$

A smaller representation as $M << 2P$

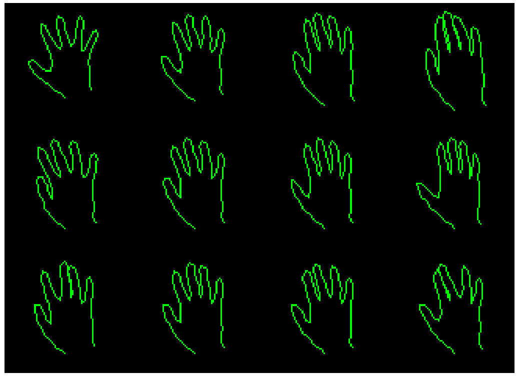
Goal: represent the essential structure variations

Approximate full shape reconstruction using $\vec{w}_i$ and (1)

Can vary $\vec{w}_i$ to vary shape

Structural model - varying the weights

Each row here varies one of top 3 eigenvectors of model from hand outlines



Visualisation of the main modes of structural variation

PDM III: The Statistical Model

If we have a good structural model, then the component weights should characterise the shape.

A family of shapes should have a distribution that characterises normal shapes (and abnormal shapes are outliers).
We assume that the distribution of normal shape weights is Gaussian.

Statistical Model:

Given a set of N component projection vectors $\{\vec{w}_i\}$

Mean vector is $\vec{t} = \frac{1}{N} \sum_i \vec{w}_i$

Covariance matrix $\mathrm{C} = \frac{1}{N-1} \sum_i (\vec{w}_i - \vec{t})(\vec{w}_i - \vec{t})'$

What We Have Learned

1. Using PDMs to model shape variation
2. Extracting the structural and statistical models