Review of 2D coordinate geometry

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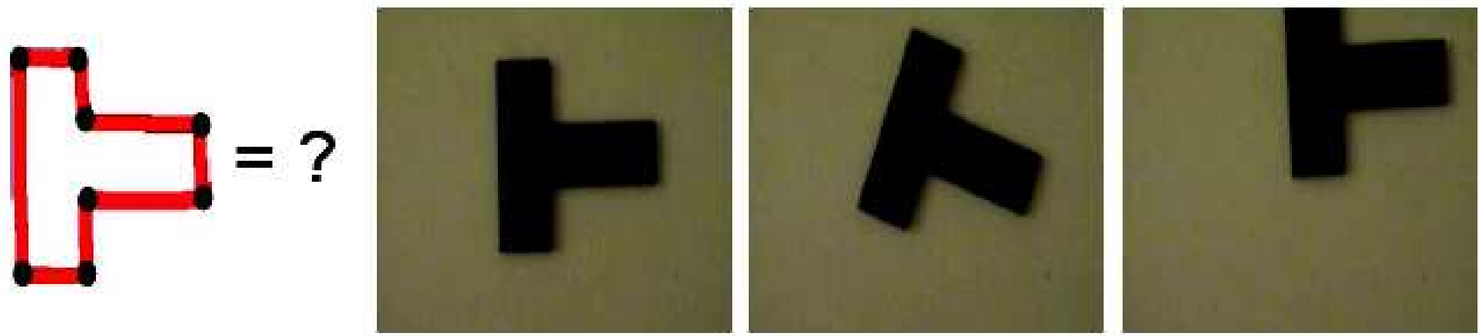
1. Object and Scene Coordinate Systems
2. Coordinate System Transformations
3. Homogeneous Coordinates I
4. Multiple Reference Frame Transformations

Object and Scene Coordinate Systems

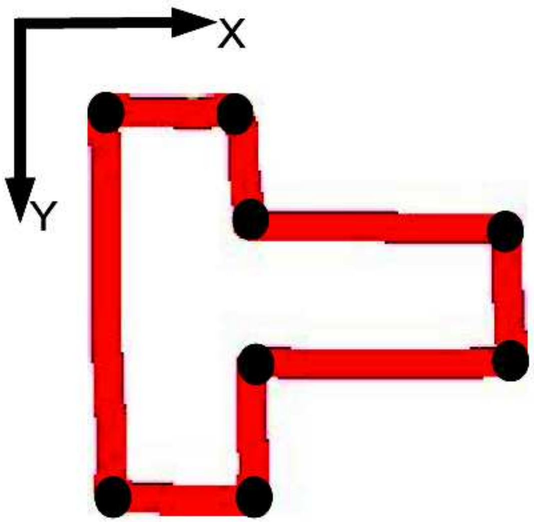
Issues:

+ Want to describe object features independently of the object's position.

+ Want to specify object position and orientation within scene



Why? Generic Model *vs* Specific Position



Generic:

Object geometric model, aligned

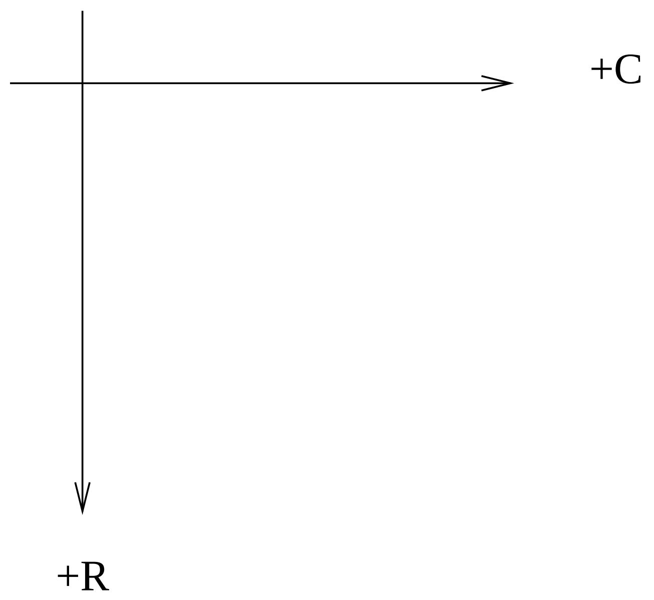
Specific:

Scene position in pixels, not aligned

Object and Scene Coordinate Systems II

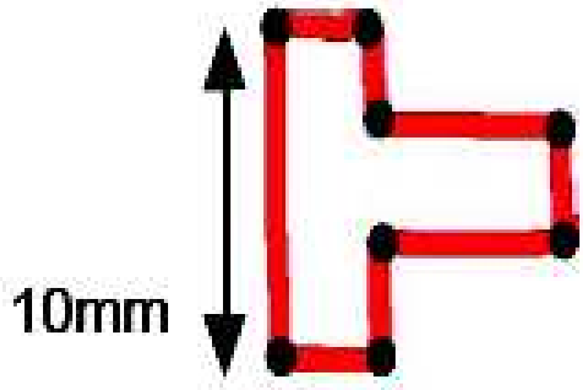
Solution: Use separate object and scene coordinate systems and link by reference frame transformations

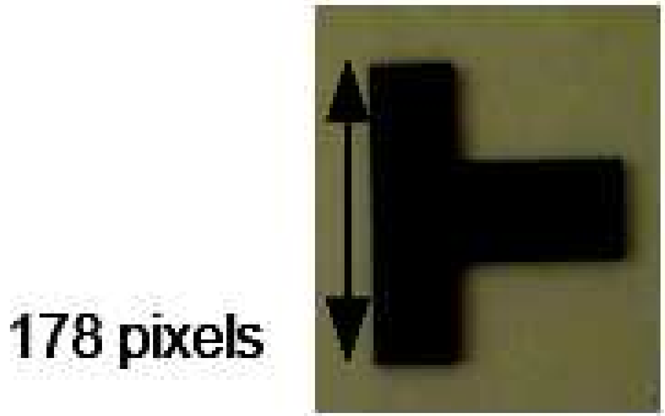
Use image coordinate system $(c, r), c \in [0, C]$, $r \in [0, R]$ for $C \times R$ image (for convenience)



Object and Scene Coordinate Systems III

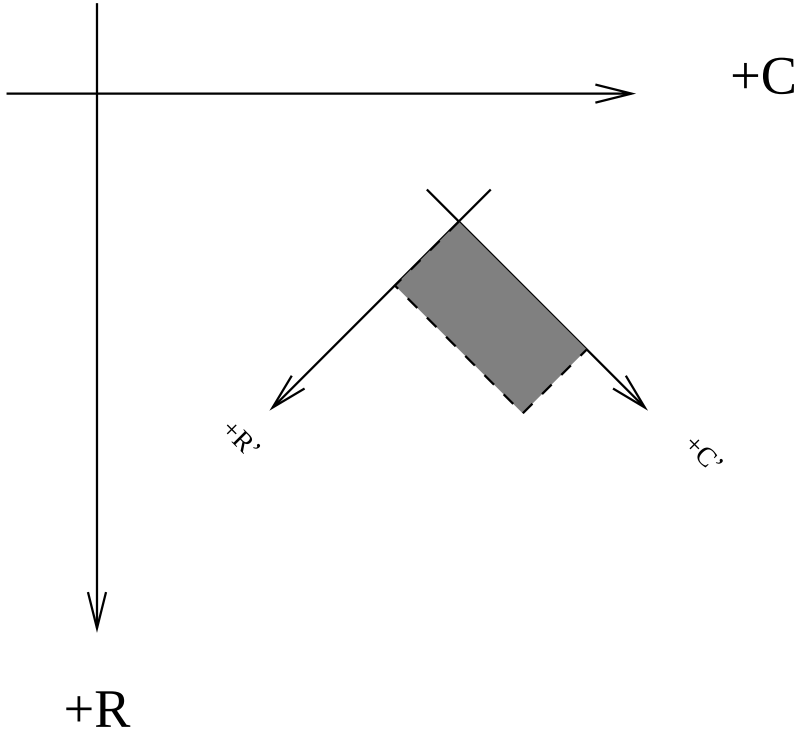
(c, r) in image (eg. in pixels) relates to (x, y) in scene (eg. in mm) using column, row scale factors ρ_c, ρ_r: $(x, y) = (\rho_c c, \rho_r r)$





Object and Scene Coordinate Systems IV

Use separate coordinate systems for object and scene

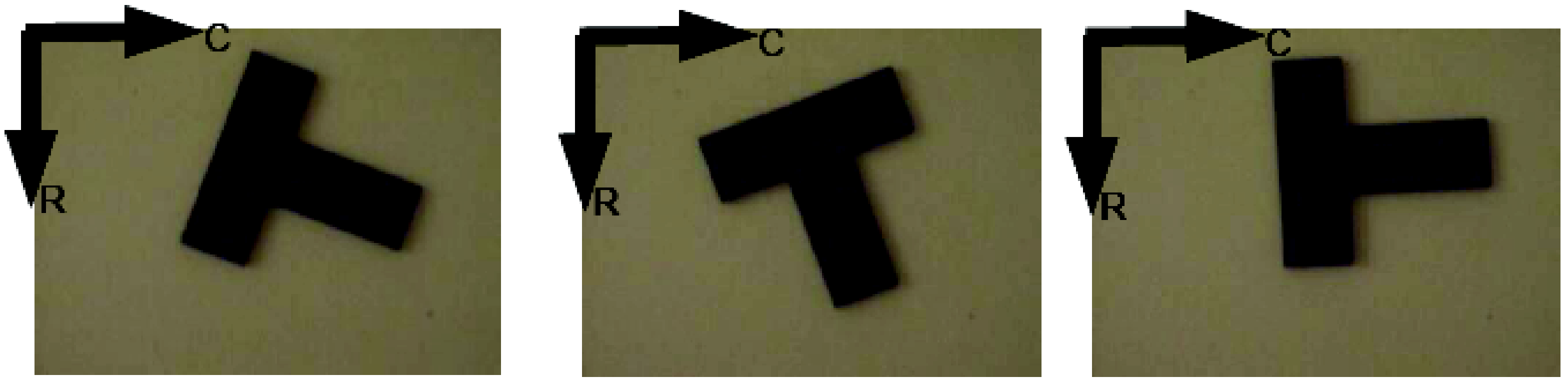


Also - image and camera coordinate systems

Coordinate System Transformations I

Placement of object relative to scene requires a coordinate system transformation

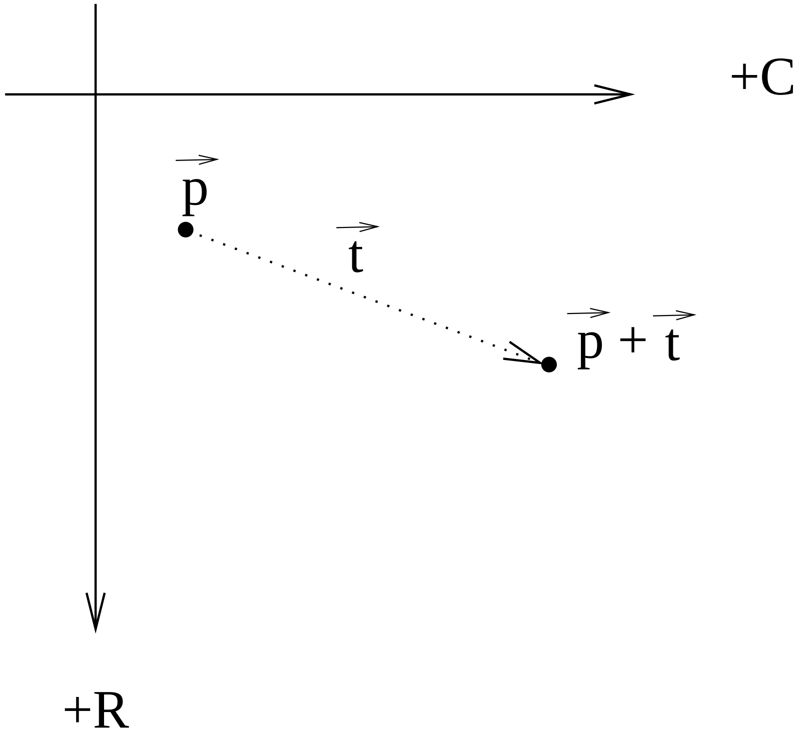
In 2D, need 1 rotation angle θ and $\vec{t} = (t_c, t_r)'$ translation (' is for transposing a row vector to a column vector and *vice versa*)

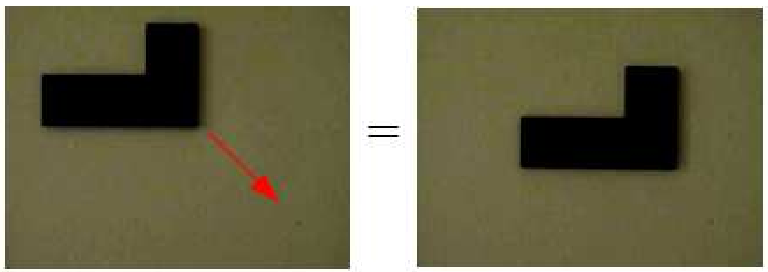


Coordinate System Transformations II

$\vec{p} = (a, b)'$ is a point in the 2D coord system

Translation of point $\vec{p} = (a, b)'$ by $\vec{t} = (t_c, t_r)'$ moves it to $\vec{p} + \vec{t} = (a + t_c, b + t_r)'$





Coordinate System Transformations III

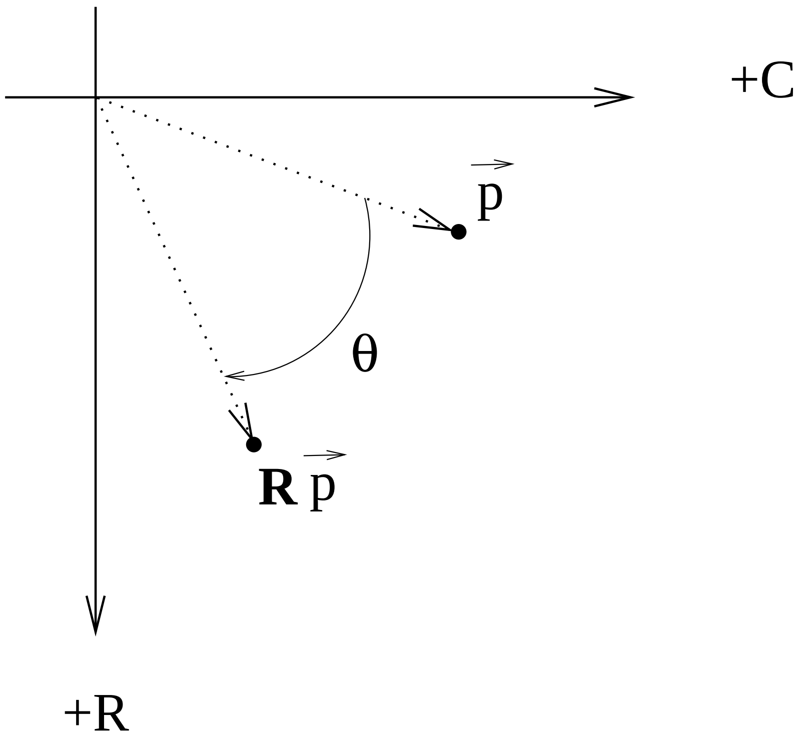
If θ is the rotation angle, let

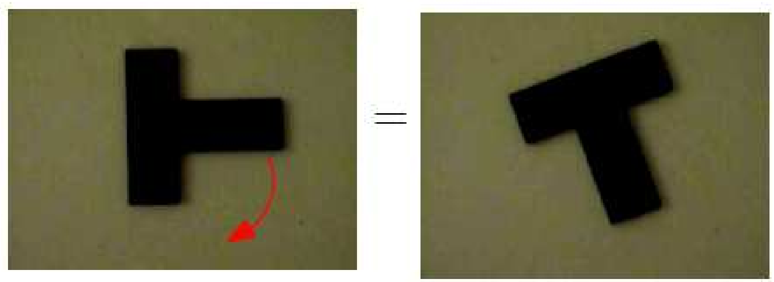
$$\mathrm{R} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

Sometime see $sin(\theta)$ and $-sin(\theta)$ swapped. A matter of convention about direction of rotation.

Coordinate System Transformations IV

Rotation of point $\vec{p} = (a, b)'$ by R moves it to $\mathrm{R}\vec{p} = (a{\cdot}cos(\theta){-}b{\cdot}sin(\theta), a{\cdot}sin(\theta){+}b{\cdot}cos(\theta))'$

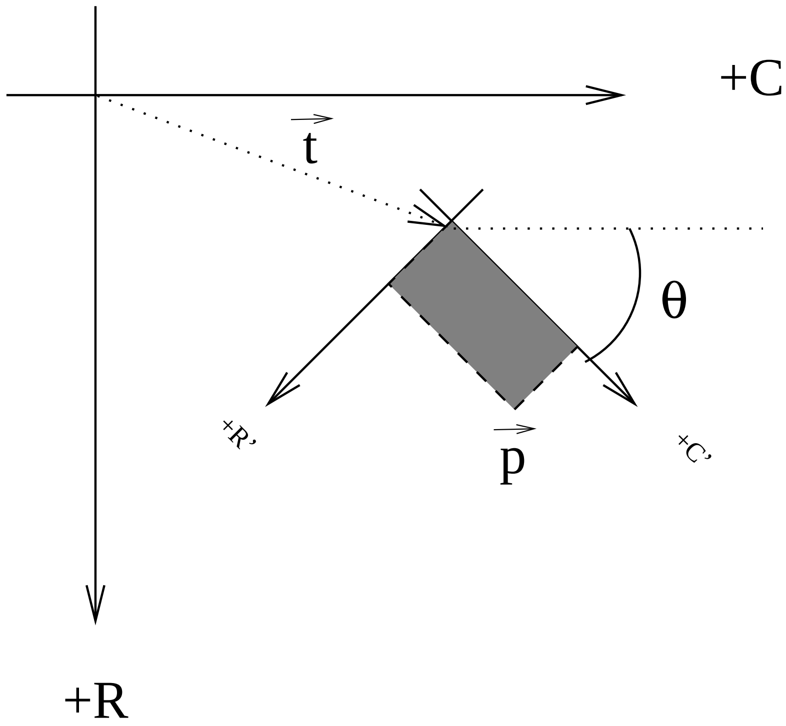




θ positive is clockwise rotation (other definition common)

Complete Transformations

Rotation & Translations: $\mathrm{R}\vec{p} + \vec{t}$



If the object local coordinate system starts at (0,0)', then the rotation & translation specify its position

Homogeneous Coordinates I

Instead of 2 operations to implement the transformation, often only one operation based on homogeneous coordinates (more advanced form in later lectures)

1) Extend points $\vec{p} = (a, b)'$ to $\vec{P} = (a, b, 1)'$

2) Extend vectors $\vec{d} = (u, v)'$ to $\vec{D} = (u, v, 0)'$

3) Combine rotation and translation into one 3×3 matrix

$$\mathrm{T} = \begin{bmatrix} \cos(\theta) & -sin(\theta) & t_c \\ sin(\theta) & cos(\theta) & t_r \\ 0 & 0 & 1 \end{bmatrix}$$

Full transformation of $\vec{p}$ is now $\mathrm{T}\vec{P}$

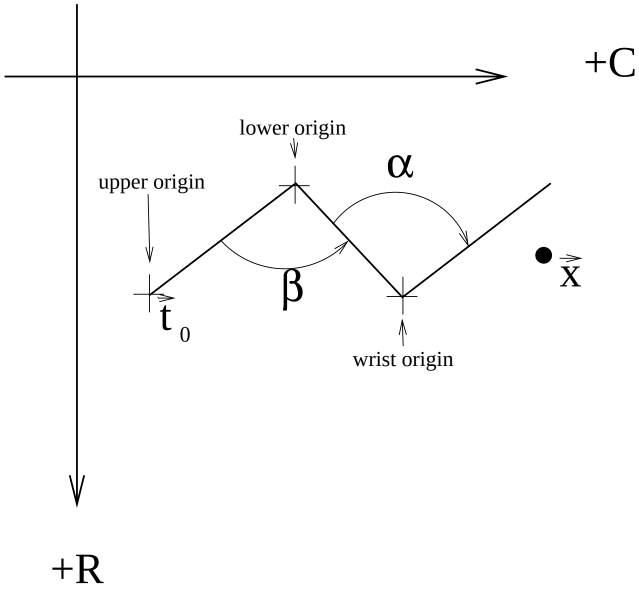
Multiple Transformations

Given 2 joint robot arm whose joint angles are α and β

$T_w(\alpha)$ is the wrist joint position relative to the lower arm

$T_l(\beta)$ is the lower arm position relative to the upper arm

The arm is at position T_0



RTX robot arm

Multiple Transformations II

Then, a wrist coordinate point $\vec{x}$ at the tip of the robot is at

$$\vec{y} = \mathrm{T}_0 \mathrm{T}_l(\beta) \mathrm{T}_w(\alpha) \vec{x}$$

Can also easily invert positions:

$$\vec{x} = (\mathrm{T}_w(\alpha))^{-1} (\mathrm{T}_l(\beta))^{-1} (\mathrm{T}_0)^{-1} \vec{y}$$

is the wrist coordinates of scene point $\vec{y}$

What We Have Learned

1. Review of Coordinate Systems Transformations

2. Introduction to Homogeneous Coordinates