

Coordinate System Review Problem

What is the position resulting from rotating the point $\vec{x} = (10, 20)'$ by $\frac{\pi}{2}$ and translating the result by $\vec{t} = (-10, 30)'$?

ANSWER: Let: $\vec{x} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$, $\vec{t} = \begin{bmatrix} -10 \\ 30 \end{bmatrix}$ and $\theta = \frac{\pi}{2}$.

Then

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

So, the result is

$$R\vec{x} + \vec{t} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix} + \begin{bmatrix} -10 \\ 30 \end{bmatrix} = \begin{bmatrix} -30 \\ 40 \end{bmatrix}$$