Stereo Geometry

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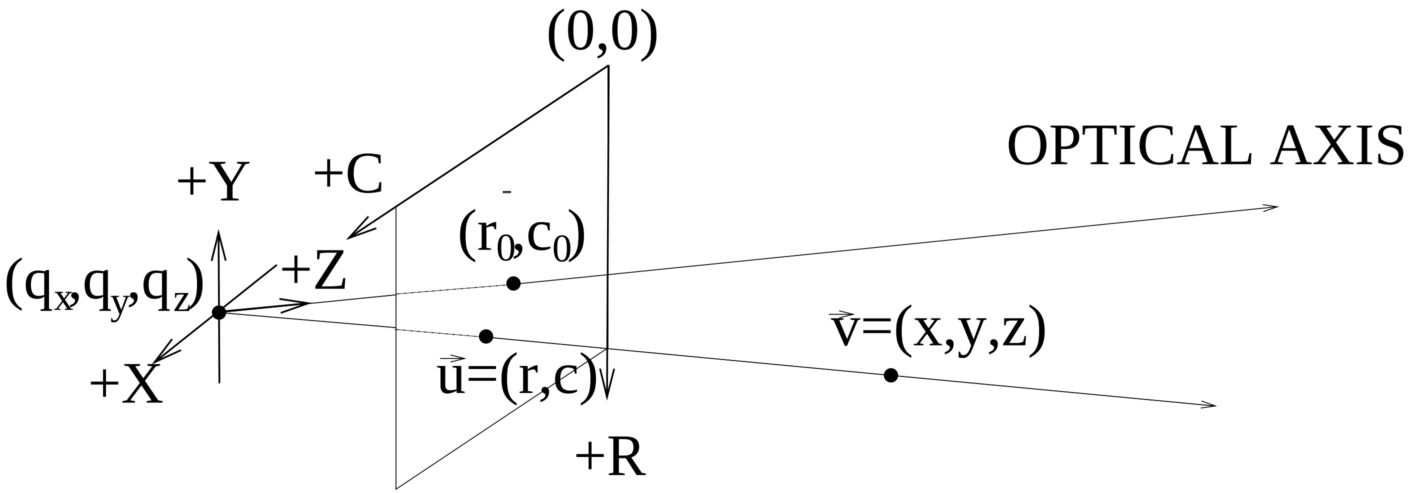
Image Projection Geometry

Pinhole camera model

Projection matrix P_i projects 3D point (in homogeneous coordinates) $\vec{v} = (x, y, z, 1)'$ onto image point $\vec{u}_i = (c_i, r_i, 1)'$ $i = L, R$.

$\lambda_i \vec{u}_i = \mathrm{P}_i \vec{v}$ so if $(\alpha, \beta, \gamma) = \mathrm{P}_i \vec{v}$, then $(c_i, r_i) = (\alpha/\gamma, \beta/\gamma)$



Projection Matrix

Projection matrix P_i decomposes as

$$\mathrm{P}_i = \mathrm{K}_i \mathrm{R}_i [I| - \vec{q}_i]$$

R_i : rotation matrix of camera (3 degrees of freedom) in world coordinates

$\vec{q}_i = (q_{xi}, q_{yi}, q_{zi})'$: camera centre position in world coordinates (3 DoF)

K_i : camera intrinsic calibration matrix

Camera Intrinsic Matrix

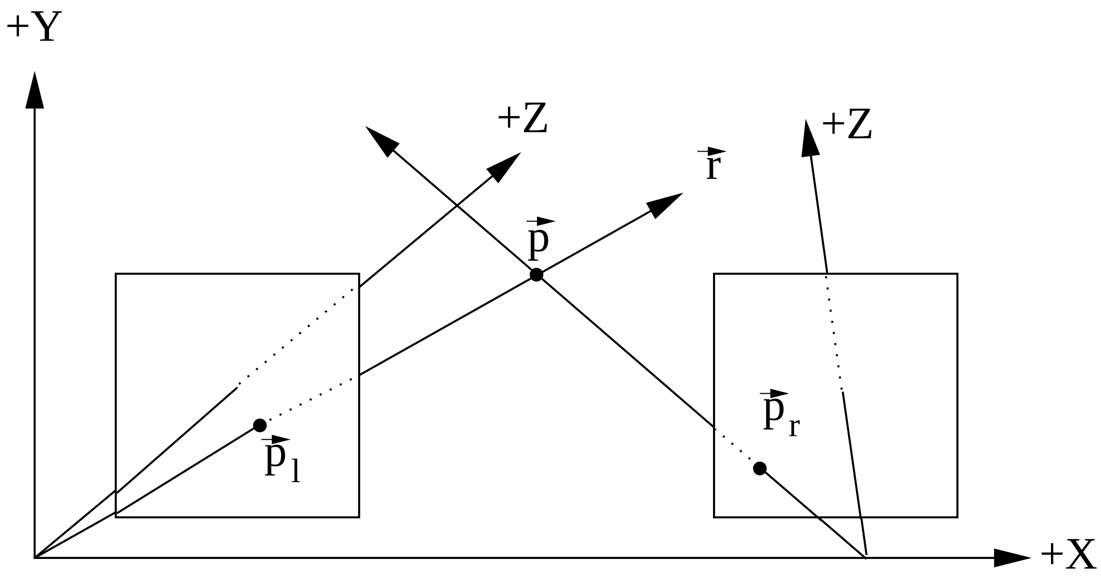
$$\mathrm{K}_i = \begin{bmatrix} f_i m_{ci} & s_i & c_{0i} \\ 0 & f_i m_{ri} & r_{0i} \\ 0 & 0 & 1 \end{bmatrix}$$

f_i : camera focal length in mm
m_{ri}, m_{ci} : row, col pixels/mm conversion on image plane
r_{0i}, c_{0i} : where optical axis hits image plane
s_i : skew factor

12 parameters (but 11 Degrees of Freedom) per camera

Epipolar Geometry

Points $\vec{p}_l$ in left image and $\vec{p}_r$ in right image are linked by common 3D point $\vec{p}$ in scene.



Images are linked by the **Fundamental matrix F**

Matched points satisfy $\vec{p}_r'\mathrm{F}\vec{p}_l = 0$

(Points are in homogeneous coordinates, $\mathbf{F}$ is 3×3)

Line Intersection

If image line is $a \times col + b \times row = d$, then homogeneous representation of line is $\vec{v}_1 = (a, b, d)'$

Given second line $\vec{v}_2$, intersection (in homogeneous coordinates) with first line is:

$$\begin{bmatrix} 0 & -d & b \\ d & 0 & -a \\ -b & a & 0 \end{bmatrix} \vec{v}_2 = [\vec{v}_1]_\times \vec{v}_2 = \vec{v}_1 \times \vec{v}_2$$

Estimating the Fundamental matrix

Assume $N \geq 8$ matched points $\vec{u}_i : \vec{v}_i, i = 1 \ldots N$ in 2 images
Each should satisfy $\vec{u}_i' \mathrm{F} \vec{v}_i = 0$
Noisy, so use a least squares algorithm. Expanding $\vec{u}_i' \mathrm{F} \vec{v}_i$ gives an equation in N variables:

$$[u_{ix}v_{ix}, u_{ix}v_{iy}, u_{ix}, u_{iy}v_{ix}, u_{iy}v_{iy}, u_{iy}, v_{ix}, v_{iy}, 1]\vec{f} = A_i\vec{f} = 0$$

when we unfold

$$\mathrm{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

into $\vec{f} = (f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33})'$.

Then we stack the A_i up as:

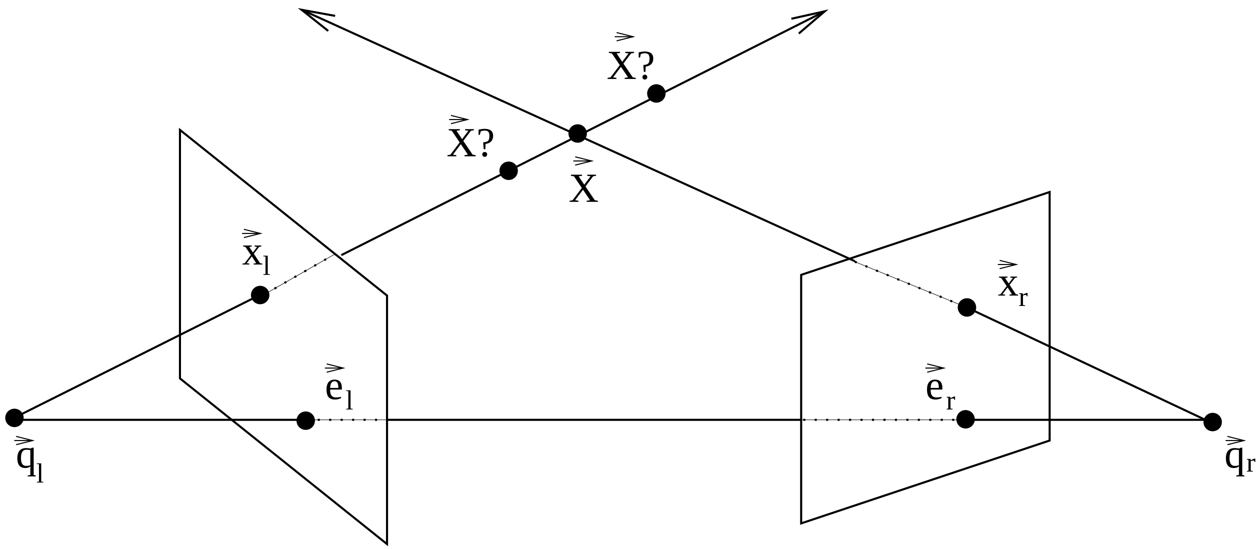
$$\mathrm{A}\vec{f} = \begin{bmatrix} A_1 \\ \dots \\ A_N \end{bmatrix} \vec{f} = 0$$

Solve for $\vec{f}$: svd(A) $= UDV'$, $\vec{f} = V(:,9)$ (plus some numerical adjustment to get det($\mathbf{F}$)=0)

Not numerically best algorithm, but simple to understand
See Hartley and Zisserman Chapter 10

Epipoles

Line connecting the 2 camera centres intersects the image planes



Estimate epipoles $\vec{e}_l$, $\vec{e}_r$, by exploiting $\vec{e}_r\ 'F = F\vec{e}_l = \vec{0}$

Solve 3 equations in 2 variables for unknown epipoles

$$(e_{rx}, e_{ry}, 1)F = F(e_{lx}, e_{ly}, 1)' = \vec{0}$$

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eL=null(F); eL = eL/eL(3) eR=null(F'); eR = eR/eR(3)
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Estimating Projection Matrices

Given left/right intrinsic parameter matrices K_L, K_R and Fundamental matrix F

Compute Essential Matrix: $\mathrm{E} = \mathrm{K}_R'\mathrm{F}\mathrm{K}_L$

Define matrix

$$\mathrm{W} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Decompose $\mathrm{E} : \mathrm{svd}(\mathrm{E}) = [\mathrm{U} \;\; \mathrm{S} \;\; \mathrm{V}]$

$\mathrm{P}_R = [\mathrm{UWV}' \mid \mathrm{U}(:, 3)]$

Note: this is 1 of 4 similar solutions for P_R; the correct one can be chosen by testing. Also, a numerical tweak is needed before SVD.

$$\mathrm{P}_L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\mathrm{P}_L, \mathrm{P}_R$ are determined up to an arbitrary projective transformation H, ie. $\mathrm{P}_L\mathrm{H}, \mathrm{P}_R\mathrm{H}$ generate the same projections.

What We Have Learned

- Pinhole camera model
- Projection and Intrinsic parameter matrices
- Epipolar geometry and Fundamental matrix
- Estimating the Epipoles, Fundamental and Projection matrices