Introduction to Kalman Filter

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Model based Tracking: Kalman filter

Why? Model can be used to

1. Predict likely position, thus reducing search
2. Integrate noisy observations, thus giving improved estimates

What's in model (here called **state**): position, velocity, shape, ...

KALMAN FILTER INTRODUCTION

“ A set of mathematical equations that provides an efficient computational (recursive) solution to the least-squares method.” [Welch & Bishop]

Most commonly used position estimator used in tracking problems

KALMAN FILTER THEORY

Assumes:

1. A changing (with t) **state** vector: $\vec{x}_t$ and its associated covariance $\mathbf{P}_t$ and state transition matrix $\mathbf{A}$

2. A **process model** that updates the state over time:

$$\vec{x}_t = \mathbf{A}\vec{x}_{t-1} + \mathbf{B}\vec{u}_{t-1} + \vec{w}_{t-1}$$

   where:

   - $\mathbf{A}$ - updates the state
   - $\mathbf{B}\vec{u}_{t-1}$ - some external action on the state at $t-1$
   - $\vec{w}_{t-1}$ - process noise: multi-variate normal distribution, mean $\vec{0}$ and covariance $\mathbf{Q}$

3. An **observation model** that relates measured data $\vec{z}_t$ to the current state:

$$\vec{z}_t = \mathbf{H}\vec{x}_t + \vec{v}_t$$

where:

- $\mathbf{H}$ - extracts observations
- $\vec{v}_t$ - observation noise: multi-variate normal distribution, mean $\vec{0}$ and covariance $\mathbf{R}$

KALMAN FILTER ALGORITHM

1. Predict likely state given what we already know: $\vec{y}_t = \mathbf{A}\vec{x}_{t-1} + \mathbf{B}\vec{u}_{t-1}$
2. Estimate covariance (error) of predicted state: $\mathbf{E}_t = \mathbf{A}\mathbf{P}_{t-1}\mathbf{A}' + \mathbf{Q}$
3. Estimate correction gain between actual and predicted observations:
   $\mathbf{K}_t = \mathbf{E}_t\mathbf{H}'(\mathbf{H}\mathbf{E}_t\mathbf{H}' + \mathbf{R})^{-1}$

4. Estimate new state given prediction and correction from observations:

   $\vec{x}_t = \vec{y}_t + \mathbf{K}_t(\vec{z}_t - \mathbf{H}\vec{y}_t)$

5. Estimate error of new state:

   $\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{H})\mathbf{E}_t$

What We Have Learned

1. What and why of Kalman Filter
2. Standard update equations