SEGMENTED IMAGES
The idea of segmentation has its roots in work by the Gestalt psychologists (e.g., Kohler), who studied the preferences exhibited by human beings in grouping or organizing sets of shapes arranged in the visual field. Gestalt principles dictate certain grouping preferences based on features such as proximity, similarity, and continuity. Other results had to do with figure/ground discrimination and optical illusions. The latter have provided a fertile ground for vision theories to post-Gestaltists such as Gibson and Gregory, who emphasize that these grouping mechanisms organize the scene into meaningful units that are a significant step toward image understanding.

In computer vision, grouping parts of a generalized image into units that are homogeneous with respect to one or more characteristics (or features) results in a segmented image. The segmented image extends the generalized image in a crucial respect: it contains the beginnings of domain-dependent interpretation. At this descriptive level the internal domain-dependent models of objects begin to influence the grouping of generalized image structures into units meaningful in the domain. For instance, the model may supply crucial parameters to segmentation procedures.

In the segmentation process there are two important aspects to consider: one is the data structure used to keep track of homogeneous groups of features; the other is the transformation involved in computing the features.

Two basic sorts of segments are natural: boundaries and regions. These can be used combined into a single descriptive structure, a set of nodes (one per region), connected by arcs representing the “adjacency” relation. The “dual” of this structure has arcs corresponding to boundaries connecting nodes representing points where several regions meet. Chapters 4 and 5 describe segmentation with respect to boundaries and regions respectively, emphasizing gray levels and gray-level differences as indicators of segments. Of course, from the standpoint of the
algorithms involved, it is irrelevant whether the features are intensity gray levels or intrinsic image values perhaps representing motion, color, or range.

Texture and motion images are addressed in Chapters 6 and 7. Each has several computationally difficult aspects, and neither has received the attention given static, nontextured images. However, each is very important in the segmentation enterprise.
Boundary Detection

4.1 ON ASSOCIATING EDGE ELEMENTS

Boundaries of objects are perhaps the most important part of the hierarchy of structures that links raw image data with their interpretation [Marr 1975]. Chapter 3 described how various operators applied to raw image data can yield primitive edge elements. However, an image of only disconnected edge elements is relatively featureless; additional processing must be done to group edge elements into structures better suited to the process of interpretation. The goal of the techniques in this chapter is to perform a level of segmentation, that is, to make a coherent one-dimensional (edge) feature from many individual local edge elements. The feature could correspond to an object boundary or to any meaningful boundary between scene entities. The problems that edge-based segmentation algorithms have to contend with are shown by Fig. 4.1, which is an image of the local edge elements yielded by one common edge operator applied to a chest radiograph. As can be seen, the edge elements often exist where no meaningful scene boundary does, and conversely often are absent where a boundary is. For example, consider the boundaries of ribs as revealed by the edge elements. Missing edge elements and extra edge elements both tend to frustrate the segmentation process.

The methods in this chapter are ordered according to the amount of knowledge incorporated into the grouping operation that maps edge elements into boundaries. "Knowledge" means implicit or explicit constraints on the likelihood of a given grouping. Such constraints may arise from general physical arguments or (more often) from stronger restrictions placed on the image arising from domain-dependent considerations. If there is much knowledge, this implies that the global form of the boundary and its relation to other image structures is very constrained. Little prior knowledge means that the segmentation must proceed more on the basis of local clues and evidence and general (domain-dependent) assumptions with fewer expectations and constraints on the final resulting boundary.
These constraints take many forms. Knowledge of where to expect a boundary allows very restricted searches to verify the edge. In many such cases, the domain knowledge determines the type of curve (its parameterization or functional form) as well as the relevant "noise processes." In images of polyhedra, only straight-edged boundaries are meaningful, and they will come together at various sorts of vertices arising from corners, shadows of corners, and occlusions. Human rib boundaries appear approximately like conic sections in chest radiographs, and radiographs have complex edge structures that can compete with rib edges. All this specific knowledge can and should guide our choice of grouping method.

If less is known about the specific image content, one may have to fall back on general world knowledge or heuristics that are true for most domains. For instance, in the absence of evidence to the contrary, the shorter line between two points might be selected over a longer line. This sort of general principle is easily built into evaluation functions for boundaries, and used in segmentation algorithms that proceed by methodically searching for such groupings. If there are no a priori restrictions on boundary shapes, a general contour-extraction method is called for, such as edge following or linking of edge elements.

The methods we shall examine are the following:

1. *Searching near an approximate location.* These are methods for refining a boundary given an initial estimate.

2. *The Hough transform.* This elegant and versatile technique appears in various guises throughout computer vision. In this chapter it is used to detect boundaries whose shape can be described in an analytical or tabular form.

3. *Graph searching.* This method represents the image of edge elements as a graph. Thus a boundary is a path through a graph. Like the Hough transform, these techniques are quite generally applicable.
4. *Dynamic programming.* This method is also very general. It uses a mathematical formulation of the globally best boundary and can find boundaries in noisy images.

5. *Contour following.* This hill-climbing technique works best with good image data.

### 4.2 Searching Near an Approximate Location

If the approximate or a priori likely location of a boundary has been determined somehow, it may be used to guide the effort to refine that boundary [Kelly 1971]. The approximate location may have been found by one of the techniques below applied to a lower resolution image, or it may have been determined using high-level knowledge.

#### 4.2.1 Adjusting A Priori Boundaries

This idea was described by [Bolles 1977] (see Fig. 4.2). Local searches are carried out at regular intervals along directions perpendicular to the approximate (a priori) boundary. An edge operator is applied to each of the discrete points along each of these perpendicular directions. For each such direction, the edge with the highest magnitude is selected from among those whose orientations are nearly parallel to the tangent at the point on the nearby a priori boundary. If sufficiently many elements are found, their locations are fit with an analytic curve such as a low-degree polynomial, and this curve becomes the representation of the boundary.

![Fig. 4.2 Search orientations from an approximate boundary location.](image)

#### 4.2.2 Non-linear Correlation in Edge Space

In this correlation-like technique, the a priori boundary is treated as a rigid template, or piece of rigid wire along which edge operators are attached like beads. The a priori representation thus also contains relative locations at which the existence of edges will be tested (Fig. 4.3). An edge element returned by the edge-operator application “matches” the a priori boundary if its contour is tangent to the template and its magnitude exceeds some threshold. The template is to be moved around the image, and for each location, the number of matches is computed. If the number of matches exceeds a threshold, the boundary location is declared to
be the current template location. If not, the template is moved to a different image point and the process is repeated. Either the boundary will be located or there will eventually be no more image points to try.

4.2.3 Divide-and-Conquer Boundary Detection

This is a technique that is useful in the case that a low-curvature boundary is known to exist between two edge elements and the noise levels in the image are low (Algorithm 8.1). In this case, to find a boundary point in between the two known points, search along the perpendiculargs of the line joining the two points. The point of maximum magnitude (if it is over some threshold) becomes a break point on the boundary and the technique is applied recursively to the two line segments formed between the three known boundary points. (Some fix must be applied if the maximum is not unique.) Figure 4.4 shows one step in this process. Divide-and-conquer boundary detection has been used to outline kidney boundaries on computed tomograms (these images were described in Section 2.3.4) [Selfridge et al. 1979].

Fig. 4.3 A template for edge-operator application.

Fig. 4.4 Divide and conquer technique.
4.3 THE HOUGH METHOD FOR CURVE DETECTION

The classical Hough technique for curve detection is applicable if little is known about the location of a boundary, but its shape can be described as a parametric curve (e.g., a straight line or conic). Its main advantages are that it is relatively unaffected by gaps in curves and by noise.

To introduce the method [Duda and Hart 1972], consider the problem of detecting straight lines in images. Assume that by some process image points have been selected that have a high likelihood of being on linear boundaries. The Hough technique organizes these points into straight lines, basically by considering all possible straight lines at once and rating each on how well it explains the data.

Consider the point \( x' \) in Fig. 4.5a, and the equation for a line \( y = mx + c \). What are the lines that could pass through \( x' \)? The answer is simply all the lines with \( m \) and \( c \) satisfying \( y' = mx' + c \). Regarding \( (x', y') \) as fixed, the last equation is that of a line in \( m - c \) space, or parameter space. Repeating this reasoning, a second point \( (x'', y'') \) will also have an associated line in parameter space and, furthermore, these lines will intersect at the point \( (m', c') \) which corresponds to the line \( AB \) connecting these points. In fact, all points on the line \( AB \) will yield lines in parameter space which intersect at the point \( (m', c') \), as shown in Fig. 4.5b.

This relation between image space \( x \) and parameter space suggests the following algorithm for detecting lines:

**Algorithm 4.1:** Line Detection with the Hough Algorithm

1. Quantize parameter space between appropriate maximum and minimum values for \( c \) and \( m \).
2. Form an accumulator array \( A(c, m) \) whose elements are initially zero.
3. For each point \( (x,y) \) in a gradient image such that the strength of the gradient