Walsh Function/O'Gorman Edge Detector

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1 Introduction

There are essentially two common types of operators to find the edges separating the regions in an image. [1] The first type is a gradient operator designed to identify places where there are large intensity changes. Most of the methods currently used correspond this. The second is a template-matching scheme, in which the template of an edge is compared with a subset of the image, and accepted as an edge if some pre-defined criterion is satisfied. The latter was first successfully used on real image by Hueckel [2] and was improved by O'Gorman using Walsh function [3].

2 Hueckel's Operator

One of the most ubiquitous problems of the traditional gradient operator is that the operator is designed on the tacit assumption, i.e. the gradient is constant within the window, which can be easily violated in the vicinity of edges and led to systematic error in the direction of the gradient. Hueckel's operator, which is not a gradient operator, was designed to avoid this problem. Hueckel approximated both the template and the picture function with truncated orthogonal expansions.

An orthogonal expansion of a function in a region R of the picture has the form

$$h(x,y) = \sum_{i=0}^{\infty} a_i f_i(x,y) \qquad (x,y) \in R \tag{1}$$

where $h(x,y)$ is the function to be expanded and f_i is the orthogonal sequence of functions.

In the case where h is the parameterized template, the coefficients a_i are also given by

$$a_i = \int\int_K h(x,y) f_i(x,y)\ dxdy \tag{2}$$

For a digitized picture function p, (2) takes the discrete form

$$A_i = \sum_{(x,y)\in R} p(x,y) f_i(x,y) \tag{3}$$

We can thus represent both the template and the picture function by vectors of coefficients, and now matching process for finding parameter values is identical to the minimizing Euclidean distance between two vectors

$$E_d = \big[\sum_i (A_i - a_i)^2\big]^{\frac{1}{2}} \tag{4}$$

Hueckel used the Fourier series as the set of orthogonal basis functions, resulted in too slow computation. O'Gorman proposed the use of a different set of orthogonal functions, i.e. the set of two dimensional Walsh functions [4], which can be calculated considerably faster than Fourier transforms.

[1] Note that there can be a different viewpoint in grouping the methods of edge detection, for example, the methods are grouped into two categories, search-based and zero-crossing based in Wikipedia [1]

3 O'Gorman Edge Detector

O'Gorman used the following two dimensional Walsh functions,

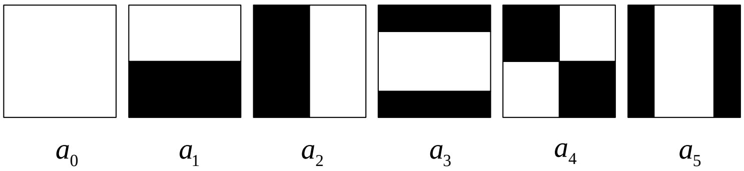


Figure 1: Walsh functions

where white represents the value +1, and black -1, and defined ideal edge function

$$h(x,y) = \begin{cases} b+d & \text{if} \quad x\,cos\theta \; + \; y\,sin\theta > 0 \\ b-d & \text{otherwise} \end{cases} \tag{5}$$

Applying (2) to (5), we obtain the corresponding six coefficients for the template

$$\begin{aligned} a_0 &= b \\ a_1 &= \frac{d}{2sc}\Big[q(s+c) - q(s-c) - 2q(c)\Big] \\ a_2 &= \frac{d}{2sc}\Big[q(s+c) + q(s-c) - 2q(s)\Big] \\ a_3 &= a_4 \quad = \quad a_5 \quad = \quad 0 \end{aligned} \tag{6}$$

where $s = sin\,\theta$, $c = cos\,\theta$ and $q(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

By substituting (6) into the distance formula (4) and taking derivatives, the analytic solution for the minimum distance can be obtained as follows

$$\begin{aligned} b &= A_0 \\ d &= \mid A_1 \mid + \mid A_2 \mid \\ tan\,\theta &= \begin{cases} d\,\mathrm{sign}(A_1)/2A_2 & \text{if } \mid A_1 \mid \geq \mid A_2 \mid \\ 2A_1\mathrm{sign}(A_1)/d & \text{if } \mid A_1 \mid < \mid A_2 \mid \end{cases} \end{aligned} \tag{7}$$

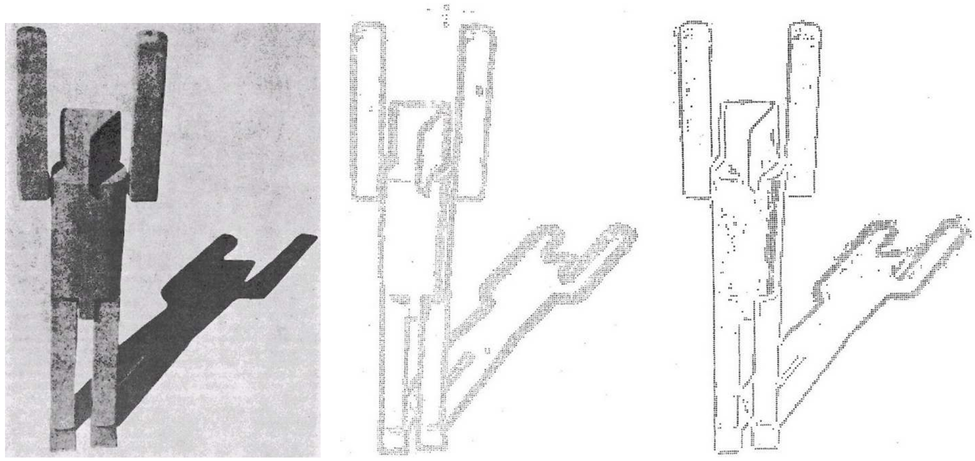
Finally, as a measure of the goodness of match, O'Gorman used the following cosine k of the angle between the vectors a_i and A_i

$$k = \left[\frac{A_1^2 + A_2^2}{A_1^2 + A_2^2 + A_3^2 + A_4^2}\right]^{\frac{1}{2}} \tag{8}$$

Figure 2 (c) shows the feature points detected using two rejection thresholds, $k < 0.9$ and $d < 1.0$. In comparison with Figure 2 (b), which was detected by 3×3 gradient operator proposed in [5], we can check significant improvement of the resolution.

4 Improvement and Discussion

O'Gorman or Walsh function edge detector is quite unique, and can be improved by substituting $f_i(x,y)$ in (1) with other orthogonal expansion algorithms. For example, Christiansen *et al* improved the performance both in accuracy and computational cost using polar Walsh



(a) Test picture (b) Gradient operator (c) Walsh function detector

Figure 2: Test Picture and the results of edge detector

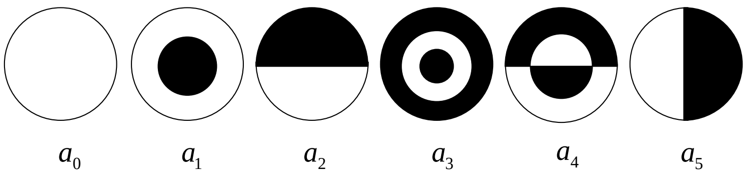


Figure 3: Polar Walsh functions

function shown in Figure 3 [6]. However, this method has hardly been adopted in computer vision for a long time, partly because of its inherent limitation, e.g. power-of-two requirement of Walsh function, and partly because of the advent of other superior detectors such as the Canny edge detector.

References

[1] http://en.wikipedia.org/wiki/Edge_detection

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