Constrained Based Matching

Kryoneriti Evangelia

The Constrained-based matching method is broadly used in Computer Vision systems. This method, like any other matching algorithm, has two main stages. In stage 1, the input images are processed in order to extract features from them. These features describe objects' attributes and relationships amongst them. Therefore, they are being used in order to create a set of constraints. Afterwards, the images are segmented into regions/parts based on their features and then the matching process takes place in stage 2.

The matching between a pair of segments/parts from a pair of images is done according to whether or not the set of constraints is satisfied and if so, matching is allowed. Among those correspondences that satisfy them, those that “minimize” the constraints are chosen as the best match.

**Constraints**

The constraints represent the similarity among the regions in comparison based on prior knowledge of the attributes (primitive or first order constraints) and the relationships (second or higher order constraints) between the objects. These constraints can take the form of a “distance” between the chosen features of the segments, e.g. if there may be a constraint that concerns the color similarity among the compared regions, that constraint calculates the difference in color between these regions and if the result is bigger than a well-chosen threshold, then the match is not allowed. Among those matches that satisfy that constraint, the ones that have a smaller value estimate are those that are more probable to match better. For instance, in the previous example, those that have almost or exactly the same color are more probable to match. But matches depend on a set of constraints, thus it is necessary to satisfy all of them and then, by calculating the overall similarity, we get the pair of segments that have the best similarity among two images. In order to define that, let's represent a (finite) set of constraints as $C$ and $c_1, c_2, ..., c_N \in C$, where $c_i, i = 1, 2, ..., N$, is a constraint. In order to get a similarity estimate, $S$, from them, we calculate a weighted sum of these constraints, $S = \sum_{i=1}^{n}w_i\cdot c_i$, where $w_i$ is a coefficient that represents the importance of the corresponding constraint.

The constraints may be formulated using global features as well as local features. In the case
of global features, such as global histogram, shape and texture, the method is easy to implement. But these features fail to describe the relationships between objects, thus the results are quite poor. This is the advantage of the local features, like fixed layout-based and salient layout-based [4] features, that this method uses mostly in order to derive very strong correlations between objects, but these constraints demand more complicated formulation.

**Weight Estimation**

Another issue of this method is the weights choice. If we pick or estimate bad weight values, the results are quite inaccurate because we get matches based on wrong importance estimation of constraints. For instance, if we have two constraints, one that concerns the color and other the shape of the segment, if we opt for weight coefficients $w_{\text{color}} >> w_{\text{shape}}$, then we will probably get incorrect matches from the method. Weight estimation is a very important aspect in this method and a good estimation is quite a difficult process. There are various methods that use the constrained-based approach for image matching, such as game theory algorithms (e.g. duality theorem[2]) and distance methods (e.g. Newton-Raphson method[6]).

**Advantages and Disadvantages**

The advantage this approach is that it is very easy to implement, especially if it based on global features. On the other hand, as mentioned before, there may be a semantic gap as a result of limited descriptions that we get from these features and the method delivers very poor results. We can overcome this problem with the use of local features but with the price of more processing and computational cost.

**Constrained Based Region Matching**

In the this section, an implementation of this approach is presented, the Constrained Based Region (CBR) [1] matching for image retrieval. All the mathematical formulas and their descriptions are derived from the paper of Tao Wang, Yong Rui, Jia-Guang Sun 2002[1]:

The CBR method is using two types of order constraints:

1. first order constraints – constraints that describe the attributes of individual objects
2. second order constraints – constraints that describe the relationships between 2 objects

The goal is to find the best possible correspondences according to a number of constraints which need to be satisfied and minimized in order to get the best possible correspondences. The CBR is using local-feature based approach and region-based in particular. Also, it uses a probabilistic approach for weight estimation.

**Constraints formation and similarity estimation:**

First the images are segmented into regions, $r_i, i = 1,..., N$, that have strong correlations with real world objects. Let's represent images 1 and 2 by region sets $R_1 = \{r_{1}, r_{2}, \ldots, r_{M}\}$ and $R_2 = \{r_{1}', r_{2}', \ldots, r_{N}'\}$ respectively. Let the similarity between the two regions $r_i, r_j'$ be $S(r_i, r_j')$ where $r_i$ is the i-th region in $R_1$ and $r_j'$ is the j-th region in $R_2$.

The similarity is calculated by using these constraints:

1. **Region property constraint**
   - Color: $S_c(r_i, r_j') = \exp(-|c_i - c_{j'}|^2 / 2\sigma_i^2)$
   - Shape: $S_s(r_i, r_j') = \exp(-|e_i - e_{j'}|^2 / 2\sigma_s^2)$
2. **Region position constraint**
   - Position: $S_p(r_i, r_j') = \exp(-|O_i - O_{j'}|^2 / 2\sigma_p^2)$
3. **Spatial relationship constraint**
   - Orientation: $S_o(r_i, r_o, r_j', r_{j'}) = \{(O_i - O_{o}) * (O_{j'} - O_{j''})/(|O_i - O_{o}| * |O_{j'} - O_{j''}| + 1)\}/2$
   - Inside/outside: $S_i(r_i, r_o, r_j', r_{j'}) = -[r_i \text{ in/out } r_o \text{ XOR } r_j' \text{ in/out } r_{j'}]$
The CRM approach is summarized as follows:

$$P(\text{estimated using the similarity})$$

The first probability is estimated using the first order similarity constraints respectively. We use the above notation to get these probabilities indicates the importance of region pair of different variation respectively.

$$\text{The total similarity}$$

$$P_r(x, y) = w_c S_c(r, r') + w_s S_s(r, r') + w_p S_p(r, r')$$ \hspace{1cm} (1)

subject to $$w_c + w_s + w_p = 1$$

The second order similarity is computed:

$$P_r(x, y) = w_o S_o(r, r')$$ \hspace{1cm} (2)

subject to $$w_o + w_l + w_r = 1$$

Proabilistic weight estimation:

The total similarity $$S(r, r')$$ between these two region sets is:

$$S(R_r, R_{r'}) = \sum_{i=1}^{l} \sum_{j=1}^{M} w_{ij} S(r, r')$$ \hspace{1cm} (3)

where $$w_c, w_s, w_p, w_o, w_l, w_r$$ and $$w_p$$ are proper weights for corresponding constraints. Also, $$w_{ij}$$ indicates the importance of region pair $$r$$ and $$r'$$ with respect to the overall similarity.

Let $$x \sim y$$ and $$x \approx y$$ denote that $$x$$ matches $$y$$ and $$x$$ based on first order and second order constraints respectively. We use the above notation to get these probabilities

$$P(r \sim r') = S_1(r, r')$$

$$P(r \sim r', r \approx r') = S_2(r, r', r', r')$$

The first probability is estimated using the first order similarity $$S_1$$ and the second is estimated using the similarity $$S_2$$. Considering all possible related regions:

$$P(r \sim r', r \approx r') = \sum_{i=1}^{l} \sum_{j=1}^{M} P(r \sim r', r \approx r')$$

$$P(r \sim r', r \approx r')$$ is the probability that region $$r$$ matches $$r'$$ based on first and second order constraints and that probability will serve as the weight $$W_{ij}$$. Thus, using Bayes rule we get

$$W_{ij} = P(r \sim r', r \approx r')$$

$$= \sum_{i=1}^{l} \sum_{j=1}^{M} \sum_{k=1}^{N} P(r \sim r', r \approx r')$$

We normalize $$W_{ij}$$ to 1 and set $$q_i$$ to region $$r_i$$, we further normalize the weight $$w_{ij}$$ as:

$$w_{ij} = q_i \cdot W_{ij} / \sum_{l=1}^{N} W_{ij}$$ \hspace{1cm} (5)

subject to $$\sum_{l=1}^{N} q_i = 1$$

The CRM approach is summarized as follows:

Input: image 1 and image 2

Output: similarity between image 1 and image 2

1. Obtain region sets $$R_1 \{r_1, r_2, ..., r_m\}$$ and $$R_2 \{r'_1, r'_2, ..., r'_n\}$$ for image 1 and image 2 respectively from the image segmentation module.
2. Extract region features of each region $$r, r'$$.
3. Compute the first order constraint $$S_1(r, r')$$ and the second order constraint $$S_2(r, r, r', r')$$ using equations (1) and (2).
4. Calculate the probability weights $$w_{ij}$$ based on both first order and second order constraints, using equations (1) and (2) respectively.
5. Calculate the total similarity $$S(R_1, R_2)$$ between region sets $$R_1, R_2$$, using equation (3).
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