Camera Models

Projection & Lens'

CS510

Lecture #2

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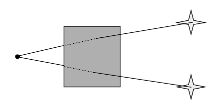
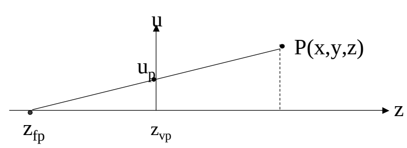
Pin-hole Camera Model

- Camera model has a *focal point* and an *image plane*
- All (and only) light rays that pass through the focal point alter the image plane.



- This models *perspective projection*

Perspective Projection



$x' = x - xt$
$y' = y - yt$
$z' = z - (z - z_{fp})t$

These equations describe all points (x', y', z') along the ray from fp to P(x,y,z)

Perspective (cont.)

To calculate the position of a point on the viewplane, set $z' = z_{vp}$

$$t = \frac{z_{vp} - z}{z_{fp} - z}$$

$$x' = u = x\frac{z_{fp} - z_{vp}}{z_{fp} - z} = x\frac{-f}{z_{fp} - z} = f\frac{x}{z - z_{fp}}$$

$$y' = v = y\frac{z_{fp} - z_{vp}}{z_{fp} - z} = y\frac{-f}{z_{fp} - z} = f\frac{y}{z - z_{fp}}$$

Perspective (III)

Setting $Z_{fp} = 0$ yields the perspective projection matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix}$$

(From H&B)

Perspective (IV)

- Not used to this the perspective matrix? There are many variations depending on your choice of coordinate system.
- All perspective projection is based on the following relations:

$$u = f\frac{X}{Z}$$

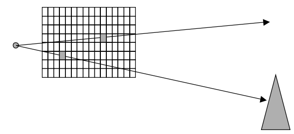
$$v = f\frac{Y}{Z}$$

Ray Casting

- Why review the pinhole camera model?
  - As a motivation for *Ray Casting*
- Ray Casting is a *rendering* algorithm
  - It produces an image from world, lighting and camera descriptions

Ray Casting (II)

- Ray casting works like H&B's perspective diagram
  - The value of any pixel is determined by casting a ray from the focal point through the pixel and out into the world

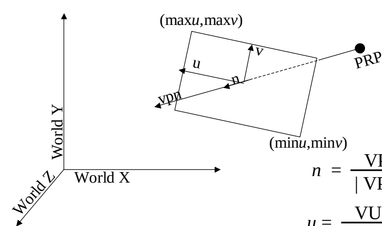


Camera Models

We had better review camera models, too…

ITEM	DESCRIPTION	Ref. Frame.
VRP	View Reference Point	World
VPN	View Plane Normal	World
VUP	Up Vector - Define V	World
PRP (a.k.a. FP)	Projection Reference Point	View Coords
window	Viewport Window (pixels)	View Plane
projection type	perspective / orthographic	N/A

Cameras in 3D



$$n = \frac{VPN}{|VPN|}$$

$$u = \frac{VUP \times VPN}{|VUP \times VPN|}$$

$$v = n \times u$$

Camera Hints:

- Remember, VRP & PRP are points in the world coordinate system
- VPN, n, u, and v are vectors in the world coordinate system
- minu, minv, maxu, maxv are 2D points in the u×v (image) plane
- All units are pixels, unless otherwise stated

Ray Casting (III)

- So the basic algorithm for ray casting is:

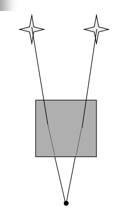
```
// udir is direction of u axis, etc.
For v from minv to maxv {
  For u from minu to maxu {
    Ray R from PRP toward (VRP+u*udir+v*vdir);
    Point P = Intersect(R, World);
    Image[u,v] = Color(P);
  }
}  // or you could play with coordinate systems…
```

Make sure this makes sense to you!

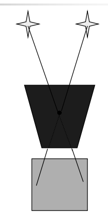
Questions

- Is it possible to build a pin-hole camera?
- In what way is a pin-hole camera not a good model of modern cameras?

True Pin-hole Cameras

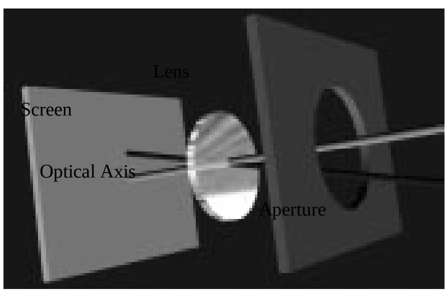


Mathematical Model
(light rays intersect plane
on way to meet at focal point)



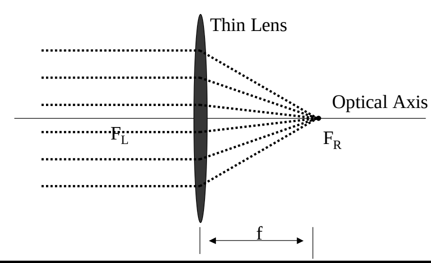
Physical Pin-hole Camera:
(light rays pass through
pin-hole, then intersect plane;
image is mirrored)

Intensity Image Formation

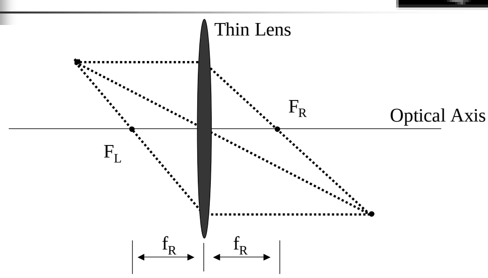


Thin Lens Model

- Rays entering parallel on one side converge at focal point.
- Rays diverging from the focal point become parallel.



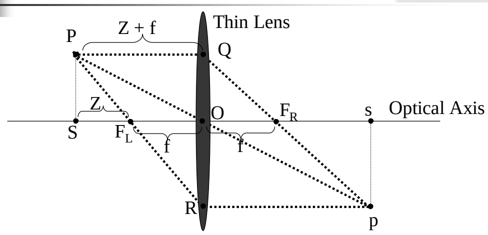
Thin Lens Model



Thin Lens Constraints

1) All rays eminating from a single point in space must converge on a single point in the image plane (*definition of focus)*
2) Any ray entering the lens parallel to the axis on one side goes through the focus point on the other side
3) Any ray entering the lens from the focus point on one side emerges parallel to the axis on the other side

Fundamental Equation of Thin Lenses

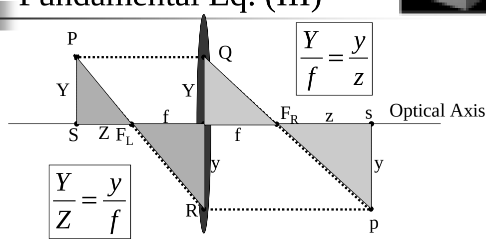


Note: P is "not too far" from optical axis

Fundamental Equation (II)

- The ray PQ (parallel to the optical axis) must be deflected to pass through F_R by property #2
- The ray PR must be deflected so that it becomes parallel to the optical axis by property #3
- After deflection, PQ & PR must intersect at p, by property #1.
- Now, use similar triangles….

Fundamental Eq. (III)

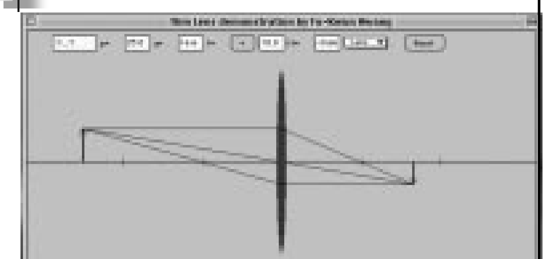


$$\frac{Y}{f} = \frac{y}{z}$$

$$\frac{Y}{Z} = \frac{y}{f}$$

Substitute for y and solve: $f^2 = zZ$, *or* $\frac{1}{Z+f} + \frac{1}{z+f} = \frac{1}{f}$

Thin Lens Model Applet



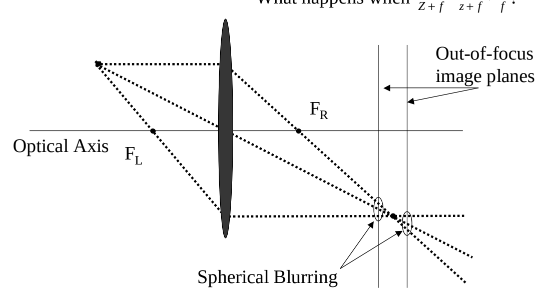
http://www.physics.gatech.edu/academics/tutorial/phys2121/Java%20Applets/ntnujava/Lens/lens_e.html

Question

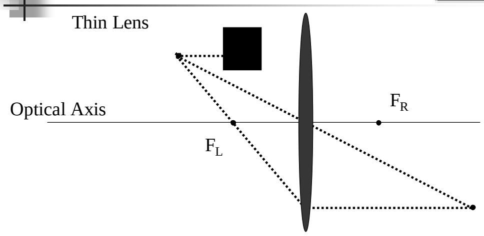
- Why (and how) does a thin lens implement perspective projection?
- How do you focus on objects that are very far away?
- You can recover the depth of a static scene with a single camera – how?

Out of Focus Images

What happens when $\frac{1}{Z+f} + \frac{1}{z+f} \neq \frac{1}{f}$?



Partial Obscuration



Partially obscured objects may be dimmer than other objects;
If out-of-focus, blurring may be non-symmetric

Ray Casting + Thin Lens

How do you do ray casting with a thin lens?

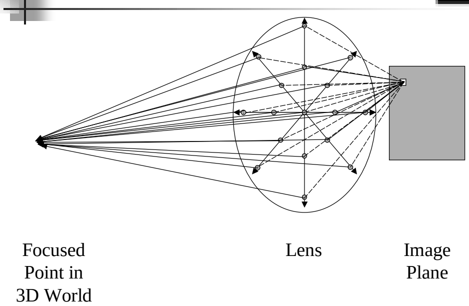
- Every point on the image plane (i.e. every pixel) generates many rays toward different points on the lens
  - How many? How much time do you have….
  - Arrange points in spherical pattern
    - Two radius' every 45 degrees + center = 17 points
- Every ray is deflected by the lens
- Compute color along every deflected ray, average the results

RC + TL (II)

How do you compute lens deflections?

- Every ray (from a single image point) deflects toward a single point in the world.
- Shoot two rays from the image point toward the lens:
  - One parallel to the optic axis
  - One through the back focal point
- These are deflected according to the fundamental equations shown earlier:
  - One through the front focal point
  - One parallel to the optic axis.

Previous Slide, Illustrated

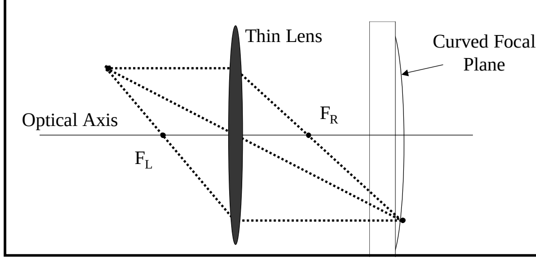


RC + TL (III)

- These two deflected rays intersect at some point in the world.
- All other rays (associated with this pixel) go from the surface of the lens toward this point.
- So fire rays from every lens surface point toward this point and average the results.
  - The two initial rays are used only for finding the point that is in focus.
  - Do not expect all rays to intersect the same surface at the same point, unless the image is in focus

Spherical Distortion

Most lenses exhibit spherical (a.k.a. radial) distortion which creates a slightly curved focal plane

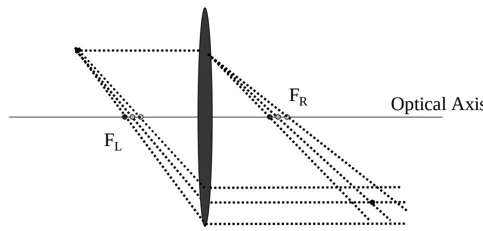


Spherical Distortion (II)

- Spherical distortion is both geometric & focal
  - Points are distorted geometrically from the true perspective position as a result of "flattening out" the plane.
    1) convert points to 2D spherical coordinates, based in the middle of the image
    2) $\rho = \rho' \cdot (\alpha\rho' + \beta\rho'^2 + \chi\rho'^3 + \ldots)$
  - Points also become more blurred as they approach the edge of the image

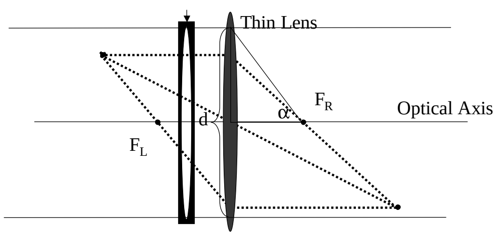
Chromatic Aberration

The focal length of a lens is a function of the wavelength of the light. Therefore, different colors of light from a single point will be out-of-focus



Field of View

Aperture: limits the effective diameter of the lens



$$FOV = \tan^{-1}\left(\frac{d}{2f}\right)$$