Fourier Matching

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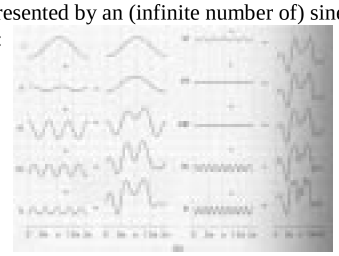
Image Matching

- To match registered images
  - one-to-one: correlation
  - one-to-many: eigenspaces
- To match unregistered images:
  - cross-correlation
    - including rotation-free cross-correlation
- *fourier methods -- how, why and when?*

Review (Lecture #8):

2D Fourier Spectrum

- Any signal that is non-zero over a finite range can be represented by an (infinite number of) sine waves:



Review:

2D Fourier (cont.)

- Mathematically, this is expressed as:

$$F(u) = \int_{-\infty}^{+\infty} f(x)\left[\cos 2\pi ux - i \sin 2\pi ux\right]dx$$

where i is the square root of -1

- Sometimes, this is written as magnitude & phase shift:

$$|F(u)| = \sqrt{R^2(u) + I^2(u)}$$

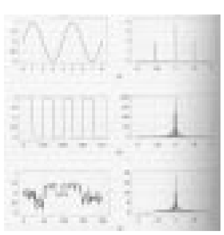
$$\phi(u) = \tan^{-1}\left(\frac{I(u)}{R(u)}\right)$$

Review:

2D Inverse Fourier

- The inverse mapping from frequencies (sines) to the spatial domain is:

$$f(x) = \int_{-\infty}^{+\infty} F(u)\left[\cos 2\pi ux + i \sin 2\pi ux\right]du$$



Matching

- The Fourier Transform converts spatial image into *frequencies* and *phases*. Can we use these to match images?
- The Fourier transform is deterministic, so if two images are identical, so are their frequencies and phases.
- Can we tell if two images are similar?

Scaling Functions

- For simplicity, let us focus on 1D functions (the same principles will apply in 2D)
- If we scale a function f(x), we get f(ax), where a is a constant

$$Fourier(f(ax)) = \int_{-\infty}^{\infty} f(ax)e^{i2\pi ux}dx = \frac{1}{|a|}F\left(\frac{u}{a}\right)$$

- What does this say?

Shifting functions

- Similarly, if we shift a function:

$$Fourier(f(x-x_0)) = \int_{-\infty}^{\infty} f(x-x_0)e^{i2\pi ux}dx = F(u)e^{i2\pi x_0 u}$$

- The last term ($e^{i2\pi xu}$) is imaginary, so:
  - it represents a phase shift (no frequency shift)
  - the amount of the phase shift is $2\pi x_0$
- The inverse is also true

$$F(s-s_0) = Fourier\left(f(x)e^{i2\pi x s_0}\right)$$

Correlation in Frequency Space

- The frequency space version of correlation can be written as:

$$h(x) = f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(u)g(x+u)du$$

  - The mask g() is assumed infinite but zero outside of a finite range.

Correlation Theorem

- The Fourier transform of the cross-correlation is:

$$Fourier(h(x)) = F(s)G^*(s)$$

  - F(s) is the fourier transform of f(x)
  - G(s) is the fourier transform of g(x)
    - G*(s) is the complex conjugate of G(s), defined as the real part of G(s) *minus* the imaginary part

So?

- Fascinating, but how do we use it?
- Go back to the shifting relation. The phase difference between two identical images shifted by (x_0,y_0) is:

$$e^{i2\pi(ux_0, vy_0)}$$

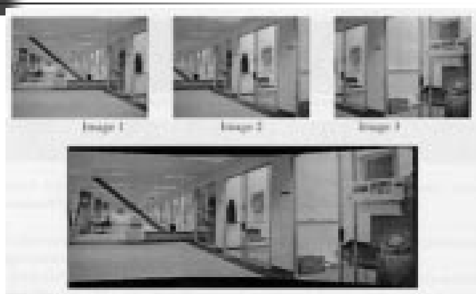
- Why is this interesting? Because:

$$\frac{F_1(u,v)F_2^*(u,v)}{\left|F_1(u,v)F_2^*(u,v)\right|} = e^{i2\pi(ux_0, vy_0)}$$

Mosaicing

- So how can we use this?
  - Image two pictures, taken by shifting the camera perpendicular to the optical axis
  - Each picture contains a piece of the scene not included in the other
  - “Mosaicing” is the process of stitching these together into one larger image
  - It requires finding the shift factor between the two images.

Example



Mosaicing (II)

- How do we Mosaic images?
  - We could do cross-correlation, but both images are large, so this is expensive.
  - Alternatively, we could take the Fourier transform of both (and the complex conjugate of one) and compute the phase shift using:

$$\frac{F_1(u,v)F_2^*(u,v)}{\left|F_1(u,v)F_2^*(u,v)\right|} = e^{i2\pi(ux_0,vy_0)}$$

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- What will be the result? If the images are identical (other than the shift) and you sum the power at each phase across frequencies u,v, you will find that all the energy lies at single phase corresponding to the shift x_0,y_0
- What if they are similar but not identical? Then most of the power lies at a single phase, so perform the same calculation and then do peak detection.
- Is this cheaper than cross-correlation?

Fourier Methods

- Mosaicing involves matching a whole to a shifted whole
  - What if we want to find a small template in a larger image?
    - Will Fourier matching work?
    - Why or why not?
  - For what other types of problems does Fourier matching work?

Back to Correlation

- I said that if two images are similar, then phase correlation says that the maximum phase value corresponds to the best shift x_0, y_0.
- What is the relationship between the height of the peak and the correlation score of the images under that shift?
- Parseval's theorem says that: $\int_{-\infty}^{\infty} h^2(x)dx = \int_{-\infty}^{\infty} |H(f)|^2 df$
- So normalizing source image normalizes frequency space, and the height of the peak is the correlation score