

# Lecture 6: Morphology: thickening, thinning, hulls, skeletons, connectivity, distance maps conditional dilation

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## Contents

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<b>6.1 Thickening and Thinning</b>	<b>1</b>
<b>6.2 Recursive Morphology</b>	<b>1</b>
6.2.1 Distance Maps	2
6.2.2 Convex Hull	2
6.2.3 Skeletons	2
<b>6.3 Conditional Dilation</b>	<b>2</b>
<b>6.4 Greyscale Morphology</b>	<b>2</b>
<b>6.5 Images, Functions, and Umbras</b>	<b>3</b>
<b>6.6 Greyscale Dilation and Erosion</b>	<b>3</b>
6.6.1 Definition	3
6.6.2 Example	3
6.6.3 Another Example	3
6.6.4 Generalizing the Examples	3
6.6.5 Properties	4
<b>6.7 Greyscale Opening and Closing</b>	<b>4</b>
6.7.1 Greyscale Opening	4
6.7.2 Greyscale Closing	5
6.7.3 Duality of Greyscale Opening and Closing	5
6.7.4 Using Greyscale Openings and Closings	5

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## Reading

SH&B, 11.5.1–11.5.3

We'll cover the remainder of Chapter 11 (from 11.5.3 on) later in the course.

### 6.1 Thickening and Thinning

We can use hit-and-miss operators to find specific points and either add them or remove them from the object. If we find such pixels and add them to the object, that constitutes a *thickening*:

$$A \odot (J, K) = A \cup (A \otimes (J, K)) \quad (6.1)$$

Similarly, if we find such pixels and remove them from the object, that constitutes a *thinning*:

$$A \ominus (J, K) = A - (A \otimes (J, K)) \quad (6.2)$$

## 6.2 Recursive Morphology

Recursive morphological operators are those that are defined in terms of themselves and are applied recursively or iteratively to the image. Examples of recursive morphological processes are distance maps, convex hull, and thinned skeletons.

### 6.2.1 Distance Maps

We discussed distance maps earlier in Lecture 2. Distance maps can be calculated using morphological operations. The set of all pixels with distance  $i$  from the object can be calculated as follows:

$$\text{Dist}_i = (A \oplus_i B) - (A \oplus_{i-1} B)$$

where  $\oplus_i$  denotes  $i$  applications of the dilation operator.

### 6.2.2 Convex Hull

The convex hull of an object is the smallest convex shape that entirely contains that object. We can calculate the convex hull of an image object by repeatedly finding and thickening convexities.

### 6.2.3 Skeletons

A skeleton of an object is a single-pixel thin set of pixels produced by repeatedly thinning the object. We can't do this using erosion alone, because we have to be able to detect when deletion of a pixel would make the object disconnected. So, we have to use hit-and-miss operators (thinning).

## 6.3 Conditional Dilation

Conditional dilation essentially involves dilation of an image followed by intersection with some other “condition” image. In other words, new pixels are added to the original

1. the pixel is in the dilation of the original, and
2. the pixel is in the condition image.

In this way, the condition image acts like a mask on the dilation. This is written as

$$A \oplus |_I B = (A \oplus B) \cap I \tag{6.3}$$

Here is an example of how we might use conditional dilation. Suppose that after applying certain morphological operations to find specific points in an image we wanted to find the original connected components within which those points were found. In essence, we want to “grow” those points out to fill the original connected component. One obvious way would be to combine connected-component labeling with morphological operations. However, you can do it with *conditional dilation*.

Let  $I$  be the original image and  $A$  the morphologically-reduced image that you want to “grow back”. Let  $B$  be a  $3 \times 3$  structuring element containing all 1s. Let  $J_0 = A$  and  $J_i = (J_{i-1} \oplus B) \cap I$ . So, at each iteration  $I$ ,  $J_i$  “grows” out one pixel *but only along the original image points*. Eventually, this will converge to  $J_i = J_{i-1}$ , and the algorithm stops. This is illustrated in Haralick, Figure 5.26.

## Vocabulary

- Thickening
- Thinning

- Convex Hull
- Skeleton
- Conditional Dilation