

Lecture 22: Texture

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Reading

SH&B, Chapter 14
Castleman, 19.4

22.1 Introduction

In previous lectures we've talked about how to describe the shape of a region. In this lecture, we'll be talking about how to describe the content of the region itself.

22.2 Intensity Descriptors

The easiest descriptors for the contents of a region describe the general intensity properties: average grey-level, medial grey-level, minimum and maximum grey-level, etc.

These descriptors are, however, subject to intensity gain or other parameters of the imaging system.

22.3 Overview of Texture

Texture is one of those words that we all know but have a hard time defining. When we see two different textures, we can clearly recognize their similarities or differences, but we may have a hard time verbalizing them.

There are three main ways textures are used:

1. To discriminate between different (already segmented) regions or to classify them,
2. To produce descriptions so that we can reproduce textures, and
3. To segment an image based on textures.

In this lecture we'll mainly discuss ways of describing textures, each of which can be applied to all three of these tasks.

There are three common ways of analyzing texture:

1. Statistical Approaches
2. Structural Approaches
3. Spectral Approaches

22.4 Statistical Approaches

Since textures may be random, but with certain consistent properties, one obvious way to describe such textures is through their statistical properties.

22.4.1 Moments of Intensity

We discussed earlier the concept of *statistical moments* and used them to describe shape.

These can also be used to describe the texture in a region. Suppose that we construct the histogram of the intensities in a region. We can then compute moments of the 1-D histogram:

- The first moment is the mean intensity, which we just discussed.
- The second central moment is the variance, which describes how similar the intensities are within the region.
- The third central moment, skew, describes how symmetric the intensity distribution is about the mean.
- The fourth central moment, kurtosis, describes how flat the distribution is.

The moments beyond this are harder to describe intuitively, but they can also describe the texture.

22.4.2 Grey-level Co-occurrence

Another statistical way to describe shape is by statistically sampling the way certain grey-levels occur in relation to other grey-levels.

For a position operator p , we can define a matrix P_{ij} that counts the number of times a pixel with grey-level i occurs at position p from a pixel with grey-level j .

For example, if we have three distinct grey-levels 0, 1, and 2, and the position operator p is “lower right”, the counts matrix P of the image

$$\begin{array}{ccccc}
 0 & 0 & 0 & 1 & 2 \\
 1 & 1 & 0 & 1 & 1 \\
 2 & 2 & 1 & 0 & 0 \\
 1 & 1 & 0 & 2 & 0 \\
 0 & 0 & 1 & 0 & 1
 \end{array} \tag{22.1}$$

is

$$P = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix} \tag{22.2}$$

If we normalize the matrix P by the total number of pixels so that each element is between 0 and 1, we get a *grey-level co-occurrence matrix* C .

Note: Different authors define the co-occurrence matrix a little differently in two ways:

- by defining the relationship operator p by an angle θ and distance d , and
- by ignoring the direction of the position operator and considering only the (bidirectional) relative relationship.

This second way of defining the co-occurrence matrix makes all such matrices symmetric. So, if $P_{\text{left}} = P_{\text{right}}^T$,

$$P_{\text{horizontal}} = P_{\text{left}} + P_{\text{right}}$$

We can get various descriptors from C by measuring various properties, including the following:

1. the maximum element of C

$$\max_{ij} (c_{ij}) \quad (22.3)$$

2. the element difference moment of order k

$$\sum_i \sum_j c_{ij} (i - j)^k \quad (22.4)$$

3. the inverse element difference moment of order k

$$\sum_i \sum_j c_{ij} / (i - j)^k \quad (22.5)$$

4. entropy

$$- \sum_i \sum_j c_{ij} \log c_{ij} \quad (22.6)$$

5. uniformity

$$\sum_i \sum_j c_{ij}^2 \quad (22.7)$$

Other similar measures are described in your text.

22.5 Structural Approaches

A second way of defining the texture in a region is to define a grammar for the way that the pattern of the texture produces structure.

Because as CS students you should be familiar with grammars by now, we won't go into great detail here. The basic scheme is to build a grammar for the texture and then parse the texture to see if it matches the grammar. The idea can be extended by defining *texture primitives*, simple patterns from which more complicated ones can be built. The parse tree for a the pattern in a particular region can be used as a descriptor.

22.6 Spectral Approaches

22.6.1 Collapsed Frequency Domains

A third way to analyze texture is in the frequency domain. If textures are periodic patterns, doesn't it make sense to analyze them with periodic functions?

The entire frequency domain is, however, as much information as the image itself. We can condense the information by collapsing a particular frequency across all orientations (by integrating around circles of fixed distance from the frequency origin) or by collapsing all frequencies in a particular orientation (by integrating along each of a unique orientation through the origin). If we express the frequency-domain coordinates in polar coordinates, these are

$$S(r) = \sum_{\theta=0}^{\pi} S(r, \theta) \quad (22.8)$$

and

$$S(\theta) = \sum_{r=0}^{N/2} S(r, \theta) \quad (22.9)$$

$S(r)$ tells us the distribution of high and low frequencies across all angles. $S(\theta)$ tells us the distribution of frequency content in specific directions. These two one-dimensional descriptors can be useful for discriminating textures.

22.6.2 Local Frequency Content

As we discussed earlier, the frequency domain contains information from all parts of the image. This makes the previous method useful for global texture analysis, but not local. If a region has already been segmented, you could pad the region with its average intensity to create a rectangular image. This doesn't, however, provide a useful way of using texture to do the segmentation.

We can define local frequency content by using some form of co-joint spatial-frequency representation. As we discussed, though, this only partially localizes the position or frequency of the information—you can't do both perfectly.

A simple way to do this would be to examine the $N \times N$ neighborhood around a point and to compute the Fourier Transform of that $N \times N$ subimage. As you move from one textured region to another, the frequency content of the window changes. Differences in the frequency content of each window could then be used as a means of segmentation.

Of course, one still needs to distill descriptors from the frequency content of each window. One such descriptor (Coggins, 1985) is to compute the total energy (squared frequency content) of the window. If you exclude the zero-frequency term, this is invariant to the average intensity. If you normalize by the zero-frequency term, it is invariant to intensity gain as well. There are, of course, other descriptors you could use as well.

22.7 Moments

We talked earlier about how moments of an intensity histogram can be used to describe a region.

We can go one step further by describing the combination of both intensity and pattern of intensity by computing moments of the two-dimensional grey-level function itself. Remember that we said that if you had enough moments, you could reconstruct the function itself? Well, this means that we can reconstruct an entire image from its moments, much like we could reconstruct the image from the transforms we've discussed. As we've done before, though, we don't really need all of the moments to be able to get good matching criteria.

However, moments themselves aren't invariant to all of the transformations we've discussed. Certain combinations of the moments can be constructed so as to be invariant to rotation, translation, scaling, and mirroring. These combinations can be used as descriptors for matching images.

Vocabulary

- Texture
- Grey-level Co-occurrence Matrix
- Spectral Energy