Generalised Fuzzy Aggregation in Estimating the Risk of Desertification of a burned forest

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Abstract

This paper investigates two aspects of using Fuzzy Logic with a GIS: The use of non-conventional aggregation operators for conjunctive and disjunctive reasoning and the use of weights when combining aggregates of different significance to the classification. Two methods of weighting are used, weights as powers of the membership functions, and the novel concept of membership functions that take values greater than 1. The two approaches are investigated in conjunction with 121 possible combinations of aggregation operators that are used to reason in the disjunctive and conjunctive level of the problem of assessing the risk of desertification after a forest fire.

Keywords: Fuzzy Sets, GIS, Aggregation, Relative importance of aggregates.

1 Introduction

In Geography and related sciences, a GIS package is often used for combining information from various sources for the ultimate purpose of region classification. Most of the commercially available GIS packages tend to simply superimpose the various information layers in a heuristic way. Apart from the obvious problem that different factors may combine in a non-linear way to influence the output classification, there is another major drawback of this approach: the information obtained from each layer may be inaccurate or uncertain to various degrees and this uncertainty should be taken into consideration when the various items of information are combined. In order to preserve the flexibility offered by a GIS in storing and retrieving geographical information, while taking into consideration both the way the different factors influence the final classification and the uncertainty in the value or class of each of them, we propose the use of a GIS system with reasoning mechanisms that can cope with the above requirements.

We shall present our ideas in the context of the problem of burned forest management. In particular, we are interested in ranking burned forests in order of risk of desertification. There is a lot of work done on mapping and monitoring forest areas [1] [24] [8] [7]

[17], assessment of vegetation change [18], fire risk assessment [22] [9] [15] and restoration of burned areas and vegetation recovery [3] [33] [25] [16]. Although the above approaches use traditional Remote Sensing and GIS techniques for spatial data integration, there are plenty of other examples where uncertainty in the available information is taken into consideration. The list of mechanisms, which have been proposed in Geographical and Geological applications, that can handle uncertainty, includes Bayesian networks [6] [21] [29], neural networks and genetic algorithms [41] [30], [31], Dempster-Shafer theory [23] [21] [19] and fuzzy logic [4] [27]. In [21], Lee and Richards discuss both a conventional statistical method and a Dempster-Shafer method for multisource data analysis and suggest that a combination of the two approaches may be well suited when there are both numerical and non-numerical data. In [6], Bonham-Carter et al use Bayesian statistics to combine various geological factors in finding areas favourable for gold exploration. Moon in [23] uses a Dempster-Shafer method to deal with incomplete information and also to integrate geophysical and geological data sets of different resolution. In [41], Zhou and Civco use an evolutionary learning algorithm in a Neural Network as a replacement and an improvement over traditional methods of GIS for suitability analysis in order to find all areas suitable for the location of a light manufacturing plant.

Of all the above soft-computing techniques, by far the most flexible one, both in terms of implementation and intuitive understanding, is fuzzy logic [40]. In the problem we are interested in, i.e., the assessment of the degree of risk of desertification of areas affected by fire, the variables involved are fuzzy with a mixture of classes. Thus, fuzzy logic has been chosen as the main reasoning mechanism. The decision making process involves implementing rules that contain linguistic variables or fuzzy sets. This demands a procedure for aggregation of fuzzy sets. There are several publications available on fuzzy aggregation operators, of which a few notable ones are [20] [2] [13] [38] [34] [14] [12] [5] [10] [39] [32] [11]. In these papers, the properties of various aggregation operators are stated. With a few exceptions, in most applications of fuzzy logic to the Environmental Sciences, the classical min and max aggregation operators are used, while the other options remain largely unexplored. (For example, Binaghi and Rampini [4] use fuzzy aggregation in fire risk determination, but mainly concentrate on the *mean* aggregation operator as an alternative to *max* and *min*). Choosing the right operator or sets of operators for a problem may not be straightforward. To deal with the example problem we use in this paper, we need a combination of two operators. If we take into consideration the nature of the various operators, we end up with 121 possibilities. We advocate here a training-based approach, during which the most appropriate set of operators is identified.

Another important issue that has to be considered, is the relative importance of the various factors that are aggregated by the fuzzy logic mechanism. There have been a few publications that say how relative importances of combined factors could be incorporated in fuzzy reasoning. Of these, the most notable ones are by Yager [34] [35] [36] [37]. In this paper, we introduce the novel idea of assigning relative importance to the aggregated factors by allowing the membership functions to take values greater than 1. It must be stressed that this is not the same as scaling the memberships functions with the help of weights, because several of the aggregation operators used are non-linear and scaling their arguments is not the same as allowing memberships functions greater than 1. For this purpose, we also propose generalised forms of these operators where their input variables are

allowed to take values greater than 1. The properties of these generalised operators are discussed in detail in [26]. The maximum value of each membership function is treated as a parameter that is determined with the help of the training data. This approach is compared with Yager's basic approach, namely that of membership functions raised to some power.
Briefly, the purpose of this paper is dual:
First, to explore a new mechanism for taking into consideration the relative importance of the various factors that influence a decision, and second to investigate this mechanism in conjunction with the various rules that have been proposed for evidence aggregation and in the context of a specific application. Hence, this work is aimed at improving a conventional GIS-based system in a decision making process that involves spatial data analysis. This improvement will be exemplified by experimental comparison with a simple GIS rule-based system.
Section-II of the paper introduces the problem. Section-III gives a brief overview of fuzzy techniques including the properties of a few aggregation operators. Section-IV brings out the issues related to the relative importances of the aggregates, reviews Yager's work and also describes our approach. Section-V describes the various experiments performed. Section-VI discusses the results obtained, justifies why the choice of aggregation operators and inclusion of relative importances matter in a decision making problem, and summarizes briefly the main points that have emerged as the outcome of the experiments and analysis done.

2 Problem Background

Forest fires are a major concern throughout the world and though some times a burned forest regenerates on its own in due course, many times the affected area becomes arid if left uncared for. Since total afforestation is virtually impractical, timely and accurate information on the sites that should have priority for reforestation could go a long way in an efficient planning of resource allocation. This could be accomplished by using an efficient reasoning model which could handle fuzzy data as well.

Since the uncertainty involved in the problem is mainly due to a mixture of classes and not due to randomness, usage of partial membership functions is more appropriate than probabilistic approaches. The data that are used in this study pertain to a few sites in Attica, Greece, which have been chosen in such a way that they represent maximum site variability.
The parameters that influence the risk of desertification as given by some experts are shown in figure 1. From this figure, it can be seen that the ultimate variables involved in the study are Ground Slope, Rock Permeability, Soil Depth and Aspect. It is true that there are several other factors that influence desertification, but over the whole study area of our project those factors could easily be assumed constant, and therefore irrelevant to the *relative* ranking of the various sites. The spatial and attribute data pertaining to Soil Depth, Rock Permeability and Digital Elevation Model (DEM) were fed into the Arc/Info GIS package and the aspect and slope data were derived as secondary data using the raster

analysis facility of Arc/Info.
Slope has been classified into the following four classes based on the degree to which it influences the risk of soil erosion.

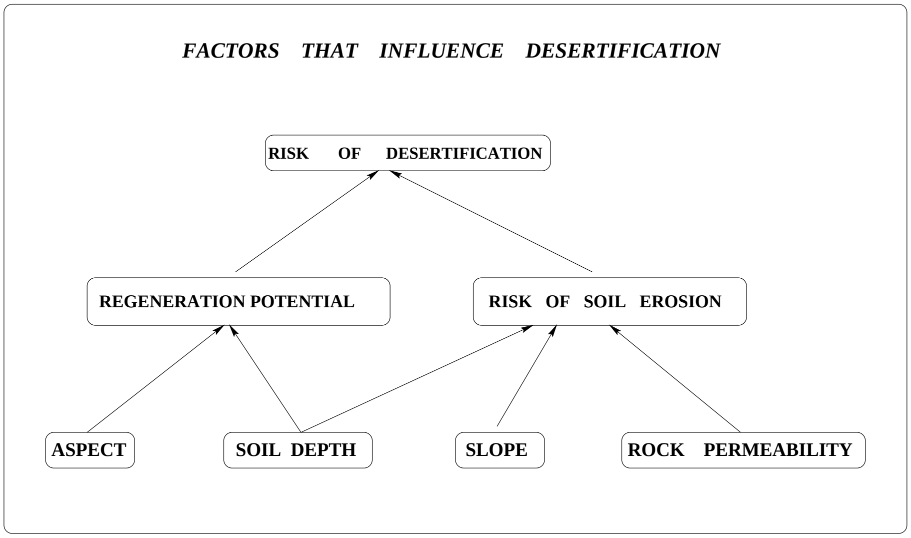

Figure 1: Factors that influence the risk of desertification of a burned forest

1. Gentle: $0 - 20\%$
2. Moderate: $21 - 40\%$
3. Steep: $> 40\%$

Slope can be expressed in degrees or percentage. When expressed as a percentage the slope is 100% when the angle is 45° and approaches ∞ as the angle approaches the vertical which is 90°.
Soil Depth has been classified into the following 3 classes.

1. Bare: $< 5cm$
2. Shallow: $5 - 30cm$
3. Deep: $> 30cm$

Aspect or orientation of a ridge can be expressed as the angle the normal to the ridge forms with the north direction. This angle could take a value from 0° to 360° and it could belong to any of the following classes.

1. North: $0 - 45^\circ$, $315 - 360^\circ$

2. East: $45 - 135°$
3. South: $135 - 225°$
4. West: $225 - 315°$

Rock Permeability refers to the ease with which water may run through the rock. The higher the rock permeability, the lower the risk of soil erosion is. The different types of rock found in the study area are Hard Limestone, Schists, Metamorphic, Calcareous tertiary deposits, Siliceous tertiary deposits and Colluvium. However, the classes used in this study were just 'permeable' and 'impermeable'.

The Arc/Info GIS package was used for performing a few intermediate spatial functions with the data, before any reasoning could be done. While the data regarding Rock Permeability, Soil Depth and DEM were provided for the entire study area, we were concerned with only a few sample sites. Hence, the required data for these sites were extracted by clipping with the sample site boundaries. Slope and aspect values were obtained from the DEM using the GRID module of Arc/Info. GRID is a rule based geo-processing system integrated with Arc/Info.

The reasoning part of the problem under consideration involved the implementation of the hard rules provided by the expert in terms of fuzzy concepts. The rules as given by the expert are shown in Table-I .

RULES FOR NATURAL REGENERATION POTENTIAL

		SOIL DEPTH		
		BARE	SHALLOW	DEEP
ASPECT	NORTH	SG	SL	NL
	EAST	SG	SL	NL
	WEST	SE	ML	SL
	SOUTH	SE	ML	SL

NL - No Limitation
SL - Slight Limitation
ML - Moderate Limitation
SG - Strong Limitation
SE - Severe limitation

RULES FOR RISK OF SOIL EROSION

PERMEABILITY & SOIL DEPTH

		PERMEABLE			IMPERMEABLE		
		BARE	SHALLOW	DEEP	BARE	SHALLOW	DEEP
SLOPE	GENTLE	*	SR	NSR	*	HR	SR
	MEDIUM	*	MR	SR	*	VHR	MR
	STEEP	*	MR	SR	*	VHR	HR

* The land with bare soil is already eroded. No further erosion can occur.
NSR - No to slight risk
SR - Slight risk
MR - Moderate risk
HR - High risk
VHR - Very high risk

RULES FOR RISK OF DESERTIFICATION

REGENERATION POTENTIAL (RP)

		NL	SL	ML	SG	SE
EROSION (SE)	NSR	NR	LR	LR	MR	MR
	SR	LR	LR	MR	MR	HR
	MR	LR	MR	MR	HR	HR
	HR	MR	MR	HR	HR	VHR
	VHR	MR	HR	HR	VHR	VHR

NR - No risk
LR - Low risk
MR - Moderate risk
HR - High risk
VHR - Very high risk

Table 1: EXPERT RULES

3 An overview of Fuzzy Techniques

Since Zadeh defined fuzzy concepts in 1965 [40], fuzzy logic has been established as the ideal method of dealing with various kinds of uncertainty and vagueness like the ones stated below.

- Experts often express their knowledge in terms of linguistic variables like shallow, medium, deep etc.
- A variable is often characterized by a measurement that takes continuous values. Forcing this variable into one of two or three classes according to the value of its measurement, is too gross and ignores the fact that the transition from one class to the other may be gradual and the boundaries between classes fuzzy.
- There is uncertainty in the measurement of variables especially when many pixel values have to be aggregated together to yield a representative figure.
- There is a mixture of classes within each individual site that is to be classified.

In the problem in question, the fact that each individual site consists of a mixture of classes has been the major motivation for using fuzzy reasoning techniques. Hence, the fuzzy membership values to any particular class of a variable, have been evaluated as the proportion of pixels falling in that class within a particular sample site.

Aggregation operations on fuzzy sets are operations by which several fuzzy sets are combined in a desirable way to produce a single fuzzy set. An aggregation operation on n fuzzy sets where $n \geq 2$ is formally defined by a function
$f : [0, 1]^n \longrightarrow$ [0,1]
When applied to fuzzy sets, this function produces an aggregate fuzzy set by operating on the membership grades of these sets [20]. The nature of aggregation of two variables say x and y, could be any of the following [5].

1. Aggregation is conjunctive if
   $f(x, y) \leq min(x, y)$
   which states that a conjunctive operator has confidence at most as high as the smallest membership value and looks for a simultaneous satisfaction of all criteria that are being combined.
2. Aggregation is disjunctive if
   $f(x, y) \geq max(x, y)$
   which states that a disjunctive operator has confidence at least as small as the greatest membership value and looks for a redundancy between the criteria that are being combined.
3. Aggregation is a compromise if
   $min(x, y) \leq f(x, y) \leq max(x, y)$
   which is a cautious behaviour.

The aggregation operators themselves fall under 4 classes, namely

- T-norms
- T-conorms
- Means
- Symmetric Sums

Functions like T-norms and T-conorms have been extensively studied in the literature [5] [2] [20]. The most commonly used T-norms and T-conorms are

Standard Intersection: min	Standard Union: max
Algebraic Product: xy	Probabilistic Sum: $x + y - xy$
Bounded Difference: $max(0, x + y - 1)$	Bounded Sum: $min(1, x + y)$

Min is the smallest T-norm and Max is the largest T-conorm and hence Min is the weakest fuzzy intersection and max is the strongest fuzzy union. T-norms are conjunctive operators and T-conorms are disjunctive operators.
Apart from these union and intersection operators, there are also other aggregation operators like *symmetrical sums* and *mean operators* which are found to be more suitable when we have to deal with aggregation of fuzzy variables refering to different concepts [28] [13] which is also the case in the problem under consideration. By different concepts, we mean aggregation of completely different variables of different scales and nature.
Symmetric Sums take the general form

$$f(x, y) = \frac{g(x,y)}{g(x,y)+g(1-x,1-y)}$$

and their behaviour as to whether they are conjunctive or disjunctive depends on the values of x and y. The symmetric sums that have been used in the present study are

$\sigma_0 \equiv \frac{xy}{1-x-y+2xy}$ corresponding to $g(x, y) = xy$
$\sigma_+ \equiv \frac{x+y-xy}{1+x+y-2xy}$ corresponding to $g(x, y) = x + y - xy$
$min_3 \equiv \frac{min(x,y)}{1-|x-y|}$ corresponding to $g(x, y) = min(x, y)$
$max_3 \equiv \frac{max(x,y)}{1+|x-y|}$ corresponding to $g(x, y) = max(x, y)$

where the nature of σ_0 and σ_+ depends on the values of x and y while min_3 and max_3 are compromise operators. Not all symmetric sums are associative.
The most commonly used mean operators are *arithmetic mean*, *harmonic mean* and *geometric mean* and all the mean operators yield a value in between the maximum and minimum and hence have a compromise behaviour.

4 Issues of relative importance

In many applications of fuzzy logic, all variables were considered equally important which may not be true in reality. For example, the two variables Soil Depth and Aspect could affect the Regeneration Potential in different degrees and moreover Soil Depth could influence Regeneration Potential and Soil Erosion also in different degrees. Hence, it may be crucial to include the relative importances of the variables as well in the reasoning process. Yager has studied the issue of inclusion of relative importance in a great detail and has suggested various ways of incorporating it in a multi-criteria decision making problem. In [37], Yager defines an aggregation operator $F : R^n \longrightarrow R$ as an ordered weighted aggregation operation of dimension n if it has associated with it a weighting vector $(w_1, w_2, ..., w_n)^T$ such that
1. $w_i \in [0, 1]$,
2. $\sum_{i=1}^{n} w_i \quad = \quad 1$ and
$F(a_1, a_2, ...a_n) \quad = \quad \sum_{j=1}^{n} b_j w_j$ where b_j is the jth largest element of $(a_1, a_2, ...a_n)$.
In [34], Yager evaluated the importance of the objectives in a multi-objective decision making problem by finding the eigenvector of the maximum eigenvalue of a matrix of pairwise comparisons of the importance of each objective. He included the relative importance in the problem by raising each objective to a power representing the respective importance obtained from the eigenvector. When a weight is more than 1, the higher the weight, the stricter the condition, while when the weight is less than 1 the condition is loosened. Hence, large membership values would be reduced to much smaller than the small ones if their weights are greater than 1, while small membership values will become larger when their weights are smaller than 1. This ensures that the membership values of less important classes are reduced more, thereby reducing the likelihood of the decision being dominated by those classes, or the membership of more important classes is increased so that the decision is dominated by these classes. In [35], [36] and [34], Yager suggests two other methods of including importances. In each of these, he suggests different ways of doing it, based on whether the nature of fusion is a conjunction or a disjunction. In [35] and [36], Yager states that, a conjunction operation could be performed as

$$min[I(\alpha_i, C_i)], i = 1, 2, ..., n$$

where min is used as the conjunction operator, α_i is the importance of criterion i and C_i is the degree of satisfaction of criterion i. I indicates the function relating importance and satisfaction and is defined as

$$I(a, b) \equiv max[(1 - a), b]$$

Similarly for a disjunction, he suggests

$$max[U(\alpha_i, C_i)], i = 1, 2, ..., n$$

where max is used as the disjunction operator and

$$U(a, b) \equiv min(a, b)$$

In [34], Yager generalises the method of inclusion of importances. He states that an ‘and’ operation could be performed as $A[I(\alpha_i, C_i)]$ where A stands for any T-norm operator and I stands for any T-conorm operator. Similarly, for an ‘or’ operation, he suggests $O[U(\alpha_i, C_i)]$, where O stands for any T-conorm operator and U stands for any T-norm operator.

In this paper, we explore a new way of incorporating relative importance of the variables: the membership values of two variables say x and y vary between 0 and w_1 and 0 and w_2 respectively rather than 0 and 1. Note that this is not the same as using multiplicative weights for the membership functions, as most of the aggregation operators used are non-linear in x and y. Hence most of the operators had to be modified appropriately in order to handle the incorporation of weights. Therefore, the definition of the aggregation function given by $f : [0,1]^n \longrightarrow [0,1]$ has been extended to $f : [0,w_1].[0,w_2]\ldots[0,w_n] \longrightarrow [0, max(w_1,w_2,\ldots w_n)]$ in order to accomodate the weights. The operators as they are originally defined and after the incorporation of weights are shown in Table - II.
For comparison, we also follow Yager and associate weights as powers of the membership values. When this method of weighting is used, the membership values vary between 0 and 1 only and $f : [0,1]^n \longrightarrow [0,1]$ and hence there was no necessity to modify the aggregation operators.
Since no quantitative information on the relative importance of the aggregates is available, in both cases, different weight combinations were experimented with. The operators were first used on a training set comprising of 39 sites and the combination of weights which gave the best results when compared with the expert's classification was used in the evaluation with the 14 test sites.

OPERATOR	GENERALISED DEFINITION(for 4 variables) $0 \leq x_i \leq w_i, \quad i = 1 \quad to \quad 4$
max_1	$maximum(x_1, x_2, x_3, x_4)$
sum	$\dfrac{\sum\limits_{i=1}^{4}(\prod\limits_{\substack{l=1\\ l\neq i}}^{4} w_l)x_i - \sum\limits_{i=1}^{4}\sum\limits_{\substack{j=1\\ j\neq i}}^{4}(\prod\limits_{\substack{l=1\\ l\neq i\\ l\neq j}}^{4} w_l)x_i x_j + \sum\limits_{i=1}^{4}\sum\limits_{\substack{j=1\\ j\neq i}}^{4}\sum\limits_{\substack{k=1\\ k\neq i\\ k\neq j}}^{4}\sum\limits_{\substack{l=1\\ l\neq i\\ l\neq j\\ l\neq k}}^{4} w_l x_i x_j x_k - \prod\limits_{i=1}^{4} x_i}{\prod\limits_{i=1}^{4} w_i / min(w_1, w_2, w_3, w_4)}$
min_2	$min_2[min_2\{min_2(x_1, x_2), x_3\}, x_4]$
min_1	$minimum(x_1, x_2, x_3, x_4)$
$prod$	$\prod\limits_{i=1}^{4} x_i$
max_2	$max_2[max_2\{max_2(x_1, x_2), x_3\}, x_4]$
am	$\frac{1}{4}\sum\limits_{i=1}^{4} x_i$
gm	$\sqrt[4]{(\prod\limits_{i=1}^{4} x_i)}$
hm	$\dfrac{4\prod\limits_{i=1}^{4} x_i}{\sum\limits_{i=1}^{4}\sum\limits_{\substack{j=1\\ j\neq i}}^{4}\sum\limits_{\substack{k=1\\ k\neq i\\ k\neq j}}^{4} x_i x_j x_k}$
min_3	$\frac{min(x_1, x_2, x_3, x_4)}{min(x_1, x_2, x_3, x_4) + min(w_1 - x_1, w_2 - x_2, w_3 - x_3, w_4 - x_4)}$
max_3	$\frac{max(x_1, x_2, x_3, x_4)}{max(x_1, x_2, x_3, x_4) + max(w_1 - x_1, w_2 - x_2, w_3 - x_3, w_4 - x_4)}$
σ_0	$\dfrac{\prod\limits_{i=1}^{4} x_i}{\prod\limits_{i=1}^{4}(w_i - x_i) + \prod\limits_{i=1}^{4} x_i}$
σ_+	$\frac{sum(x_1, x_2, x_3, x_4)}{sum(x_1, x_2, x_3, x_4) + sum(w_1 - x_1, w_2 - x_2, w_3 - x_3, w_4 - x_4)}$

Table 2: Generalised Definition of operators

5 Application

It is obvious from table 1, that there are two levels of aggregation of variables, namely *and* and *or* and hence two operators have to be combined in various ways to create the composite operator needed: one operator to be used for combining the membership values to the various classes of the independent variables, say for example, Soil Depth and Aspect, and one operator to be used for combining the confidences of the various rules

that lead to the same classification of the dependent variable, say for example Regeneration Potential. The nature of fusion of information in these two levels is disjunctive and conjunctive respectively i.e., the fusion of information is a conjunction when we combine conditions for certain situations to arise and it is a disjunction when we combine evidence from different rules to lead to the same conclusion. This is schematically represented in figure 2. When the variables involved are fuzzy, these two levels of aggregation could be dealt with fuzzy union and intersection operators.
In this paper, a comparison of the behaviour of all types of aggregation operator is performed. The operators with a conjunctive (disjunctive) nature are applied in the conjunctive (disjunctive) level of the expert rules, and the symmetrical sums and the mean operators are used in both levels. Thus, we have 10 operators each for the conjunctive and the disjunctive levels, creating a total of 100 combination operators when we use the power weighting method. When we use the weighting method proposed here we have 11 operators each creating a total of 121 combination operators as the operators sum and $prod$ behave like a compromise operator and a symmetric sum respectively when generalised. Moreover, since some of the rules involve 3 variables, the operators are generalised to take care of that. All these operators are applied to evaluate the Risk of Soil Erosion, Natural Regeneration Potential and Risk of Desertification independently.

This approach presents two options: We may train the system so that for each sub-

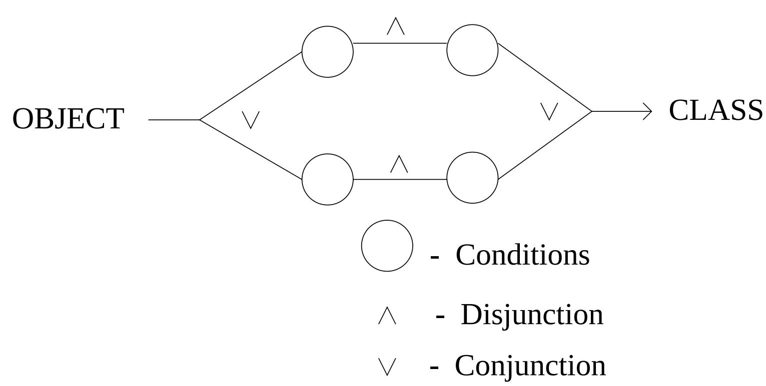

Figure 2: Conjunctive and Disjunctive levels of reasoning

classificaton the best combination of operators is used. This combination could be different for each of the three subproblems. Alternatively, we train the system using for all three subproblems at any one time the same combination of operators for the conjunctive and the disjunctive reasoning. We call these methods Methods I and II respectively.
In yet another approach, the rules supplied by the expert may be combined to form rules that directly relate the attributes of a site with the risk of desertification, omitting the intermediate assessment of risk of erosion and regeneration potential. These composite rules are given in table 3. We call this method Method III.
Each of the three methods was applied with weights used either as membership function powers or as non-unit maximal values of the membership functions, as proposed here. In every case, the weight of one of the variables was fixed to be 1 and different combinations of weights ranging from 0.1 to 10 were experimented with, with the rest of the variables. The operators and the weights that gave the best results in each case with the 39 training

sites when compared with the expert's classification were then used on the 14 test sites.

Soil depth	Aspect	Rock Permeability	Slope	ROD
Deep	North	Permeable	Gentle	NR
Deep	East	Permeable	Gentle	NR
Deep	South	Permeable	Medium	LR
Deep	West	Permeable	Medium	LR
Deep	South	Permeable	Steep	LR
Deep	West	Permeable	Steep	LR
Deep	South	Impermeable	Gentle	LR
Deep	West	Impermeable	Gentle	LR
Shallow	South	Permeable	Medium	MR
Shallow	West	Permeable	Medium	MR
Shallow	South	Permeable	Steep	MR
Shallow	West	Permeable	Steep	MR
Deep	South	Impermeable	Steep	MR
Deep	West	Impermeable	Steep	MR
Shallow	South	Impermeable	Gentle	HR
Shallow	West	Impermeable	Gentle	HR
Shallow	South	Impermeable	Medium	VHR
Shallow	West	Impermeable	Medium	VHR
Shallow	South	Impermeable	Steep	VHR
Shallow	West	Impermeable	Steep	VHR

ROD - Risk of Desertification
NR - No risk
LR - Low risk
MR - Moderate risk
HR - High risk
VHR - Very high risk

Table 3: Combined rules for Risk Of Desertification

5.1 Method I:

The results of method I are presented in tables 4, 5 and 6. The operators and the weights that gave the best results with the training sites were used for the testing sites with no further adjustment. The first column of each set of results gives the number of sites that end up with exactly the same classification as that obtained by the expert, while in the second column we also include sites which were $\pm$ 1 class (out of five possible classes) off from the expert's classification.
In table 4 we list the combination of disjunctive and conjunctive operators and the cor-

responding weights that produced the best results for the Natural Regeneration Potential. It is interesting to note that both weighting approaches placed higher significance to Soil Depth than to Aspect.

WEIGHTING PROCEDURE	OPERATOR		WEIGHTS		TRAINING SITES		TEST SITES	
	DISJUNCTIVE	CONJUNCTIVE	SD	AS	CCS	CCSE	CCS	CCSE
PROPOSED HERE	min_2	sum	1	0.1,0.2,0.4,0.8	21	36	8	14
	sum	sum	1	0.1,0.2,0.4	21	36	8	14
	min_2	max_3	1	10	19	29	8	12
	sum	max_3	1	7-10	13	27	7	12
POWER	sum	max_3	1	2	21	37	7	14
	min_2	max_3	1	2	21	37	7	14

SD - Soil Depth, AS - Aspect, CCS - Correctly Classified Sites, CCSE - Correctly Classified Sites with ± 1 class error

Table 4: Method I: Results for Limitation to Natural Regeneration Potential

For the risk of soil erosion (table 5), the weighting method we propose gave the best results with the operators max_3 and sum for the disjunctive and the conjunctive level respectively, while with power weighting the best results were obtained with max_1 and max_3 operators. For comparison, we also include in the table the results obtained when the combination of operators that worked best for one of the weighting methods was used with the other weighting method. However, while the operator sum behaved as the best conjunctive operator in our weighting method, it is not used in the power weighting method for comparison because the standard sum operator is only a disjunctive operator. We notice that the weighting method proposed here gave slightly better results with the test sites, even though the power method gave slightly better results with the training sites. Relative importances given by the two methods are not consistent. Soil depth appears to be the least important in the weighting method proposed here, while it appears as the most important in the power method.

The results obtained by using the best aggregation operators and weights for Natural Regeneration Potential and Soil Erosion were then used to evaluate the ROD (Risk of Desertification) and the results are given in table 6. The best set of operators for our weighting method was $min_2\sigma_+$ with Regeneration Potential considered 5 times more important than Risk of Soil Erosion, while the best set of operators for the power weighting method was $max_1 max_3$ with Risk of Soil Erosion considered more important than Regeneration Potential. For comparison, we also present in table 6 the results obtained when we exchanged the set of best operators between the two weighting methods. The results show that there is not a significant difference between the results obtained by our weighting

WEIGHTING PROCEDURE	OPERATOR		WEIGHTS			TRAINING SITES		TEST SITES	
	DISJUNCTIVE	CONJUNCTIVE	SD	Rp	SL	CCS	CCSE	CCS	CCSE
PROPOSED HERE	max_3	sum	1	9	9	27	35	7	8
	max_1	max_3	1	0.9-7	0.1	20	36	3	10
POWER	max_1	max_3	1	5-10	4-(Rp-1)	29	39	6	12

SD - Soil Depth, ASP - Aspect, Rp - Rock Permeability, SL - Slope, CCS - Correctly Classified Sites, CCSE - Correctly Classified Sites with ± 1 class error

Table 5: Method I: Results for Risk of Soil Erosion

method and the power method.

WEIGHTING PROPOSED HERE

ROD				NRP		SE		TRAINING		TEST	
DISJ	CONJ	WEIGHTS NRP	SE	DISJ	CONJ	DISJ	CONJ	CCS	CCSE	CCS	CCSE
min_2	σ_+	1	0.2	min_2	sum	max_3	sum	26	33	7	14
max_1	min_1	1	0.2	min_2	max_3	max	max_3	23	33	5	10

ROD				NRP		SE		TRAINING		TEST	
DISJ	CONJ	DISJ NRP	CONJ SE	WEIGHTS		DISJ	CONJ	CCS	CCSE	CCS	CCSE
max_1	max_3	1	0.7	sum	max_3	max_1	max_3	25	37	7	13
max_1	min_1	1	0.3	sum	max_3	max_1	max_3	25	37	7	13
max_1	σ_+	1	6	sum	max_3	max_1	max_3	25	37	7	13

SD - Soil Depth, Rp - Rock Permeability, SL - Slope, CCS - Correctly Classified Sites, CCSE - Correctly Classified Sites with 1 class error, ROD - Risk of Desertification, NRP - Natural Regeneration Potential, SE - Risk of Soil Erosion

Table 6: Method I: Results for Risk of Desertification

5.2 Method II:

In this method, the same set of aggregation operators was used at all levels of conjunctive/disjunctive reasoning everytime. The best results obtained are given in table 7. The results obtained are much better than those obtained with Method I. It can be noticed that our weighting approach gave better results than the power method by correctly classifying 9 out of the 14 test sites. Guided from the performance with the training data, one would choose min_2am combination of operators for our weighting approach and the min_2max_3 combination for the power approach and achieve 9 and 6 correctly classified sites out of the 14 test sites respectively. The combination min_2max_3 that performed the best with power method gave better results with our weighting method. In this case, it is not possible to draw any conclusions about the relative importance of the various factors, as the reasoning is performed in two levels that involve non-linear processing. Note that the Soil Depth attribute was incorporated with separate weights when entered through the Regeneration Potential and Soil Erosion.

5.3 Method III:

In this method, instead of evaluating the Risk of Desertification from Regeneration Potential and Soil Erosion, the rules of Regeneration Potential and Soil Erosion were combined to evaluate the Risk of Desertification directly from the fuzzy memberships of the four

WEIGHTING METHOD PROPOSED HERE

OPERATOR		WEIGHTS							TRAINING SITES		TEST SITES	
DISJ.	CONJ.	SD_1	ASP	Rp	SL	SD_2	NRP	SE	CCS	CCSE	CCS	CCSE
min_2	am	1	0.3	0.1	1	0.5	1	1	37	39	9	12
min_2	am	1	0.3	0.1	1	0.6	1	10	38	39	9	13
min_2	max_3	1	0.1	3	1	0.1	1	10	32	39	8	12

POWER WEIGHTING

OPERATOR		WEIGHTS							TRAINING SITES		TEST SITES	
DISJ.	CONJ.	SD_1	ASP	Rp	SL	SD_2	NRP	SE	CCS	CCSE	CCS	CCSE
min_2	max_3	1	2	6	1	4	1	1	35	38	6	13
min_2	am	1	0.5	0.5	1	6	1	10	33	39	5	14

SD_1 - Soil Depth as a factor of NRP, ASP - Aspect, SD_2 - Soil Depth as a factor of SE, Rp - Rock Permeability, SL - Slope, NRP - Limitation to Natural Regeneration Potential, SE - Risk of Soil Erosion, CCS - Correctly Classified Sites, CCSE - Correctly Classified Sites with 1 class error

Table 7: Method II: Results for Risk of Desertification

basic variables, according to the rules shown in table 3. The results are given in table 8. The generalised aggregation operators for four variables presented in table 2 were used in this case. From table 8, it can be seen that the maximum number of correctly classified sites obtained in the training phase is as low as 22 sites out of 39, with our weighting approach and 24 sites with the power approach. This method did not produce good results with the test sites either. Further, the generalisation of the system is not very good. There is also some inconsistency in the ranking of the various factors in order of decreasing importance: Our weighting method gives highest importance to Rock Permeability followed by equal importance to Soil Depth and Aspect followed by Slope. The power weighting gives maximum importance to Slope.

6 Discussion

It can be observed that of the above three methods, Method II gave the maximum number of correctly classified sites, when compared with the expert's opinion, but with not very good generalization capabilities. The weighting method proposed here gave marginally better results than the power weighting method. By far, the worst properties were exhibited by Method III. From the fact that Method III did not perform well, we conclude

WEIGHTING PROCEDURE	OPERATOR		WEIGHTS				TRAINING SITES		TEST SITES	
	DISJ	CONJ	SD	ASP	Rp	SL	CCS	CCSE	CCS	CCSE
PROPOSED HERE	sum	am	1	1	4	0.1	22	25	3	8
	min_2	σ_+	1	1	1	0.1	17	34	4	10
POWER	min_2	σ_+	3	1	2	0.1	24	31	4	8
	sum	am	0.1	1	0.1	0.1	13	28	0	6

SD - Soil Depth, ASP - Aspect, Rp - Rock Permeability, SL - Slope, CCS - Correctly Classified Sites, CCSE - Correctly Classified Sites with 1 class error
* - many combinations of weights give the same result

Table 8: Method III: Results for Risk of Desertification

that, at least the expert who evaluated the data we used, used the intermediate steps of evaluating Regeneration Potential and Soil Erosion in his reasoning on the problem, and the non-linearities of these two steps of reasoning cannot be modelled satisfactorily with the type of nonlinearity we introduce with our operators and the weighting process.
The importance of using weighting is assessed by presenting in table 9 the results obtained by all three methods when all factors are given equal weight (i.e., by using the classical fuzzy logic approach). The results presented in the table are the best from each method over all possible combinations of operators. It can be seen that the best result achieved with equal weights was by using Method I, that classified correctly 23 out of the 39 training sites and 5 out of the 14 test sites.These results have to be compared with the weighted Method II results: A total of 37 out of 39 training sites were correctly classified and all 39 sites were correctly classified if we allow an error of 1 class deviation, by method II and by the weighting procedure proposed in this paper. For the testing sites one could achieve the correct classification of 9 out of the 14 sites. This proves that the use of weighting greatly improves the performance of the approach.
Since Method II seems to be so much better than all others, we investigated it in more detail. In all the work presented here, memberships are calculated as fractional class components of composite sites. For example, if a site consists of 150 pixels, 30 of which have slope attribute in class ‘steep’ and 120 have slope attribute ‘medium’, the site is given 20% membership to class ‘steep’ and 80% to class ‘medium’. It has been explained elsewhere [27], how the membership functions can be calculated as integrals of the Gaussianly distributed errors in the measurements. The mean and variance of each Gaussian is estimated from the (numerical) attributes of each region. We used this type of membership function in combination with all possible operators and both ways of weighting, for Method II. We present the results obtained in detail in tables 10 and 11 for the two weighting approaches respectively. The numbers in brackets are the results obtained with membership functions calculated as described in [27]. In general these membership func-

tions improved the results without altering the conclusion as to which operators are best. With the proposed weighting with min_2am operators, 38 (instead of 37) out of the 39 training sites were correctly classified and 11 (instead of 9) out of the 14 test sites. With the power weighting, the best achieved was with min_2max_3 operators with 37 (instead of 35) training sites correctly classified and 7 (instead of 6) testing sites.

It can also be seen from table 10 that all compromise operators perform quite badly both as disjunctive and conjunctive operators, except am, a compromise operator which gives good results when used as a conjunctive operator. Though am as conjunctive operator performs reasonably well combined with most of the other operators, it gives the best results when combined with min_2 as the disjunctive operator. Symmetrical sums do not seem to perform well in either level. Though some of the combinations of disjunctive and conjunctive operators perform reasonably, none of them performs as well as min_2am i.e., with min_2, in the disjunctive level and am in the conjunctive level. When the weights are used as power of membership values, max_3 performs well as a conjunctive operator, though it gives the best results when combined with min_2. If we allow one class error, then min_2am, $min_2\sigma_+$ and min_2max_3 correctly classify all the sites in both cases. Though the same disjunctive operator min_2 works well in the disjunctive level of both cases, two different compromise operators perform well in the conjunctive level.

METHOD	RISK	OPERATOR		TRAINING SITES		TEST SITES	
		DISJUNCTIVE	CONJUNCTIVE	CCS	CCSE	CCS	CCSE
I	NRP	min_2	$mean$	18	27	8	12
		min_2	am	18	27	8	12
		min_2	σ_+	18	27	8	12
		min_2	max_3	18	27	8	12
	SE	gm	$mean$	20	27	4	5
		gm	am	20	27	4	5
		hm	$mean$	20	27	4	5
		hm	am	20	27	4	5
	ROD	min_2	gm	23	32	3	12
		min_2	σ_+	23	32	5	14
II	ROD (prop)	sum	$mean$	17	33	3	10
		sum	am	17	33	3	10
	ROD (gauss)	sum	max_3	16	32	3	13
III	ROD	min_2	σ_+	17	34	4	10
		min_2	max_3	17	34	4	10

CCS - Correctly Classified Sites, CCSE - Correctly Classified Sites with 1 class error

Table 9: Results with all variables considered equally important

RESULTS WITH DIFFERENT COMBINATIONS OF AGGREGATION OPERATORS
- (with proposed weights)

OPERS	TR		TT		OPERS	TR		TT	
	CCS	CCSE	CCS	CCSE		CCS	CCSE	CCS	CCSE
max_1sum	24 (27)	33 (35)	5 (8)	11 (14)	$hmmax_3$	9 (9)	22 (23)	0 (1)	3 (7)
max_1am	18 (12)	30 (35)	4 (5)	8 (8)	$hmmin_3$	0 (0)	22 (23)	0 (0)	1 (1)
max_1gm	9 (8)	22 (27)	4 (3)	8 (7)	$hmprod$	0 (0)	22 (23)	0 (0)	1 (1)
max_1hm	9 (9)	22 (27)	4 (3)	8 (7)	$hmmin_1$	0 (0)	22 (23)	0 (0)	1 (1)
$max_1\sigma_0$	10 (14)	26 (29)	6 (5)	8 (7)	$hmmax_2$	3 (3)	29 (28)	1 (2)	5 (8)
$max_1\sigma_+$	15 (17)	30 (28)	5 (4)	11 (9)	σ_0sum	26 (31)	35 (38)	3 (3)	9 (13)
max_1max_3	18 (18)	32 (36)	6 (4)	11 (11)	σ_0am	20 (17)	31 (35)	2 (4)	8 (10)
max_1min_3	11 (14)	26 (29)	6 (5)	8 (7)	σ_0gm	0 (0)	31 (35)	0 (0)	1 (1)
max_1prod	9 (8)	22 (27)	4 (3)	8 (7)	σ_0hm	1 (1)	3 (2)	0 (0)	1 (1)
max_1min_1	12 (12)	26 (29)	5 (4)	9 (7)	$\sigma_0\sigma_0$	0 (0)	3 (2)	0 (0)	1 (1)
max_1max_2	17 (11)	31 (32)	3 (3)	8 (10)	$\sigma_0\sigma_+$	20 (19)	28 (31)	3 (2)	6 (7)
min_2sum	32 (34)	34 (36)	7 (8)	13 (14)	σ_0max_3	19 (20)	28 (33)	2 (3)	6 (9)
min_2am	37 (38)	39 (39)	9 (11)	12 (14)	σ_0min_3	1 (1)	3 (2)	0 (0)	1 (1)
min_2gm	10 (11)	25 (24)	4 (3)	8 (6)	σ_0prod	0 (1)	3 (2)	0 (0)	1 (1)
min_2hm	10 (10)	22 (26)	3 (2)	7 (6)	σ_0min	0 (0)	3 (2)	0 (0)	1 (1)
$min_2\sigma_0$	10 (13)	27 (29)	6 (5)	8 (7)	σ_0max_2	1 (1)	6 (4)	0 (0)	1 (1)
$min_2\sigma_+$	35 (32)	39 (39)	10 (6)	12 (14)	σ_+sum	20 (20)	37 (38)	2 (2)	11 (14)
min_2max_3	19 (19)	39 (39)	9 (8)	14 (14)	σ_+am	18 (19)	32 (32)	1 (4)	9 (9)
min_2min_3	11 (13)	27 (29)	6 (5)	8 (7)	σ_+gm	8 (9)	24 (24)	3 (2)	7 (6)
min_2prod	10 (11)	22 (24)	5 (3)	7 (6)	σ_+hm	8 (9)	23 (27)	4 (3)	8 (7)
min_2min_1	11 (12)	23 (29)	4 (3)	9 (7)	$\sigma_+\sigma_0$	7 (9)	20 (25)	4 (2)	6 (6)
min_2max_2	18 (17)	32 (35)	2 (3)	7 (10)	$\sigma_+\sigma_+$	22 (18)	30 (30)	2 (1)	7 (8)
$sumsum$	26 (29)	39 (37)	0 (2)	9 (12)	σ_+max_3	18 (19)	26 (29)	0 (2)	6 (7)
$sumam$	25 (24)	34 (33)	4 (5)	9 (8)	σ_+min_3	8 (10)	21 (25)	4 (2)	6 (6)
$sumgm$	9 (9)	23 (26)	4 (3)	8 (6)	σ_+prod	9 (9)	23 (24)	4 (2)	8 (6)
$sumhm$	10 (9)	22 (27)	4 (3)	8 (7)	σ_+min	10 (9)	23 (27)	3 (2)	7 (6)
$sum\sigma_0$	10 (13)	26 (29)	6 (5)	8 (7)	σ_+max_2	7 (11)	24 (27)	2 (1)	6 (9)
$sum\sigma_+$	20 (18)	32 (34)	4 (3)	10 (10)	max_3sum	15 (20)	32 (32)	2 (2)	10 (12)
$summax_3$	22 (23)	36 (37)	5 (3)	11 (11)	max_3am	16 (17)	29 (32)	1 (3)	8 (9)
$summin_3$	11 (14)	26 (29)	6 (5)	8 (7)	max_3gm	9 (8)	22 (26)	3 (2)	7 (7)

contd ...

TR - Training Sites
TT - Test Sites
CCS - Correctly Classified Sites
CCSE - Correctly Classified Sites with 1 class error

OPERS	TR		TT		OPERS	TR		TT	
	CCS	CCSE	CCS	CCSE		CCS	CCSE	CCS	CCSE
$sumprod$	10 (9)	22 (23)	4 (3)	8 (7)	max_3hm	10 (9)	22 (27)	4 (3)	8 (7)
$summin_1$	11 (12)	23 (29)	4 (3)	9 (7)	$max_3\sigma_0$	7 (9)	20 (24)	4 (2)	6 (6)
$summax_2$	16 (18)	27 (35)	1 (4)	7 (11)	$max_3\sigma_+$	16 (18)	29 (26)	1 (1)	7 (6)
$amsum$	16 (18)	33 (31)	2 (2)	11 (12)	max_3max_3	14 (17)	28 (32)	0 (3)	7 (9)
$amam$	21 (21)	30 (31)	1 (2)	5 (9)	max_3min_3	8 (10)	21 (25)	4 (2)	6 (5)
$amgm$	5 (7)	21 (24)	3 (2)	7 (6)	max_3prod	9 (8)	22 (27)	4 (3)	8 (7)
$amhm$	7 (9)	21 (24)	3 (2)	7 (6)	max_3min_1	11 (10)	22 (26)	4 (2)	7 (6)
$am\sigma_0$	6 (8)	20 (25)	3 (2)	7 (6)	max_3max_2	5 (5)	22 (22)	2 (4)	3 (6)
$am\sigma_+$	19 (18)	28 (27)	1 (2)	5 (7)	min_3sum	19 (22)	37 (34)	5 (5)	11 (8)
$ammax_3$	18 (19)	26 (30)	1 (2)	5 (7)	min_3am	18 (15)	29 (32)	3 (5)	6 (9)
$ammin_3$	7 (9)	21 (25)	3 (2)	7 (6)	min_3gm	1 (1)	3 (2)	0 (0)	1 (1)
$amprod$	7 (8)	21 (26)	3 (2)	7 (6)	min_3hm	1 (1)	3 (2)	0 (0)	1 (1)
$ammin_1$	8 (9)	21 (25)	3 (2)	7 (6)	$min_3\sigma_0$	0 (0)	3 (2)	0 (0)	1 (1)
$ammax_2$	5 (3)	23 (21)	0 (1)	5 (6)	$min_3\sigma_+$	17 (19)	29 (35)	1 (2)	9 (10)
$gmsum$	16 (18)	32 (31)	2 (3)	11 (12)	min_3max_3	14 (16)	26 (33)	0 (4)	6 (9)
$gmam$	15 (18)	23 (28)	0 (2)	4 (8)	min_3min_3	1 (1)	3 (2)	0 (0)	1 (1)
$gmgm$	0 (0)	23 (28)	0 (0)	1 (1)	min_3prod	1 (1)	3 (2)	0 (0)	1 (1)
$gmhm$	0 (1)	23 (2)	0 (0)	1 (1)	min_3min_1	1 (1)	3 (2)	0 (0)	1 (1)
$gm\sigma_0$	0 (0)	23 (2)	0 (0)	1 (1)	min_3max_2	0 (0)	3 (2)	0 (0)	0 (0)
$gm\sigma_+$	18 (19)	25 (28)	1 (2)	5 (7)	$prodsum$	14 (13)	32 (29)	1 (2)	10 (7)
$gmmax_3$	17 (19)	23 (30)	1 (2)	5 (7)	$prodam$	21 (23)	30 (35)	2 (4)	6 (8)
$gmmin_3$	1 (1)	3 (2)	0 (0)	1 (1)	$prodgm$	0 (1)	30 (2)	0 (0)	1 (1)
$gmprod$	0 (0)	3 (2)	0 (0)	1 (1)	$prodhm$	0 (1)	30 (2)	0 (0)	1 (1)
$gmmin_1$	1 (1)	3 (2)	0 (0)	1 (1)	$prod\sigma_0$	0 (0)	30 (2)	0 (0)	1 (1)
$gmmax_2$	0 (1)	3 (4)	0 (0)	0 (1)	$prod\sigma_+$	5 (6)	19 (19)	3 (3)	6 (7)
$hmsum$	13 (13)	31 (29)	3 (2)	5 (7)	$prodmax_3$	0 (3)	19 (22)	0 (2)	4 (9)
$hmam$	18 (18)	27 (29)	2 (3)	6 (7)	$prodmin_3$	0 (0)	19 (22)	0 (0)	1 (1)
$hmgm$	0 (0)	27 (29)	0 (0)	1 (1)	$prodprod$	0 (1)	19 (2)	0 (0)	1 (1)
$hmhm$	0 (1)	27 (2)	0 (0)	1 (1)	$prodmin_1$	0 (0)	19 (2)	0 (0)	1 (1)
$hm\sigma_0$	0 (0)	27 (2)	0 (0)	1 (1)	$prodmax_2$	11 (11)	32 (32)	3 (5)	5 (8)
$hm\sigma_+$	12 (12)	25 (23)	1 (2)	4 (7)					

TR - Training Sites
TT - Test Sites
CCS - Correctly Classified Sites
CCSE - Correctly Classified Sites with 1 class error

Table 10: Results for different combinations of aggregation operators (proposed weights)

RESULTS WITH DIFFERENT COMBINATIONS OF AGGREGATION OPERATORS
- (with power weights)

OPERS	TR		TT		OPERS	TR		TT	
	CCS	CCSE	CCS	CCSE		CCS	CCSE	CCS	CCSE
max_1am	15 (9)	29 (37)	4 (5)	9 (9)	$hmam$	3 (9)	16 (24)	1 (1)	8 (7)
max_1gm	9 (9)	22 (27)	4 (3)	8 (6)	$hmgm$	0 (0)	16 (24)	0 (0)	1 (1)
max_1hm	9 (9)	22 (27)	4 (3)	8 (6)	$hmhm$	0 (0)	16 (24)	0 (0)	1 (1)
$max_1\sigma_0$	12 (15)	27 (29)	5 (4)	8 (7)	$hm\sigma_0$	1 (1)	3 (2)	0 (0)	1 (1)
$max_1\sigma_+$	23 (25)	33 (34)	2 (4)	10 (12)	$hm\sigma_+$	17 (18)	33 (29)	3 (2)	8 (7)
max_1max_3	26 (26)	37 (36)	7 (8)	11 (10)	$hmmax_3$	3 (6)	13 (22)	1 (1)	7 (7)
max_1min_3	11 (14)	27 (29)	3 (5)	8 (7)	$hmmin_3$	2 (2)	4 (3)	0 (0)	1 (1)
max_1prod	9 (9)	22 (27)	4 (3)	8 (6)	$hmprod$	0 (0)	4 (3)	0 (0)	1 (1)
max_1min_1	12 (11)	24 (29)	4 (4)	9 (7)	$hmmin_1$	0 (0)	4 (3)	0 (0)	1 (1)
max_1max_2	17 (16)	29 (26)	6 (5)	8 (9)	$hmmax_2$	0 (1)	4 (3)	0 (0)	1 (1)
min_2am	12 (13)	39 (39)	7 (6)	14 (14)	σ_0am	11 (15)	29 (35)	3 (4)	7 (10)
min_2gm	10 (9)	26 (27)	4 (3)	8 (7)	σ_0gm	0 (1)	29 (2)	0 (0)	1 (1)
min_2hm	10 (9)	25 (27)	4 (3)	8 (6)	σ_0hm	0 (1)	29 (2)	0 (0)	1 (1)
$min_2\sigma_0$	12 (14)	26 (28)	7 (4)	8 (7)	$\sigma_0\sigma_0$	1 (1)	3 (2)	0 (0)	1 (1)
$min_2\sigma_+$	33 (33)	39 (39)	8 (12)	11 (14)	$\sigma_0\sigma_+$	19 (16)	28 (30)	3 (2)	7 (8)
min_2max_3	35 (37)	38 (39)	6 (7)	13 (14)	σ_0max_3	19 (20)	31 (37)	2 (4)	9 (12)
min_2min_3	15 (14)	28 (26)	5 (3)	8 (7)	σ_0min_3	2 (2)	4 (2)	0 (0)	1 (1)
min_2prod	9 (9)	24 (27)	4 (3)	8 (6)	σ_0prod	0 (0)	4 (2)	0 (0)	1 (1)
min_2min_1	12 (11)	24 (29)	4 (3)	9 (7)	σ_0min_1	0 (1)	4 (2)	0 (0)	1 (1)
min_2max_2	21 (12)	29 (38)	2 (4)	8 (10)	σ_0max_2	1 (1)	6 (3)	0 (0)	4 (1)
$sumam$	17 (15)	33 (38)	3 (2)	10 (13)	σ_+am	13 (12)	26 (27)	1 (2)	7 (8)
$sumgm$	9 (9)	24 (27)	4 (3)	8 (6)	σ_+gm	8 (9)	22 (24)	3 (2)	7 (6)
$sumhm$	9 (9)	24 (27)	4 (3)	8 (6)	σ_+hm	9 (9)	22 (26)	3 (2)	7 (6)
$sum\sigma_0$	12 (14)	27 (29)	6 (5)	9 (7)	$\sigma_+\sigma_0$	9 (12)	22 (23)	3 (1)	6 (5)
$sum\sigma_+$	24 (23)	29 (31)	2 (3)	7 (13)	$\sigma_+\sigma_+$	23 (22)	36 (31)	3 (4)	9 (9)

contd ...

TR - Training Sites
TT - Test Sites
CCS - Correctly Classified Sites
CCSE - Correctly Classified Sites with 1 class error

OPERS	TR		TT		OPERS	TR		TT	
	CCS	CCSE	CCS	CCSE		CCS	CCSE	CCS	CCSE
$summax_3$	26 (24)	36 (36)	2 (3)	10 (13)	$\sigma_+ max_3$	26 (24)	36 (35)	2 (3)	10 (12)
$summin_3$	11 (13)	22 (28)	1 (5)	5 (6)	$\sigma_+ min_3$	11 (14)	25 (25)	4 (4)	7 (6)
$sumprod$	9 (9)	23 (27)	4 (3)	8 (6)	$\sigma_+ prod$	10 (9)	22 (27)	4 (3)	7 (6)
$summin_1$	12 (11)	24 (29)	4 (3)	9 (7)	$\sigma_+ min_1$	10 (10)	22 (29)	4 (3)	8 (7)
$summax_2$	16 (8)	31 (34)	4 (3)	7 (8)	$\sigma_+ max_2$	10 (6)	28 (29)	3 (1)	7 (6)
$amam$	12 (12)	27 (27)	1 (3)	7 (9)	$max_3 am$	13 (12)	25 (27)	1 (3)	7 (9)
$amgm$	8 (9)	22 (24)	3 (2)	7 (6)	$max_3 gm$	8 (9)	22 (24)	3 (2)	7 (5)
$amhm$	9 (9)	22 (26)	3 (2)	7 (5)	$max_3 hm$	9 (9)	22 (26)	3 (2)	7 (6)
$am\sigma_0$	9 (10)	24 (24)	4 (2)	7 (6)	$max_3 \sigma_0$	10 (12)	23 (25)	4 (2)	8 (5)
$am\sigma_+$	22 (20)	35 (32)	0 (3)	6 (12)	$max_3 \sigma_+$	20 (20)	27 (30)	4 (2)	6 (8)
$ammax_3$	17 (20)	28 (30)	2 (2)	8 (9)	$max_3 max_3$	18 (17)	28 (32)	1 (3)	7 (9)
$ammin_3$	11 (12)	22 (24)	4 (5)	8 (5)	$max_3 min_3$	10 (10)	20 (23)	3 (2)	6 (4)
$amprod$	10 (9)	22 (26)	3 (2)	7 (5)	$max_3 prod$	10 (9)	22 (27)	4 (3)	7 (6)
$ammin_1$	10 (9)	22 (27)	3 (2)	7 (5)	$max_3 min_1$	11 (10)	22 (29)	4 (4)	7 (7)
$ammax_2$	8 (5)	24 (25)	2 (1)	7 (7)	$max_3 max_2$	10 (6)	28 (30)	3 (1)	7 (6)
$gmam$	11 (12)	27 (26)	0 (1)	6 (7)	$min_3 am$	11 (14)	24 (27)	2 (2)	5 (9)
$gmgm$	1 (1)	3 (2)	0 (0)	1 (1)	$min_3 gm$	1 (1)	3 (2)	0 (0)	1 (1)
$gmhm$	1 (1)	3 (2)	0 (0)	1 (1)	$min_3 hm$	0 (1)	3 (2)	0 (0)	1 (1)
$gm\sigma_0$	1 (1)	3 (2)	0 (0)	1 (1)	$min_3 \sigma_0$	3 (2)	6 (3)	0 (0)	1 (1)
$gm\sigma_+$	19 (19)	28 (26)	0 (1)	6 (6)	$min_3 \sigma_+$	20 (21)	29 (29)	2 (1)	6 (6)
$gmmax_3$	16 (16)	28 (32)	2 (2)	7 (8)	$min_3 max_3$	16 (18)	28 (35)	2 (3)	4 (12)
$gmmin_3$	2 (2)	3 (3)	0 (0)	1 (1)	$min_3 min_3$	5 (12)	10 (17)	2 (1)	5 (3)
$gmprod$	1 (1)	3 (2)	0 (0)	1 (1)	$min_3 prod$	0 (1)	10 (2)	0 (0)	1 (1)
$gmmin_1$	1 (1)	3 (2)	0 (0)	1 (1)	$min_3 min_1$	1 (1)	3 (2)	0 (0)	1 (1)
$gmmax_2$	0 (1)	3 (3)	0 (0)	0 (1)	$min_3 max_2$	0 (1)	3 (4)	0 (0)	0 (1)

TR - Training Sites
TT - Test Sites
CCS - Correctly Classified Sites
CCSE - Correctly Classified Sites with 1 class error

Table 11: Results for different combinations of aggregation operators (power weights)

As a final comparison, we would like to compare all the above discussed results with those that would have been obtained if the fuzzy nature of the problem had been totally ignored. The Arc/Info GIS was used for a simple rule-based reasoning. A Digital Elevation Model (DEM) was used to create the Slope and Apsect layers with the GRID analysis facility of Arc/Info. The layers on Aspect and Soil depth were integrated using the overlay facility and the rules given in table 1 were used to create the output layer reflecting the ranking of ‘Limitation to Natural Regeneration Potential’. Similarly, the layers on Soil Depth, Slope and Rock Permeability were integrated and the rules were used to create the layer on ‘Risk of Soil Erosion’. The two layers on ‘Limitation to Natural Regeneration Potential’ and ‘Risk of Soil Erosion’ were integrated and the rules for Risk of Desertification were used to create the output layer on ‘Risk of Desertification’. Since there is no training procedure involved, all 53 sites were lumped together and experimented without distinguishing between the training and the test sites. The comparison showed that only 17 out of 53 sites were correctly classified.

7 Conclusions

The results presented in this paper advocate the following points:
1. The use of Fuzzy Logic when reasoning with a GIS, as opposed to a rule-based approach. This conjecture is proved by the fact that when a rule-based approach was used with the GIS data, only 17 out of 53 sites could be correctly classified.
2. The use of integrals of Gaussians as membership functions [27] in preference to the aggregate membership functions calculated as fractions of pixels of each site that belong to a certain class. The former method gave marginally better results than the latter.
3. The use of weights of importance for the various aggregates. This point was proved by comparing the results with those obtained by the classical no-weighting approach.
4. The use of operators other than the conventional min and max operators. The importance of this can be judged by the variety of results obtained with different operators.
The last two points introduce the necessity of a training stage for each problem, when the best operators and the best set of weights can be chosen.
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