Visual servoing

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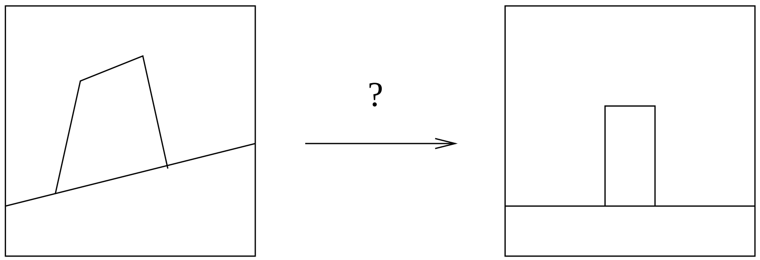
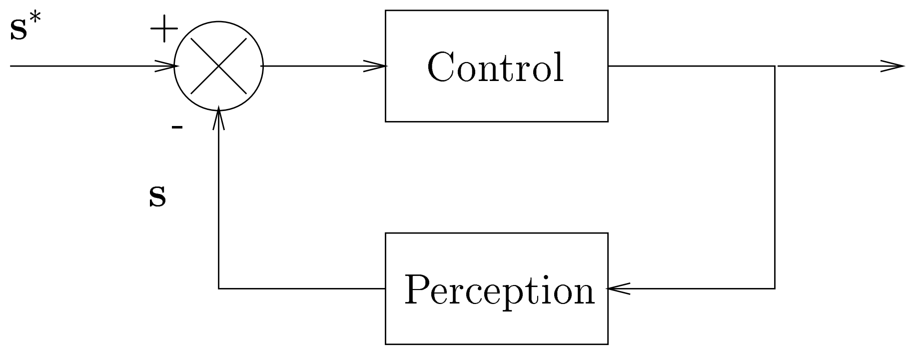
Aim of visual servoing: to realize robotics tasks (i.e. to control robot motion) using visual data embedded in a closed-loop system.

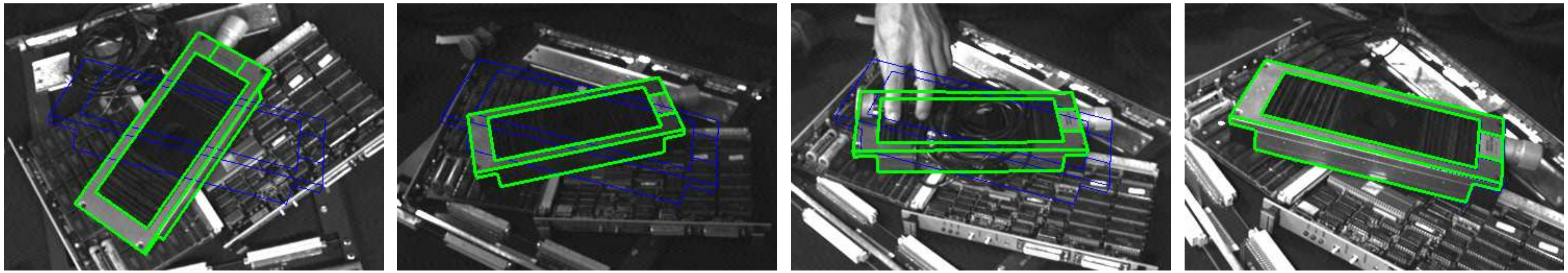




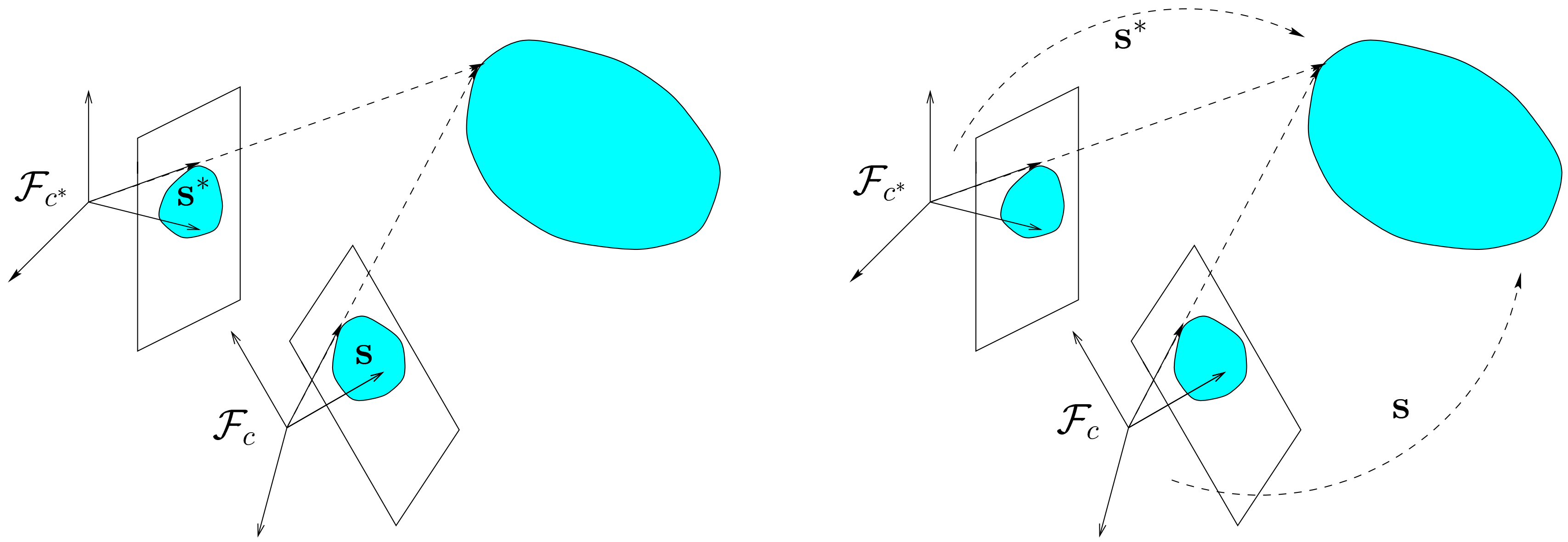
Necessary steps: Select k adequate visual features s
to control m dof ($k \geq m, m \leq 6$)
Determine the goal $\mathbf{s}^*$
Regulate the error $(\mathbf{s} - \mathbf{s}^*)$ to $\mathbf{0}$
Image processing (matching and tracking
near video rate)

Pedestrian tracking using a pan/tilt camera

Positioning/grasping task using a 6 dof robot arm



Positioning task



2D visual features | 3D visual features

Visual features: $\mathbf{s} = \mathbf{s}(\mathbf{p}(t)) \quad \Rightarrow \quad \dot{\mathbf{s}} = \mathbf{L_s}\ \mathbf{v}$ where:

- $\mathbf{L_s}$ = interaction matrix (similar to a jacobian matrix)
- $\mathbf{v} = (\boldsymbol{\upsilon}, \boldsymbol{\omega})$ = instantaneous velocity (or kinematic screw)
  with 3 translational and 3 rotational components

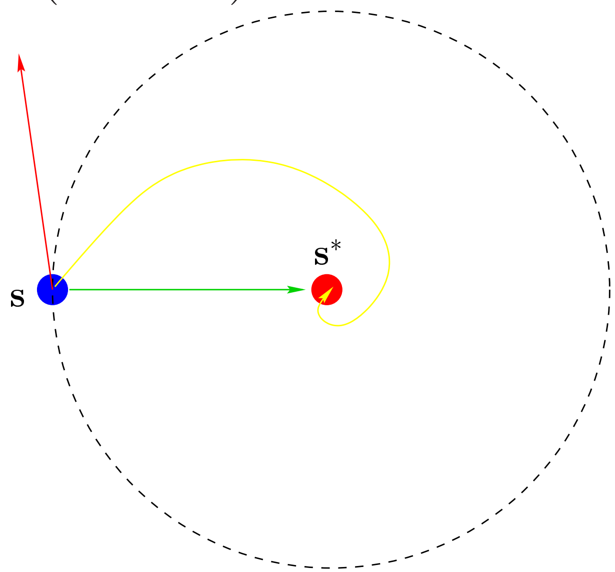
Principle of the control law

If we want $\dot{\mathbf{s}} = -\lambda(\mathbf{s} - \mathbf{s}^*)$ (exponential decoupled decrease):

$$\mathbf{v} = -\lambda\, \widehat{\mathbf{L_s}}^+ (\mathbf{s} - \mathbf{s}^*) \text{ with } \widehat{\mathbf{L_s}}_{(\mathbf{s},\mathbf{p},\mathbf{a})}$$

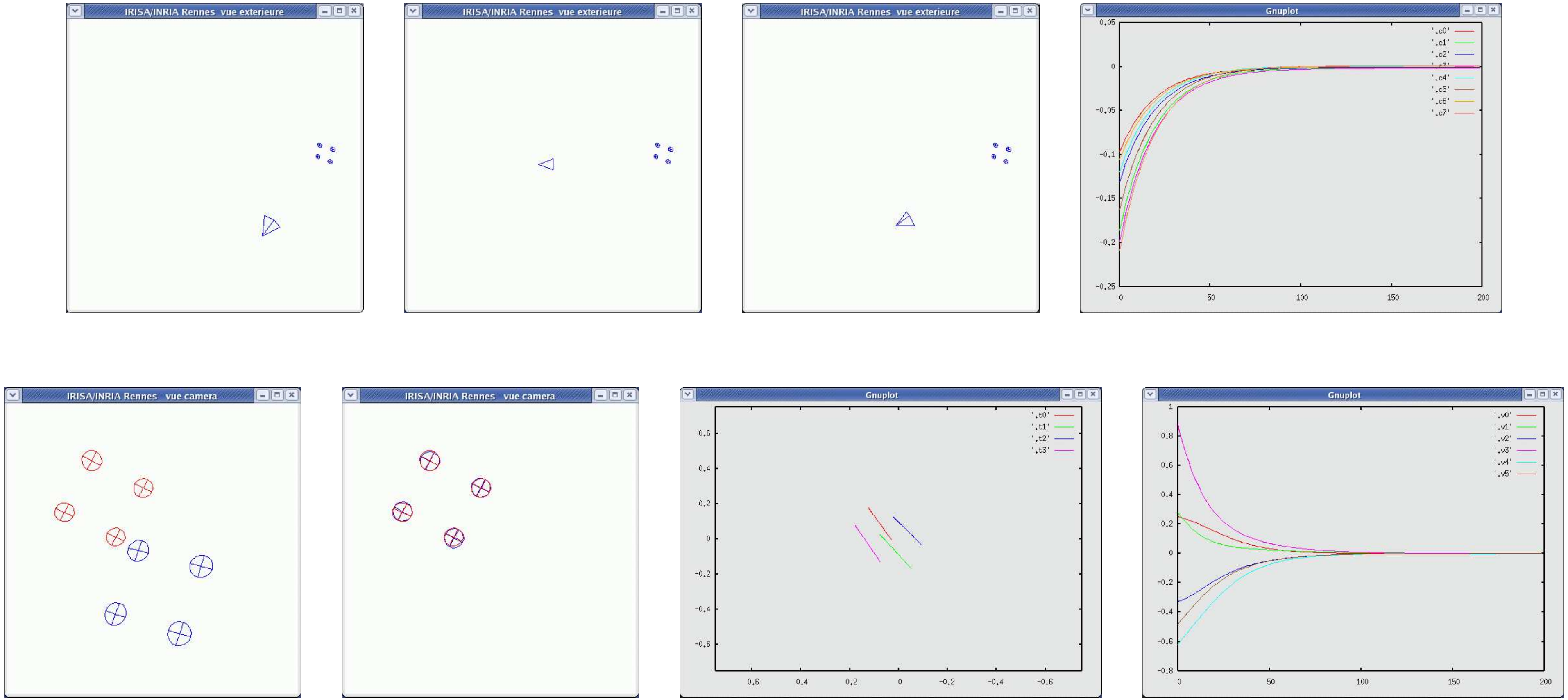
Closed-loop system: $\dot{\mathbf{s}} = \mathbf{L_s}\, \mathbf{v} = -\lambda\, \mathbf{L_s} \widehat{\mathbf{L_s}}^+ (\mathbf{s} - \mathbf{s}^*)$

- if $\mathbf{L_s}\widehat{\mathbf{L_s}}^+ = \mathbf{I}$, perfect behavior
- if $\mathbf{L_s}\widehat{\mathbf{L_s}}^+ > 0$, $\|\mathbf{s} - \mathbf{s}^*\|$ decreases
- if $\mathbf{L_s}\widehat{\mathbf{L_s}}^+ < 0$, $\|\mathbf{s} - \mathbf{s}^*\|$ increases...

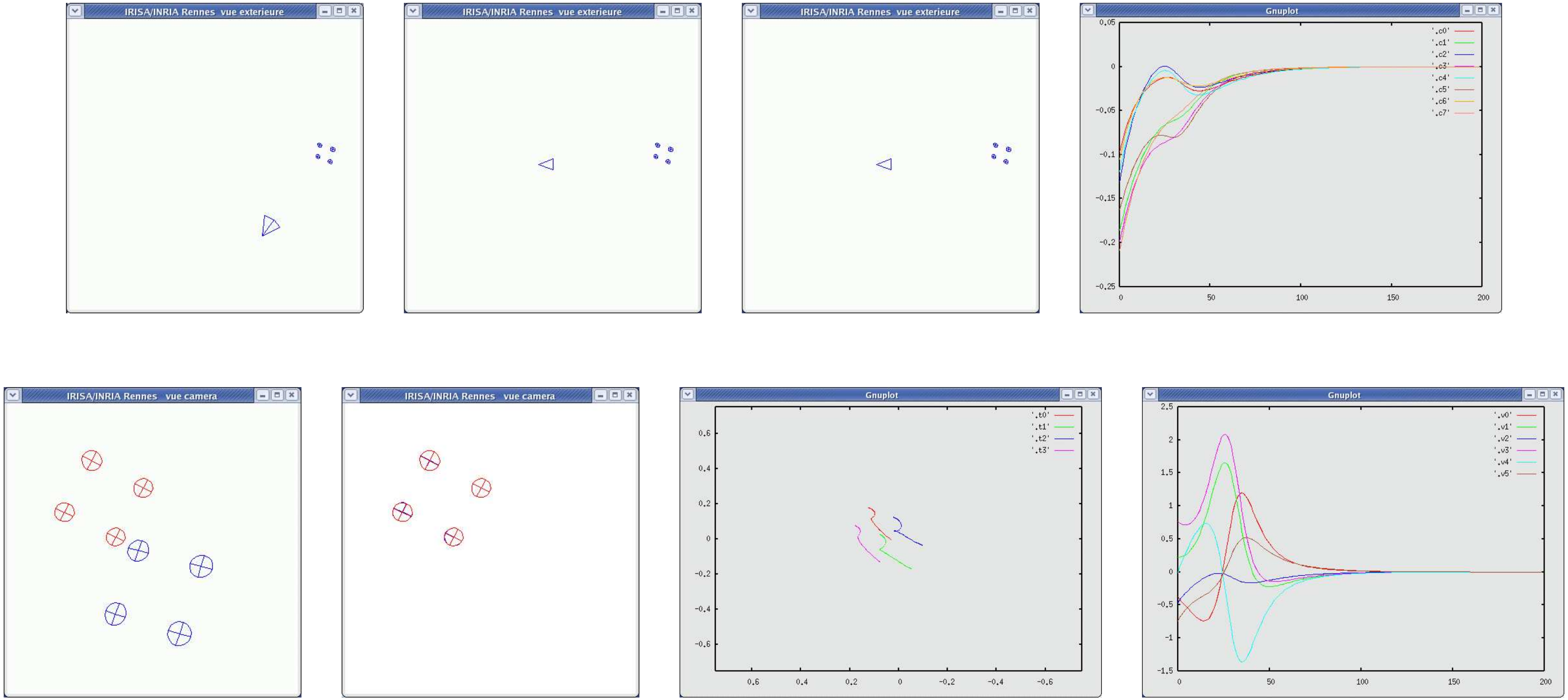


2D and 3D behavior directly linked by the choice of s (through $\mathbf{L_s}$ and $\widehat{\mathbf{L_s}}$)

Example 1: reaching a local minimum using $\widehat{\mathbf{L_s}}^{+}$



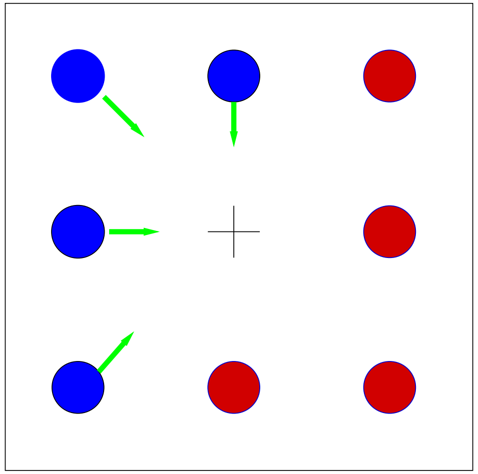
Example 2: reaching the global minimum using $\widehat{\mathbf{L_s}}^{+}_{|\mathbf{s}=\mathbf{s}^*}$



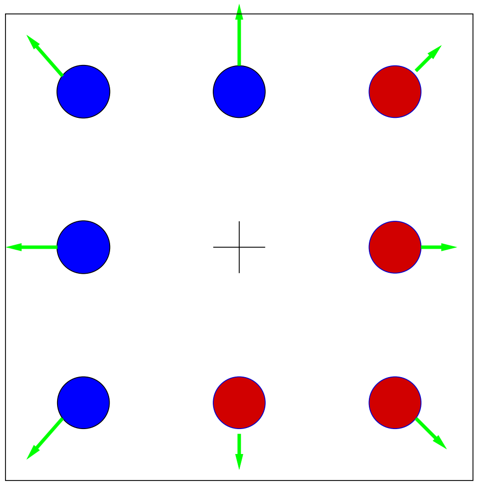
Example 3: reaching a singularity of $\mathbf{L_s}$

Example : rotation of 180° around the optical axis

s composed of image points coordinates

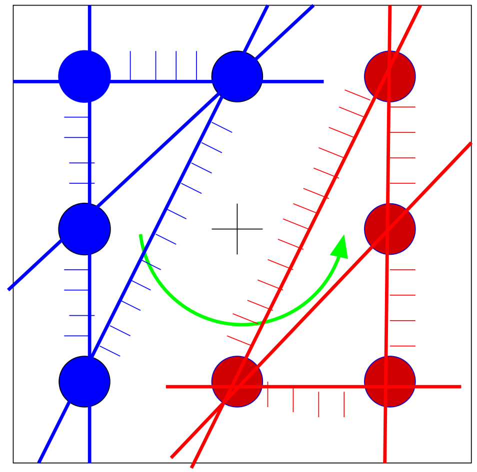


Using $\widehat{\mathbf{L_s}}^+$



Using $\widehat{\mathbf{L_s}_{|\mathbf{s}=\mathbf{s}^*}}^+$

Other choice



Using $\widehat{\mathbf{L_s}}^+$ or $\widehat{\mathbf{L_s}_{|\mathbf{s}=\mathbf{s}^*}}^+$

At singularity, rank $\mathbf{L_s}$ = 2.

Perfect behavior if s is composed of 2D straight lines parameters

(or composed of exact pose parameters)

Main goal: select adequate s for a given task

At least : select s such that rank $\mathbf{L_s} = m$ around $\mathbf{s}^*$
and such that Ker $\mathbf{L_s} = \mathcal{S}^*$ (virtual linkage:
plane-to-plane, bearing, ball joint, etc.)

At most (yet a dream for 6 dof!) : select s such that $\mathbf{L_s} = \mathbf{I}_6$

- one feature for each robot dof
- perfect decoupling, same behavior of s and **v**
- none singularity nor local minima (global stability)
- ideal condition number
- control of a linear system

⇒ 1) Modeling issues

- ▷ Basics
- ▷ 2D visual features
- ▷ 3D visual features
- ▷ Omni-directional vision sensor, vision + structured light

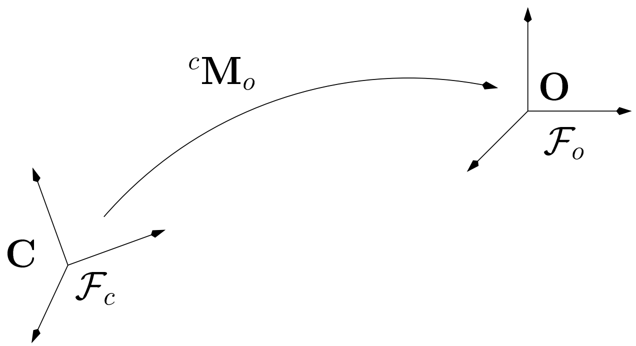
2) Control issues

3) Visual tracking

4) Applications

Change of frames

pose $\mathbf{p} \in SE_3$



$$\mathbf{X}_c = {}^c\mathbf{R}_o \, \mathbf{X}_o + {}^c\mathbf{t}_o$$

$\mathbf{X}_c$: coordinates of $\mathbf{X}$ in $\mathcal{F}_c$

$\mathbf{X}_o$: coordinates of $\mathbf{X}$ in $\mathcal{F}_o$

${}^c\mathbf{t}_o$: position of $\mathbf{O}$ in $\mathcal{F}_c$

${}^c\mathbf{R}_o$: rotation matrix between $\mathcal{F}_c$ and $\mathcal{F}_o$

$$\mathbf{R} = \cos\theta \, \mathbf{I}_3 + \sin\theta \, [\mathbf{u}]_\times + (1 - \cos\theta) \, \mathbf{u}\mathbf{u}^\top$$

$\mathbf{u}$: rotation axis ($\|\mathbf{u}\| = 1$) $\qquad$ θ : rotation angle around $\mathbf{u}$

$[\mathbf{u}]_\times$: skew symetric matrix related to $\mathbf{u}$: $[\mathbf{u}]_\times = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix}$

Kinematic screw (instantaneous velocity)

$\mathbf{v} = (\boldsymbol{\upsilon}, \boldsymbol{\omega})$: kinematic screw between the camera and the scene expressed at $\mathbf{C}$ in $\mathcal{F}_c$

$\boldsymbol{\omega}$: rotational velocity : $\quad [\boldsymbol{\omega}]_\times = {}^o\mathbf{R}_c^\top \, {}^o\dot{\mathbf{R}}_c = -{}^o\dot{\mathbf{R}}_c^\top \, {}^o\mathbf{R}_c$

$\boldsymbol{\upsilon}$: translational velocity at $\mathbf{C}$: $\quad \boldsymbol{\upsilon}(\mathbf{X}) = \boldsymbol{\upsilon}(\mathbf{C}) + [\boldsymbol{\omega}]_\times \, \mathbf{CX}$

To express $\mathbf{v}$ at $\mathbf{O}$ in $\mathcal{F}_o$: ${}^o\mathbf{v} = {}^o\mathbf{V}_c \, \mathbf{v}$ with ${}^o\mathbf{V}_c = \begin{bmatrix} {}^o\mathbf{R}_c & [{}^o\mathbf{t}_c]_\times {}^o\mathbf{R}_c \\ \mathbf{0}_3 & {}^o\mathbf{R}_c \end{bmatrix}$

We can decompose $\mathbf{v}$ as $\mathbf{v} = \mathbf{v}_c - \mathbf{v}_o$

where $\mathbf{v}_c$: camera kinematic screw, expressed at $\mathbf{C}$ in $\mathcal{F}_c$

$\mathbf{v}_o$: object kinematic screw, expressed at $\mathbf{C}$ in $\mathcal{F}_c$

The interaction matrix

A set $\mathbf{s}$ of k visual features is given by a function from SE_3 to $\mathbb{R}^k$:

$$\mathbf{s} = \mathbf{s}(\mathbf{p}(t))$$

where $\mathbf{p}(t)$ is the pose between the camera and the scene.
We get

$$\dot{\mathbf{s}} = \frac{\partial \mathbf{s}}{\partial \mathbf{p}} \, \dot{\mathbf{p}} = \mathbf{L}_{\mathbf{s}} \, \mathbf{v}$$

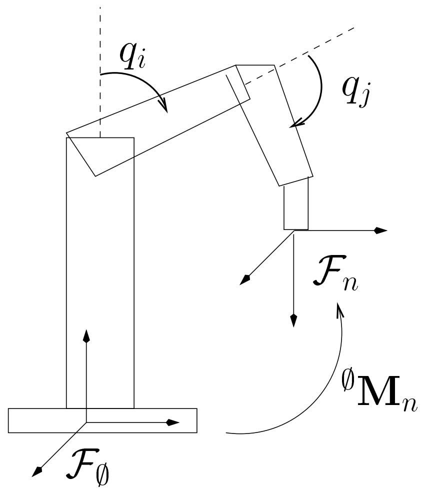
where $\mathbf{L}_{\mathbf{s}}$ is the **interaction matrix** related to $\mathbf{s}$
(Jacobian $\frac{\partial \mathbf{s}}{\partial \mathbf{p}} \approx \mathbf{L}_{\mathbf{s}}$ since $\mathbf{v} = \mathbf{M}_{\mathbf{p}} \, \dot{\mathbf{p}}$ with $\mathbf{M}_{\mathbf{p}} \approx \mathbf{I}_6$)

Using $\mathbf{v}_c$ and $\mathbf{v}_o$, we obtain :

$$\dot{\mathbf{s}} = \mathbf{L}_{\mathbf{s}} \, (\mathbf{v}_c - \mathbf{v}_o)$$

Robot Jacobian

Geometry of a robot arm defined by kinematics equations : $\mathbf{p}(t) = \mathbf{f}(\mathbf{q}(t))$



$\mathbf{q}$: joint positions ($\mathbf{q} \in \mathbb{R}^n$)
$\mathbf{p} \sim {}^{\emptyset}\mathbf{M}_n$: end-effector pose ($\mathbf{p} \in SE_3$)

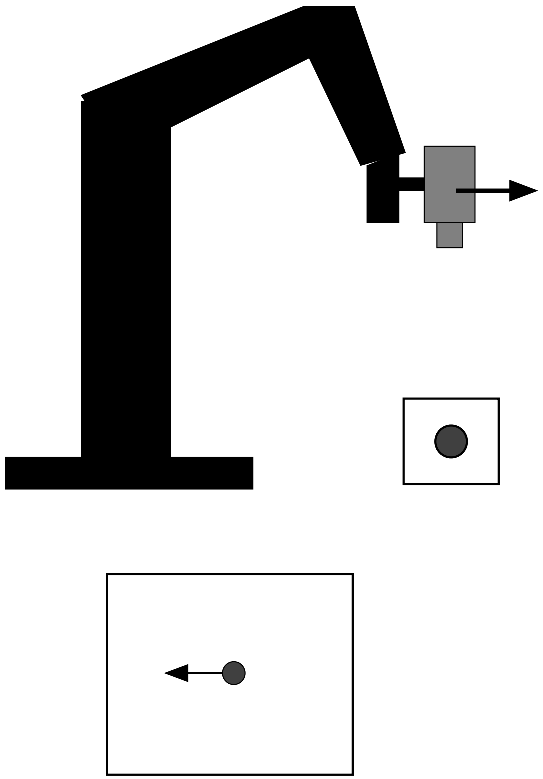
End-effector kinematic screw given by :

$$\mathbf{v}_n = {}^n\mathbf{J}_n(\mathbf{q})\ \dot{\mathbf{q}} \text{ where } {}^n\mathbf{J}_n(\mathbf{q}) = \mathbf{M}_{\mathbf{p}} \frac{\partial \mathbf{p}}{\partial \mathbf{q}} \text{ is the robot jacobian}$$

For velocity control, one computes $\dot{\mathbf{q}}^* = {}^n\mathbf{J}_n(\mathbf{q}^*)^{-1}\ \mathbf{v}_n^*$

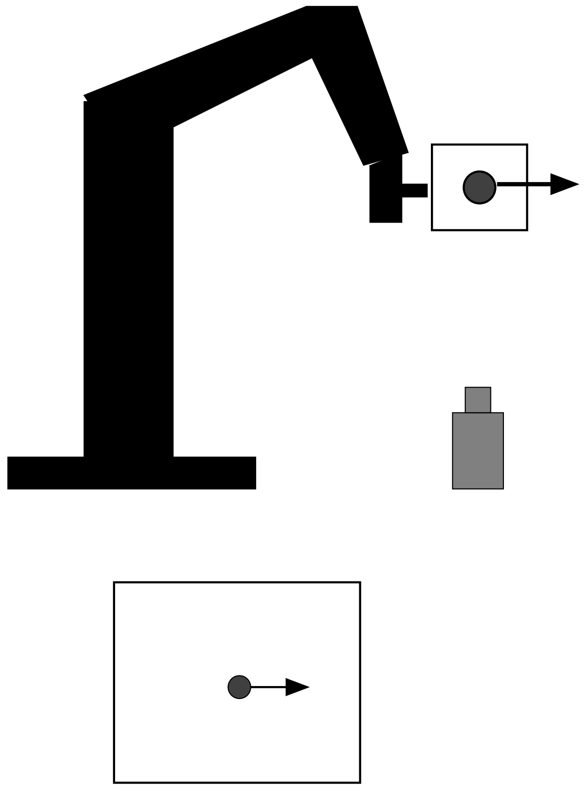
Robot singularities = $\{\mathbf{q}_s, \det\left({}^n\mathbf{J}_n(\mathbf{q}_s)\right) = 0\}$

Eye-in-Hand system



$$\dot{\mathbf{s}} = \mathbf{L_s}\,{}^c\mathbf{V}_n\,{}^n\mathbf{J}_n(\mathbf{q})\,\dot{\mathbf{q}} + \frac{\partial \mathbf{s}}{\partial t}$$

Eye-to-Hand system



$$\begin{aligned}\dot{\mathbf{s}} &= -\mathbf{L_s}\,{}^c\mathbf{V}_n\,{}^n\mathbf{J}_n(\mathbf{q})\,\dot{\mathbf{q}} + \frac{\partial \mathbf{s}}{\partial t}\\ &= -\mathbf{L_s}\,{}^c\mathbf{V}_\emptyset\,{}^\emptyset\mathbf{V}_n\,{}^n\mathbf{J}_n(\mathbf{q})\,\dot{\mathbf{q}} + \frac{\partial \mathbf{s}}{\partial t}\end{aligned}$$

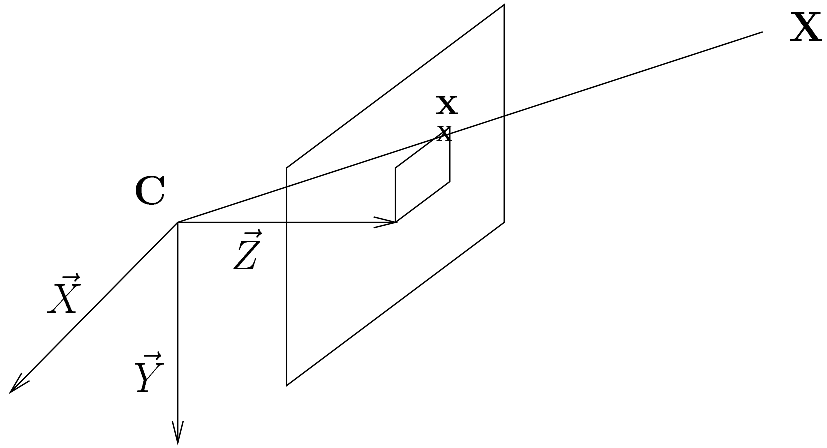
- Modeling issues
  - ▷ Basics
  - ⇒ 2D visual features
  - ▷ 3D visual features
  - ▷ Omni-directional vision sensor, vision + structured light

2D visual features: image point coordinates

Perspective projection : $\mathbf{x} = (x, y)$

$$x = X/Z \ , y = Y/Z$$

$$\Rightarrow \begin{cases} \dot{x} = [\ 1/Z \quad 0 \quad -X/Z^2\ ]\ \dot{\mathbf{X}} \\ \dot{y} = [\ 0 \quad 1/Z \quad -Y/Z^2\ ]\ \dot{\mathbf{X}} \end{cases}$$



Using a mobile camera and a fixed point:

$$\dot{\mathbf{X}} = \boldsymbol{v}(\mathbf{X}) = -\boldsymbol{v}(\mathbf{C}) - [\boldsymbol{\omega}]_\times \mathbf{CX} = \begin{bmatrix} -\mathbf{I}_3 & [\mathbf{X}]_\times \end{bmatrix}\ \mathbf{v}$$

We obtain:

$$\dot{\mathbf{x}} = \mathbf{L_x}\ \mathbf{v} \text{ where } \mathbf{L_x} = \begin{bmatrix} -1/Z & 0 & x/Z & xy & -(1+x^2) & y \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{bmatrix}$$

2D visual features: image point coordinates

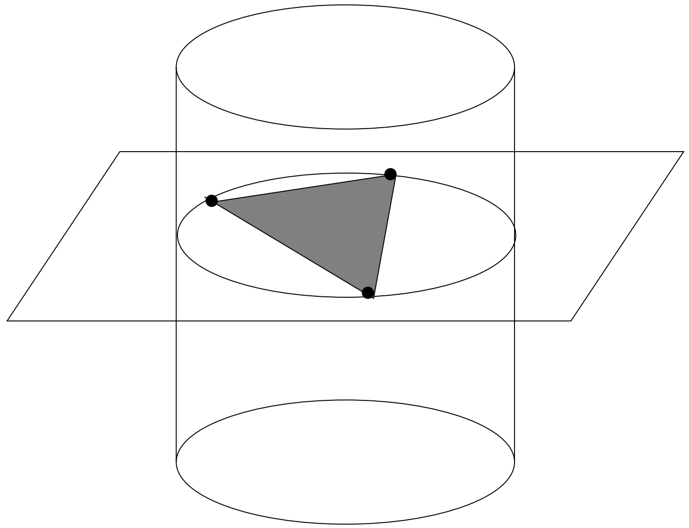
When $x = y = 0$ (principal point):

$$\mathbf{L_x} = \begin{bmatrix} -1/Z & 0 & 0 & 0 & -1 & 0 \\ 0 & -1/Z & 0 & 1 & 0 & 0 \end{bmatrix}$$

A single point is adequate to control v_x or ω_y and v_y or ω_x

Using several points (at least 3) allows to control the 6 dof.

$$\mathbf{s} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} \Rightarrow \mathbf{L_x} = \begin{bmatrix} \mathbf{L}_{\mathbf{x}_1} \\ \vdots \\ \mathbf{L}_{\mathbf{x}_n} \end{bmatrix}$$



Be careful to singularities in the interaction matrix $(\Rightarrow n \geq 4)$

Image point expressed in pixels

$$\mathbf{x} = (\; x \;,\; y \;) = \text{ image point coordinates expressed in meters}$$
$$\mathbf{x}_p = (\; x_p, y_p \;) = \text{ image point coordinates expressed in pixels}$$

$$x_p = x_c + f_x\, x \quad , \quad y_p = y_c + f_y\, y$$

where $\mathbf{x}_c = (x_c, y_c)$ = principal point

and f_x, f_y = ratio between focal length f and pixel size.

$$\Rightarrow \mathbf{L}_{\mathbf{x}_p} = \begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix} \mathbf{L}_{\mathbf{x}}$$
$$= \begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix} \begin{bmatrix} -1/Z & 0 & x/Z & xy & -(1+x^2) & y \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{bmatrix}$$

where $x = (x_p - x_c)/f_x \quad , \quad y = (y_p - y_c)/f_y$

Useful for stability analysis wrt. calibration errors

Image point expressed in pixels

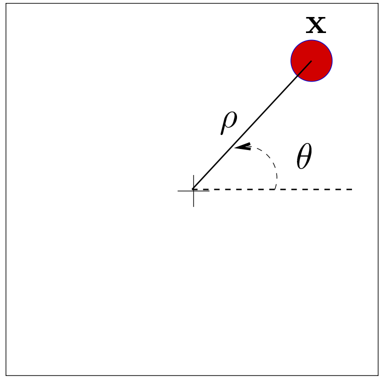
when focal length is an available supplementary dof

$$x_p = x_c + \frac{f}{l_x}\, x \quad , \quad y_p = y_c + \frac{f}{l_y}\, y$$

$$\begin{aligned}\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} &= \mathbf{L}_{\mathbf{x}_p}\, \mathbf{v} + \begin{bmatrix} x/l_x \\ y/l_y \end{bmatrix} \dot{f} = \mathbf{L}_{\mathbf{x}_p}\, \mathbf{v} + \begin{bmatrix} (x_p - x_c)/f \\ (y_p - y_c)/f \end{bmatrix} \dot{f} \\ &= \begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix} \begin{bmatrix} -1/Z & 0 & x/Z & xy & -(1+x^2) & y & x/f \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x & y/f \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \dot{f} \end{bmatrix}\end{aligned}$$

Useful redundancy wrt. motion along the optical axis v_z

Image point in cylindrical coordinates [Iwatsuki 02]



Use of (ρ, θ) for an image points instead of (x, y):

$$\rho = \sqrt{x^2 + y^2}\ ,\ \ \theta = \arctan \frac{y}{x}$$

Corresponding interaction matrix:

$$\mathbf{L}_\rho = \left[\ \frac{-\cos\theta}{Z} \quad \frac{-\sin\theta}{Z} \quad \frac{\rho}{Z} \quad (1+\rho^2)\sin\theta \quad -(1+\rho^2)\cos\theta \quad 0\ \right]$$

$$\mathbf{L}_\theta = \left[\ \frac{\sin\theta}{\rho Z} \quad \frac{-\cos\theta}{\rho Z} \quad 0 \quad \frac{\cos\theta}{\rho} \quad \frac{\sin\theta}{\rho} \quad -1\ \right]$$

Better decoupling between v_z and ω_z

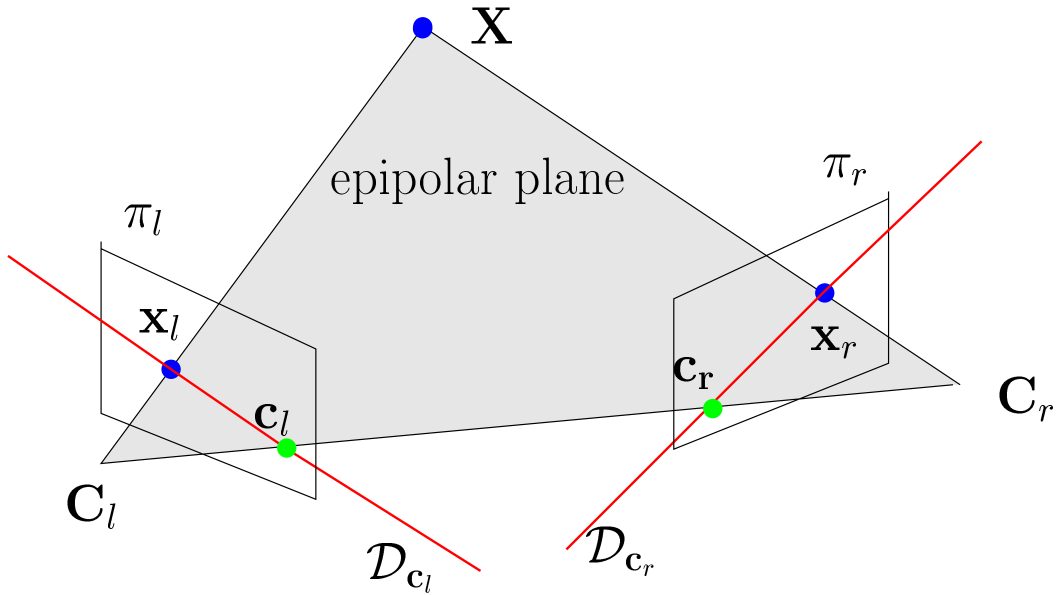
Be careful for the principal point ($x = y = \rho = 0,\ \theta$ undefined)

Image point for a stereovision system

$$\dot{\mathbf{x}}_l = \mathbf{L}_{\mathbf{x}_l}\,\mathbf{v}_l$$
$$\dot{\mathbf{x}}_r = \mathbf{L}_{\mathbf{x}_r}\,\mathbf{v}_r$$

$$\Rightarrow \begin{bmatrix} \dot{\mathbf{x}}_l \\ \dot{\mathbf{x}}_r \end{bmatrix} = \mathbf{L}_{\mathbf{x}_l\mathbf{x}_r}\,\mathbf{v}_c$$

where $\mathbf{L}_{\mathbf{x}_l\mathbf{x}_r} = \begin{bmatrix} \mathbf{L}_{\mathbf{x}_l}\,{}^l\mathbf{V}_c \\ \mathbf{L}_{\mathbf{x}_r}\,{}^r\mathbf{V}_c \end{bmatrix}$



$\mathbf{L}_{\mathbf{x}_l\mathbf{x}_r}$ is of rank 3 because of the epipolar constraint

- Generalization to multi-cameras systems immediate

2D visual features: geometrical primitives

P_o : configuration of an *object feature* parameterized by $\mathbf{P_o}$

$p_i = \pi(P_o)$: configuration of an *image feature* parameterized by $\mathbf{p_i}$

Noting $\mathbf{P_o} = \varphi(P_o)$ and $\mathbf{p_i} = \psi(p_i)$, we get

$$\mathbf{p_i} = \nu(\mathbf{P_o}) = \psi \circ \pi \circ \varphi^{-1}(\mathbf{P_o})$$

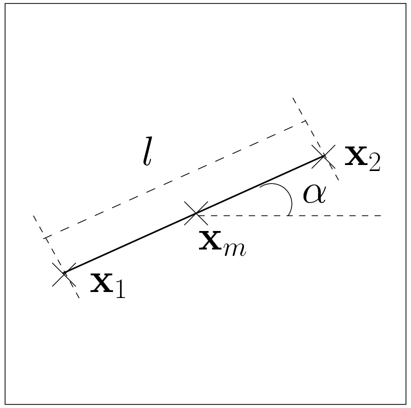
We also have $\mathbf{P_o} = \varphi \circ \delta(\mathbf{p}) \Rightarrow \mathbf{p_i} = \psi \circ \pi \circ \delta(\mathbf{p}) = \nu \circ \varphi \circ \delta(\mathbf{p})$

$$\begin{array}{ccccccc}
W \subseteq SE_3 & \longrightarrow & U \subseteq \mathcal{P}_o & \longrightarrow & V \subseteq \mathcal{P}_i & & \\
(\mathbf{p}) & \delta & (P_o) & \pi & (p_i) & & \\
 & & \Big\downarrow \varphi & & \Big\downarrow \psi & & \\
 & & \mathbb{R}^n & \longrightarrow & \mathbb{R}^m & \longrightarrow & \mathbb{R}^k \\
 & & (\mathbf{P_o}) & \nu = \psi \circ \pi \circ \varphi^{-1} & (\mathbf{p_i}) & \sigma & (\mathbf{s})
\end{array}$$

Finally $\mathbf{s} = \sigma(\mathbf{p_i}) \Rightarrow \mathbf{L_s} = \frac{\partial \mathbf{s}}{\partial \mathbf{p_i}} \frac{\partial \mathbf{p_i}}{\partial \mathbf{P_o}} \mathbf{L_{P_o}}$

2D visual features: case of a segment

$$\mathbf{s} = \begin{bmatrix} x_m \\ y_m \\ l \\ \alpha \end{bmatrix} \text{ with } \begin{cases} x_m = (x_1 + x_2)/2 \\ y_m = (y_1 + y_2)/2 \\ l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ \alpha = \arctan(y_1 - y_2)/(x_1 - x_2) \end{cases}$$



$$\begin{bmatrix} \mathbf{L}_{x_m} \\ \mathbf{L}_{y_m} \\ \mathbf{L}_{l} \\ \mathbf{L}_{\alpha} \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ \Delta x/l & \Delta y/l & -\Delta x/l & -\Delta y/l \\ -\Delta x/l^2 & \Delta x/l^2 & \Delta y/l^2 & -\Delta x/l^2 \end{bmatrix} \begin{bmatrix} \mathbf{L}_{x_1} \\ \mathbf{L}_{y_1} \\ \mathbf{L}_{x_2} \\ \mathbf{L}_{y_2} \end{bmatrix}$$

with $\Delta x = x_1 - x_2$ and $\Delta y = y_1 - y_2$.

Using $\begin{cases} x_1 = x_m + l\cos\alpha/2, \; y_1 = y_m + l\sin\alpha/2 \\ x_2 = x_m - l\cos\alpha/2, \; y_2 = y_m - l\sin\alpha/2 \end{cases}$, we get $\mathbf{L_s}(\mathbf{s}, Z_1, Z_2)$.

2D visual features: case of a segment

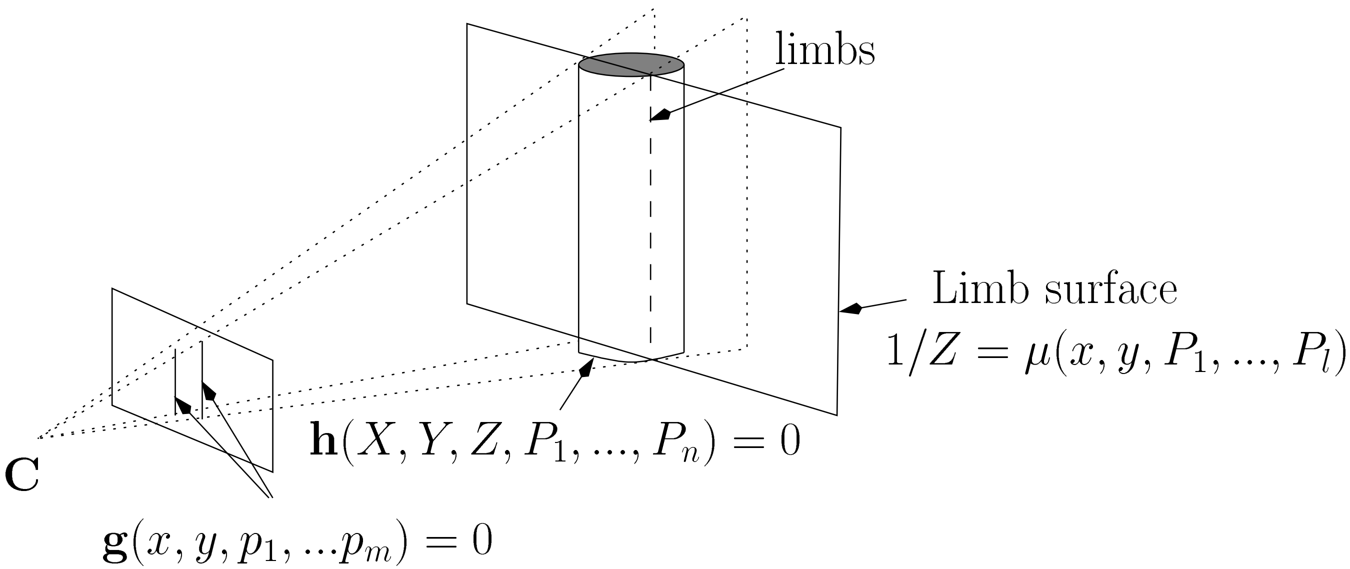
$$\begin{array}{lllllllll}
\mathbf{L}_{x_m} & = [& -1/Z_m & 0 & x_m/Z_m + D\epsilon_x & x_m w_x & -1 - x_m w_y & y_m &] \\
\mathbf{L}_{y_m} & = [& 0 & -1/Z_m & y_m/Z_m + D\epsilon_y & 1 + y_m w_x & y_m w_y & -x_m &] \\
\mathbf{L}_{l} & = [& -Dc & -Ds & l/Z_m + D\epsilon_l & l w_x & l w_y & 0 &] \\
\mathbf{L}_{\alpha} & = [& Ds/l & -Dc/l & D\epsilon_\alpha & \alpha w_x & \alpha w_y & -1 &]
\end{array}$$

with $1/Z_m = (1/Z_1 + 1/Z_2)/2$ and $D = 1/Z_1 - 1/Z_2$

Nice triangular form for a segment parallel to the image plane ($D = 0$)

Exercise: Better parameterization: $\mathbf{s} = (x_m/l, y_m/l, 1/l, \alpha)$

Modeling a geometrical primitive



3D primitive : $\mathbf{h}(\mathbf{X}, \mathbf{P_o}) = 0$

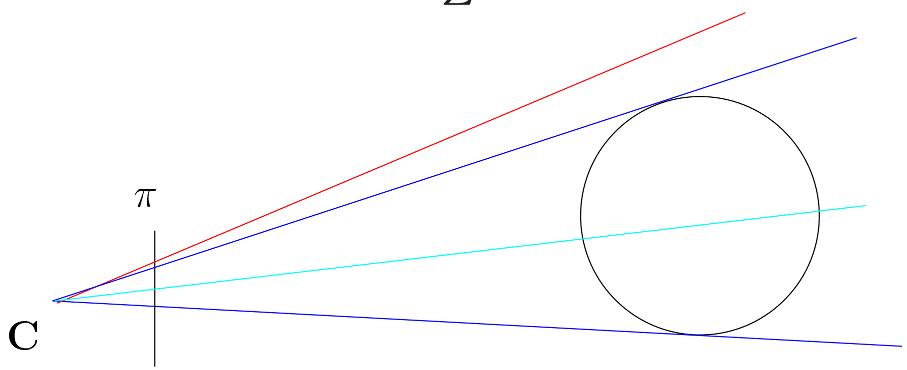
2D primitive : $\mathbf{g}(\mathbf{x}, \mathbf{p_i}) = 0$

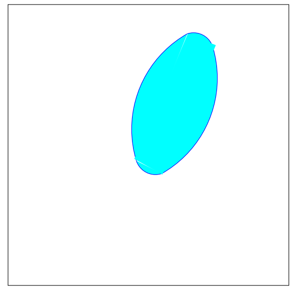
Limb surface : $\Rightarrow 1/Z = \mu(\mathbf{x}, \mathbf{P_o})$

2D visual features : case of the sphere

3D primitive : $\mathbf{h}(\mathbf{X}, \mathbf{P_o}) = (X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2 - R^2 = 0$

$x = X/Z \;,\; y = Y/Z \Rightarrow K\frac{1}{Z^2} - 2(X_0 x + Y_0 y + Z_0)\frac{1}{Z} + x^2 + y^2 + 1 = 0$





$\Delta = 0 \Rightarrow \frac{1}{Z} = \mu(\mathbf{x}, \mathbf{P_o}) = \frac{X_0}{K}x + \frac{Y_0}{K}y + \frac{Z_0}{K}$ (eq. of a 3D plane)

$\Delta = 0 \Leftrightarrow (X_0 x + Y_0 y + Z_0)^2 - K(x^2 + y^2 + 1) = 0$

$\Leftrightarrow \mathbf{g}(\mathbf{x}, \mathbf{p_i}) = x^2 + a_1 y^2 + 2a_2 xy + 2a_3 x + 2a_4 y + a_5 = 0$

Image of a sphere = ellipse (circle if $X_0 = Y_0 = 0$)

Direct computation of the interaction matrix

$$\mathbf{L_{p_i}} = \frac{\partial \mathbf{p_i}}{\partial \mathbf{P_o}} \mathbf{L_{P_o}}$$

$$\left\{ \begin{array}{l} a_1 = (R^2 - X_0^2 - Z_0^2)/(R^2 - Y_0^2 - Z_0^2) \\ \\ a_2 = X_0 Y_0/(R^2 - Y_0^2 - Z_0^2) \\ ... \end{array} \right. \Rightarrow \frac{\partial \mathbf{p_i}}{\partial \mathbf{P_o}} : \left\{ \begin{array}{l} \dot{a}_1 = (-2X_0\dot{X}_0 - 2Z_0\dot{Z}_0)/(R^2 - Y_0^2 - Y_0^2) - ... \\ \dot{a}_2 = ... \\ ... \end{array} \right.$$

$$\mathbf{L_{P_o}} : \ \dot{\mathbf{X}}_0 = \begin{bmatrix} \dot{X}_0 \\ \dot{Y}_0 \\ \dot{Z}_0 \end{bmatrix} = \left[\ -\mathbb{I}_3 \ \ [\mathbf{X}_0]_\times \ \right] \ \mathbf{v} \ \Rightarrow \ \mathbf{L_{P_o}} = \left[\ -\mathbb{I}_3 \ \ [\mathbf{X}_0]_\times \ \right]$$

We always have rank $\mathbf{L_{p_i}}$ = rank $\mathbf{L_{P_o}}$ = 3

Results in $\mathbf{L_{p_i}}$ are function of 3D data $\mathbf{P_o} = (X_0, Y_0, Z_0, R)$

Other (better) method

$$\mathbf{g}(\mathbf{x}, \mathbf{p_i}) = 0 \Rightarrow \dot{\mathbf{g}}(\mathbf{x}, \mathbf{p_i}) = 0 \Leftrightarrow \frac{\partial \mathbf{g}}{\partial \mathbf{p_i}}(\mathbf{x}, \mathbf{p_i})\ \dot{\mathbf{p_i}} = -\frac{\partial \mathbf{g}}{\partial \mathbf{x}}(\mathbf{x}, \mathbf{p_i})\ \dot{\mathbf{x}}\ ,\ \forall \mathbf{x} \in p_i$$

We have $\dot{\mathbf{x}} = \mathbf{L}_{xy}(\mathbf{x}, 1/Z)\ \mathbf{v} = \mathbf{L}_{xy}(\mathbf{x}, \mathbf{P_o})\ \mathbf{v}$

$$\Rightarrow \frac{\partial \mathbf{g}}{\partial \mathbf{p_i}}(\mathbf{x}, \mathbf{p_i})\ \dot{\mathbf{p_i}} = -\frac{\partial \mathbf{g}}{\partial \mathbf{x}}(\mathbf{x}, \mathbf{p_i})\ \mathbf{L}_{xy}(\mathbf{x}, \mathbf{P_o})\ \mathbf{v}\ ,\ \forall \mathbf{x} \in p_i$$

If dim($\mathbf{p_i}$) = dim(p_i) = m, using m points of p_i,
we obtain a $m \times m$ linear system:

$$\mathbf{L_{p_i}}(\mathbf{p_i}, \mathbf{P_o}) = \begin{bmatrix} \boldsymbol{\alpha}_1(\mathbf{p_i}) \\ \vdots \\ \boldsymbol{\alpha}_m(\mathbf{p_i}) \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{\beta}_1(\mathbf{p_i}, \mathbf{P_o}) \\ \vdots \\ \boldsymbol{\beta}_m(\mathbf{p_i}, \mathbf{P_o}) \end{bmatrix} \text{ with } \begin{cases} \boldsymbol{\alpha}_i(\mathbf{p_i}) = \frac{\partial \mathbf{g}}{\partial \mathbf{p_i}}(\mathbf{x}_i, \mathbf{p_i}), i = 1 \text{ to } m \\ \boldsymbol{\beta}_i(\mathbf{p_i}, \mathbf{P_o}) = -\frac{\partial \mathbf{g}}{\partial \mathbf{x}}(\mathbf{x}_i, \mathbf{p_i})\mathbf{L}_{xy}(\mathbf{x}_i, \mathbf{P_o}), \\ \qquad i = 1 \text{ to } m \end{cases}$$

2D visual features : case of straight lines

$$\mathbf{h}(\mathbf{X}, \mathbf{P_o}) = \begin{cases} h_1 = A_1 X + B_1 Y + C_1 Z = 0 \\ h_2 = A_2 X + B_2 Y + C_2 Z + D_2 = 0 \end{cases}$$

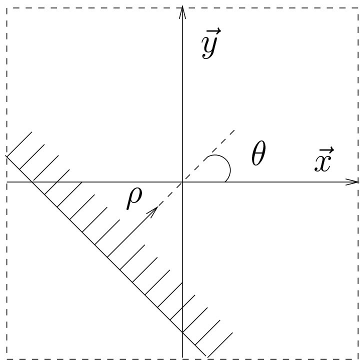
We obtain :

- function $\frac{1}{Z} = \mu(\mathbf{x}, \mathbf{P_o})$ from h_2 : $1/Z = Ax + By + C$
  with $A = -A_2/D_2, B = -B_2/D_2, C = -C_2/D_2$
- 2D straight line $\mathcal{D}$: $A_1 x + B_1 y + C_1 = 0$

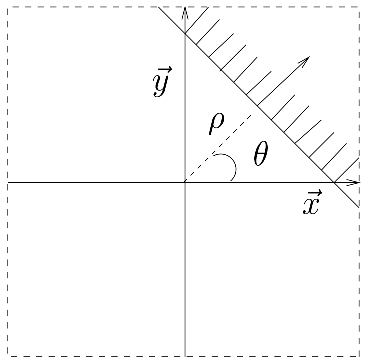
Minimal parameterization $\mathbf{p_i} = (\rho, \theta)$

$\mathbf{g}(\mathbf{x}, \mathbf{p_i}) = x \cos\theta + y \sin\theta - \rho = 0$

with $\theta = \arctan\left(B_1/A_1\right)$
and $\rho = -C_1/\sqrt{A_1^2 + B_1^2}$.



$\rho < 0$



$\rho > 0$

Computation of the interaction matrix

$$\dot{\mathbf{g}}(\mathbf{x}, \mathbf{p_i}) = 0 \Rightarrow \dot\rho + (x \sin\theta - y\cos\theta)\,\dot\theta = \dot x \cos\theta + \dot y \sin\theta\ ,\ \forall \mathbf{x} \in \mathcal{D}$$

From $\mathbf{g}(\mathbf{x}, \mathbf{p_i}) = 0$, we write $x = f(y, \rho, \theta)$ to get :

$$(-\dot\theta/\cos\theta)\ y + (\dot\rho + \rho\tan\theta\ \dot\theta) = y\ K_1(\mathbf{p_i}, \mathbf{P_o})\ \mathbf{v} + K_2(\mathbf{p_i}, \mathbf{P_o})\ \mathbf{v},\ \forall y \in \mathbb{R}$$

We obtain $\left\{ \begin{array}{rcl} \dot\theta & = & -K_1(\mathbf{p_i}, \mathbf{P_o})\ \cos\theta\ \mathbf{v} \\ \dot\rho & = & (K_2(\mathbf{p_i}, \mathbf{P_o}) + K_1(\mathbf{p_i}, \mathbf{P_o})\rho\sin\theta)\ \mathbf{v} \end{array} \right.$

$$\Rightarrow \begin{array}{l} \mathbf{L}_\rho = [\ \lambda_\rho c\theta\ \ \lambda_\rho s\theta\ \ -\lambda_\rho \rho\ \ (1+\rho^2)s\theta\ \ -(1+\rho^2)c\theta\ \ 0\ ] \\ \mathbf{L}_\theta = [\ \lambda_\theta c\theta\ \ \lambda_\theta s\theta\ \ -\lambda_\theta \rho\ \ -\rho c\theta\ \ -\rho s\theta\ \ -1\ ] \end{array}$$

with $\lambda_\rho = -(A\rho c\theta + B\rho s\theta + C)$ and $\lambda_\theta = Bc\theta - As\theta$

Exercise : obtain the same result using 2 points of $\mathcal{D}$,

for example $(\rho\cos\theta, \rho\sin\theta)$ and $(\rho\cos\theta + \sin\theta, \rho\sin\theta - \cos\theta)$

2D visual features : case of a circle

$$\mathbf{h}(\mathbf{X},\mathbf{P_o}) = \begin{cases} h_1 = (X-X_0)^2+(Y-Y_0)^2+(Z-Z_0)^2-R^2=0 \\ h_2 = \alpha(X-X_0)+\beta(Y-Y_0)+\gamma(Z-Z_0)=0 \end{cases}$$

From $h_2 : \dfrac{1}{Z} = Ax + By + C$ with $\begin{cases} A &= \alpha/(\alpha X_0+\beta Y_0+\gamma Z_0) \\ B &= \beta/(\alpha X_0+\beta Y_0+\gamma Z_0) \\ C &= \gamma/(\alpha X_0+\beta Y_0+\gamma Z_0) \end{cases}$

Using $h_1 : \mathbf{g}(\mathbf{x},\mathbf{p_i}) = x^2 + a_1 y^2 + 2a_2 xy + 2a_3 x + 2a_4 y + a_5 = 0$

Image of a circle = **ellipse** and a circle if $a_1 = 1$ et $a_2 = 0$, that is

$A = B = 0$ or $\begin{cases} A = 2X_0/(X_0^2+Y_0^2+Z_0^2-R^2)\ , \\ B = 2Y_0/(X_0^2+Y_0^2+Z_0^2-R^2) \end{cases}$

2D visual features : case of a circle

Better parameterization for ellipses : $\mathbf{p_i} = \left(x_g, y_g, \mu_{20}, \mu_{11}, \mu_{02}\right)$

Centered **moments** : $\mu_{ij} = \int\int_{\mathcal{D}(t)} (x - x_g)^i (y - y_g)^j dxdy$

$\mathbf{L_{p_i}}$ is always of full rank 5, but for the centered circle ($x_g = y_g = \mu_{11} = A = B = 0, \mu_{20} = \mu_{02} = r^2$) where:

$$\mathbf{L_{p_i}} = \begin{bmatrix} -1/Z_0 & 0 & 0 & 0 & -1-r^2 & 0 \\ 0 & -1/Z_0 & 0 & 1+r^2 & 0 & 0 \\ 0 & 0 & 2r^2/Z_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2r^2/Z_0 & 0 & 0 & 0 \end{bmatrix}$$

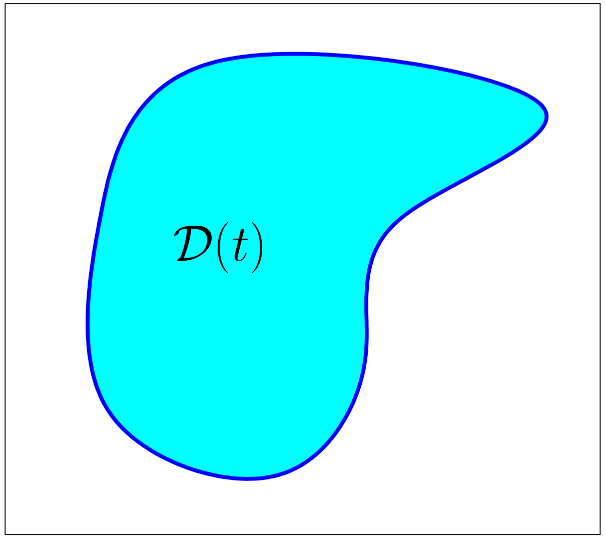
Summary

3D primitives	2D primitives	Parameterization
point	point	(x, y) or (ρ, θ)
segment	segment	(x_1, y_1, x_2, y_2) $(x_m/l, y_m/l, 1/l, \alpha)$
straight line	straight line	(ρ, θ)
circle	ellipse	$(x_g, y_g, \mu_{20}, \mu_{11}, \mu_{02})$
sphere	ellipse	$(x_g, y_g, a = \pi r^2)$
cylinder	2 straight lines	$(\rho_1, \theta_1, \rho_2, \theta_2)$

$\mathbf{L_s}$ also available for distance from a point to a straight line, angle between two straight lines, etc.

Moments definition

moments: $m_{ij} = \iint_{\mathcal{D}(t)} x^i y^j \, dx dy$



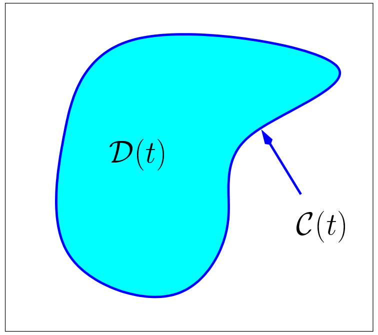
widely used in pattern recognition [Hu 1962]

related to intuitive features:

- area a : m_{00}
- center of gravity $\mathbf{x_g}$: from m_{10} and m_{01}
- object orientation α and inertial axes : from m_{20}, m_{11}, and m_{02}
- skewness : from m_{30} and m_{03}

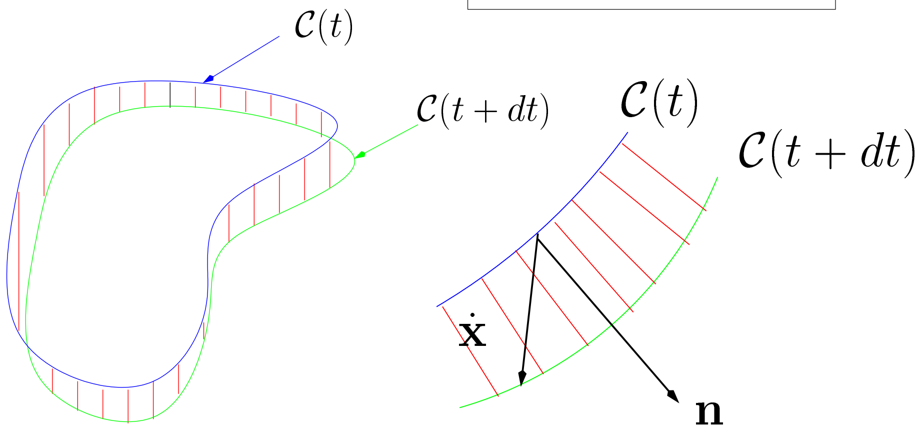
Interest in visual servoing: objects of complex or simple shape

Computation of the interaction matrix $\mathbf{L}_{m_{ij}}$



$$m_{ij}(t) = \iint_{\mathcal{D}(t)} f(x,y)dxdy \quad (f(x,y) = x^i y^j)$$

$$\Rightarrow \dot{m}_{ij} = \oint_{\mathcal{C}(t)} f(x,y)\ \dot{\mathbf{x}}^\top \mathbf{n}\ dl$$



Using Green's theorem :

$$\dot{m}_{ij} = \iint_{\mathcal{D}(t)} \mathrm{div}[f(x,y)\dot{\mathbf{x}}]\ dxdy$$

$$\Rightarrow \dot{m}_{ij} = \iint_{\mathcal{D}} [\frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y} + f(x,y)(\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y})]dxdy$$

$$= \iint_{\mathcal{D}} [ix^{i-1}y^j\dot{x} + jx^iy^{j-1}\dot{y} + x^iy^j(\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y})]dxdy$$

Computation of the interaction matrix $\mathbf{L}_{m_{ij}}$

$$\dot{m}_{ij} = \iint_{\mathcal{D}} [i x^{i-1} y^j \dot{x} + j x^i y^{j-1} \dot{y} + x^i y^j (\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y})] dx dy$$

$$\begin{cases} \dot{x} = \begin{bmatrix} -1/Z & 0 & x/Z & xy & -1-x^2 & y \end{bmatrix} \mathbf{v} \\ \dot{y} = \begin{bmatrix} 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{bmatrix} \mathbf{v} \end{cases}$$

For planar object: $1/Z = Ax + By + C$ from which we deduce:

$$\begin{cases} \frac{\partial \dot{x}}{\partial x} = \begin{bmatrix} -A & 0 & (2Ax + By + C) & y & -2x & 0 \end{bmatrix} \mathbf{v} \\ \\ \frac{\partial \dot{y}}{\partial y} = \begin{bmatrix} 0 & -B & (Ax + 2By + C) & 2y & -x & 0 \end{bmatrix} \mathbf{v} \end{cases}$$

($A = B = 0$ when the object is parallel to the image plane)

Interaction matrix for moments

$$\mathbf{L}_{m_{ij}} = \left[\, m_{vx} \ m_{vy} \ m_{vz} \ m_{wx} \ m_{wy} \ m_{wz} \,\right]$$

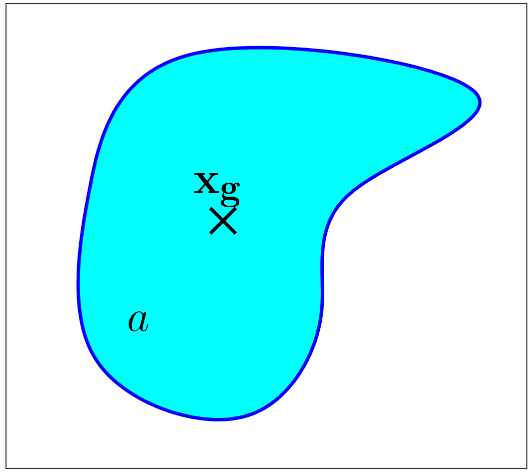
with
$$\begin{cases} m_{vx} = -i\left(Am_{ij} + Bm_{i-1,j+1} + Cm_{i-1,j}\right) - Am_{ij} \\ m_{vy} = -j\left(Am_{i+1,j-1} + Bm_{ij} + Cm_{i,j-1}\right) - Bm_{ij} \\ m_{vz} = (i+j+3)(Am_{i+1,j} + Bm_{i,j+1} + Cm_{ij}) - Cm_{ij} \\ m_{wx} = \quad (i+j+3)\, m_{i,j+1} + j\, m_{i,j-1} \\ m_{wy} = -(i+j+3)\, m_{i+1,j} - i\, m_{i-1,j} \\ m_{wz} = i\, m_{i-1,j+1} - j\, m_{i+1,j-1} \end{cases}$$

$\mathbf{L}_{m_{ij}}$ can be computed from moments of order less than $i+j+2$ and from plane parameters A, B and C for translational components.

Area, Center of gravity

Area $a = m_{00}$

$$\mathbf{L}_a = [\ -aA \quad -aB \quad a(3/Z_g - C) \quad 3ay_g \quad -3ax_g \quad 0\ ]$$



Object cog: $x_g = m_{10}/m_{00}, y_g = m_{01}/m_{00}$

$$\mathbf{L}_{x_g} = [\ -1/Z_g \quad 0 \quad x_g/Z_g + \epsilon_1 \quad x_g y_g + 4n_{11} \quad -(1 + x_g^2 + 4n_{20}) \quad y_g\ ]$$
$$\mathbf{L}_{y_g} = [\ 0 \quad -1/Z_g \quad y_g/Z_g + \epsilon_2 \quad 1 + y_g^2 + 4n_{02} \quad -x_g y_g - 4n_{11} \quad -x_g\ ]$$

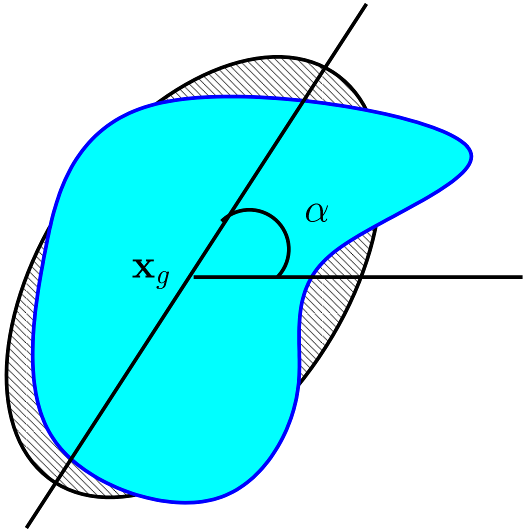
(generalization of the pure point case)

$$\left\{\begin{array}{l} 1/Z_g = Ax_g + By_g + C \\ \epsilon_1 = 4(An_{20} + Bn_{11}) \\ \epsilon_2 = 4(An_{11} + Bn_{02}) \end{array}\right. \quad n_{ij} = \mu_{ij}/a \text{ with } \left\{\begin{array}{l} \mu_{20} = m_{20} - ax_g^2 \\ \mu_{02} = m_{02} - ay_g^2 \\ \mu_{11} = m_{11} - ax_g y_g \end{array}\right.$$

Centered moments $\mu_{ij} = \iint_{\mathcal{D}} (x - x_g)^i (y - y_g)^j \, dx dy$

$$\mathbf{L}_{\mu_{ij}} = \left[\mu_{vx} \;\; \mu_{vy} \;\; \mu_{vz} \;\; \mu_{wx} \;\; \mu_{wy} \;\; \mu_{wz} \right]$$

$$\mu_{vx} = \mu_{vy} = 0 \text{ when } A = B = 0$$



Object orientation $\alpha = \frac{1}{2} \arctan\left(\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right)$

$$\mathbf{L}_{\alpha} = \left[\alpha_{vx} \;\; \alpha_{vy} \;\; \alpha_{vz} \;\; \alpha_{wx} \;\; \alpha_{wy} \;\; -1 \right]$$

$$\begin{cases} \alpha_{vx} = \alpha_{vy} = \alpha_{vz} = 0 \text{ when } A = B = 0 \\ \alpha_{wx} = 0 \text{ when } x_g = y_g = 0 \text{ and } \mu_{03} = \mu_{12} = \mu_{21} = 0 \\ \alpha_{wy} = 0 \text{ when } x_g = y_g = 0 \text{ and } \mu_{30} = \mu_{21} = \mu_{12} = 0 \end{cases}$$

Cooking moments

• Normalization of $\mathbf{s} = (x_g, y_g, a)$:

$$\mathbf{s}_n = (x_n, y_n, a_n) \text{ with } a_n = 1/\sqrt{a}, x_n = x_g/\sqrt{a}, y_n = y_g/\sqrt{a}$$

$$\Rightarrow \mathbf{L}^{||}_{\mathbf{x}_n} = \begin{bmatrix} -\kappa & 0 & 0 & a_n\epsilon_{11} & -a_n(1+\epsilon_{12}) & y_n \\ 0 & -\kappa & 0 & a_n(1+\epsilon_{21}) & -a_n\epsilon_{11} & -x_n \\ 0 & 0 & -\kappa & -3y_n/2 & 3x_n/2 & 0 \end{bmatrix} \quad (A{=}B{=}0)$$

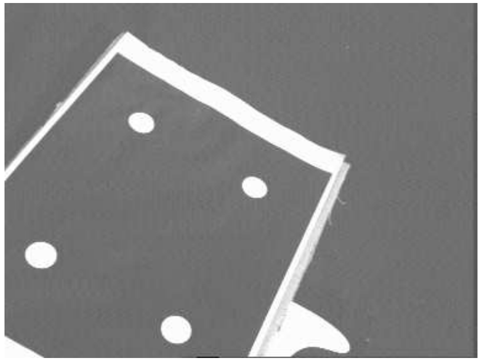
Pure image-based, but so near from position-based...

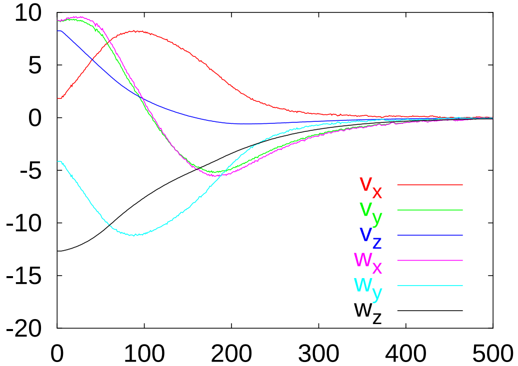
• Moment invariants: some combinations of moments are invariant to 2D translations, scale, and 2D rotation, so that by selecting adequately two of such combinations r_i and r_j:

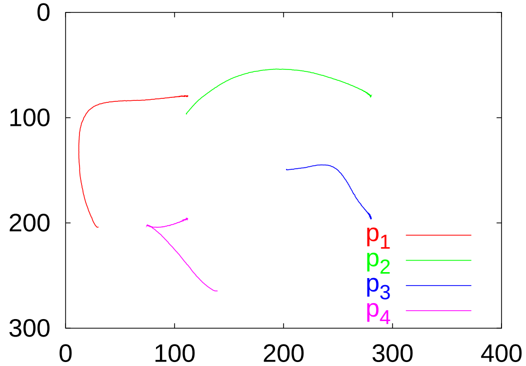
$$\mathbf{L}^{||}_{r_i r_j} = \begin{bmatrix} 0 & 0 & 0 & r_{i_{wx}} & r_{i_{wy}} & 0 \\ 0 & 0 & 0 & r_{j_{wx}} & r_{j_{wy}} & 0 \end{bmatrix} \quad (A{=}B{=}0)$$

Interest of cooking visual features

Using the coordinates of 4 points for s
(cond $\mathbf{L_s} \approx 180$)

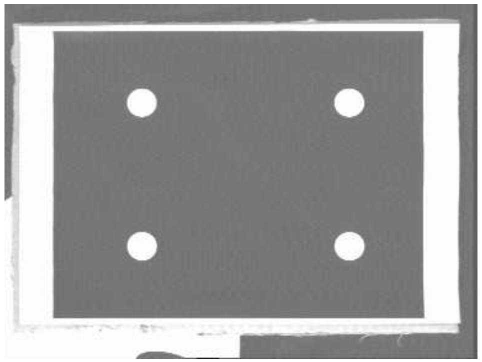


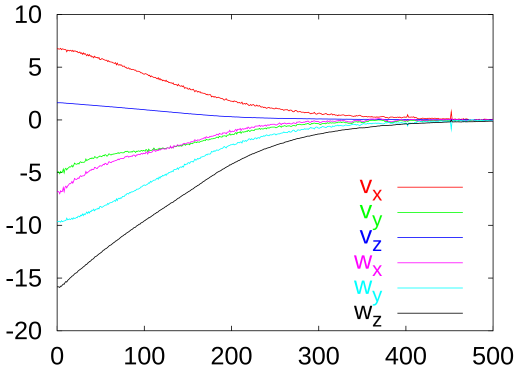


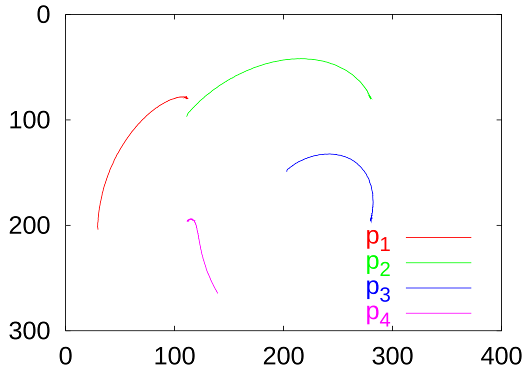


Initial image

Using adequate moments for s (cond $\mathbf{L_s} \approx 2$)



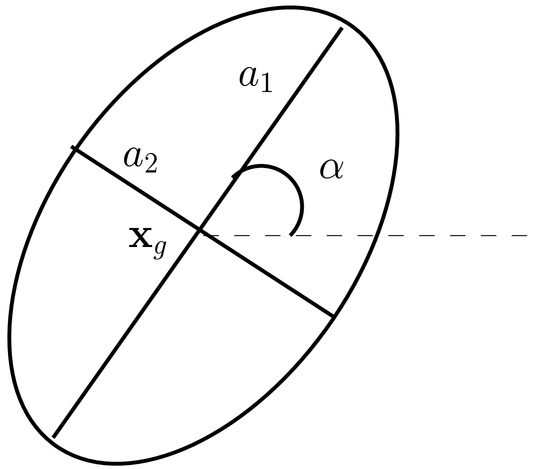




Desired image | Camera velocity | Image trajectories

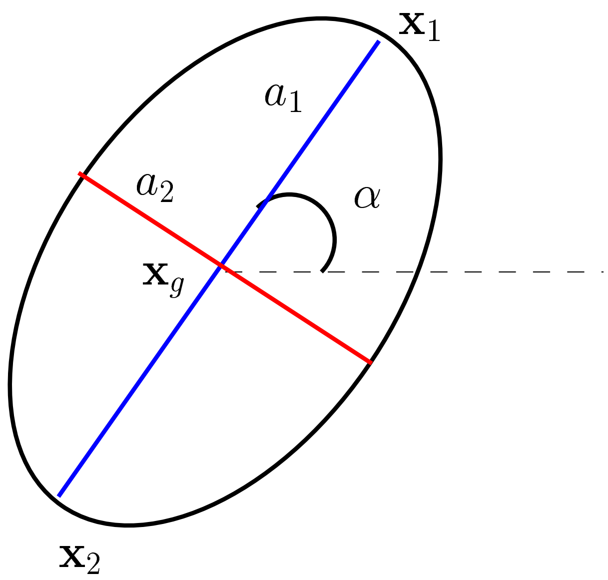
From moments to simple objects: ellipses

Ellipse defined by moments of order less than 3
(moments of order 3 are all 0)



- Image of a circle or an ellipse:
  - Select $\mathbf{s}_{\mathcal{E}} = (x_n, y_n, a_n, \alpha, n_{20} - n_{02})$
  - $\mathbf{L_s}$ always of rank 5,
    but when a centered circle appears in the image
    where $\mathbf{L_s}$ is of rank 3.
- Image of a sphere: - $\mathbf{L}_{\mathbf{s}_{\mathcal{E}}}$ always of rank 3
  - Select $\mathbf{s}_{\mathcal{S}} = (x_n, y_n, a_n)$

From moments to simple objects: segments



A segment is a particular ellipse with:

- length of minor axis $a_2 = 0$
- length of major axis $a_1 = l/2$

Segment parameterized by:

$$\mathbf{s} = (x_g, y_g, l, \alpha) \quad (l = 4\sqrt{n_{20} + n_{02}})$$

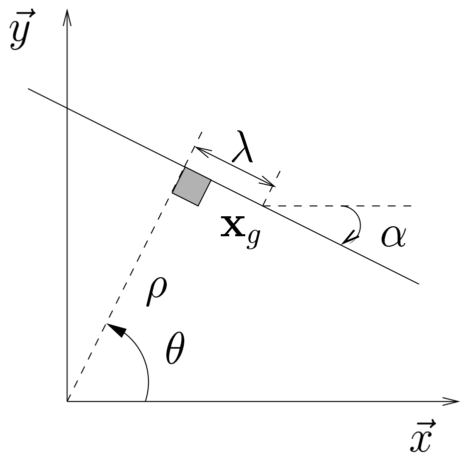
$$\begin{array}{lll}
\mathbf{L}_{x_g} & = [& -1/Z_g \quad 0 \quad x_{g_{vz}} \quad x_{g_{wx}} \quad -1 - x_{g_{wy}} \quad y_g \;] \\
\mathbf{L}_{y_g} & = [& 0 \quad -1/Z_g \quad y_{g_{vz}} \quad 1 + y_{g_{wx}} \quad y_{g_{wy}} \quad -x_g \;] \\
\mathbf{L}_l & = [& -Dc \quad -Ds \quad l/Z_g + Dl_1 \quad l_{wx} \quad l_{wy} \quad 0 \;] \\
\mathbf{L}_\alpha & = [& Ds/l \quad -Dc/l \quad D\alpha_1 \quad \alpha_{wx} \quad \alpha_{wy} \quad -1 \;]
\end{array}$$

($D = 0$ for a segment parallel to the image plane)

Better parameterization: $\mathbf{s} = (x_g/l, y_g/l, 1/l, \alpha)$

From moments to simple objects: straight lines

A straight line is a particular segment
(with infinite length)
parameterized with $\mathbf{s} = (\rho, \theta)$



$$\mathcal{D} : x \cos\theta + y \sin\theta - \rho = 0$$

$$\text{where} \begin{cases} \theta = \alpha + \frac{\pi}{2} \\ \rho = x_g \cos\theta + y_g \sin\theta \end{cases}$$

$$\begin{aligned} \mathbf{L}_\rho &= [\lambda_\rho c\theta \;\; \lambda_\rho s\theta \;\; -\lambda_\rho \rho \;\; (1+\rho^2)s\theta \;\; -(1+\rho^2)c\theta \;\; 0] \\ \mathbf{L}_\theta &= [\lambda_\theta c\theta \;\; \lambda_\theta s\theta \;\; -\lambda_\theta \rho \;\; -\rho c\theta \;\; -\rho s\theta \;\; -1] \end{aligned}$$

where $\lambda_\rho = -(A\rho c\theta + B\rho s\theta + C)$ and $\lambda_\theta = Bc\theta - As\theta$.

($\lambda_\theta = 0$ for a straight line parallel to the image plane)

2D visual features : unknown complex objects

Analytical form of $\mathbf{L_s}$ not available

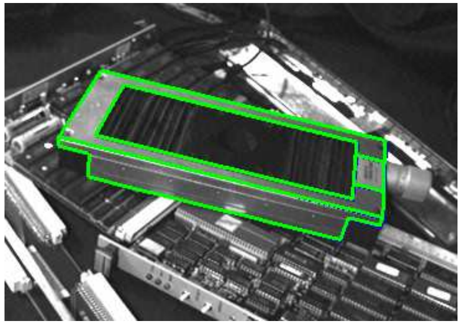
Off-line learning: **eigenspace** for **appearance** [Nayar 96, Deguchi 97]
neural networks [Suh 93, Wells 96]

Off-line or on-line learning also possible for geometrical features [Hosoda 94, Jägersand 94, Piepmeier 02, Lapresté 04]

In that case, theoretical stability impossible to analyse
(since no analytical form of $\mathbf{L_s}$ available)

- Modeling issues
  - ▷ Basics
  - ▷ 2D visual features
  - ⇒ 3D visual features
  - ▷ Omni-directional vision sensor, vision + structured light

3D visual features with one camera



Based on pose estimation $\hat{\mathbf{p}}(t)$ from $\mathcal{F}_c$ to $\mathcal{F}_o$ using

- an image of the object: $\mathbf{x}(t)$
- the knowledge of the object 3D CAD model: $\mathbf{X}$
- an estimation of the camera intrinsic parameters: x_C, y_C, f_x, f_y

$$\hat{\mathbf{p}}(t) = \hat{\mathbf{p}}(\mathbf{x}(t), \mathbf{X}, x_C, y_C, f_x, f_y)$$

Pose estimation problem $\sim$ camera calibration problem

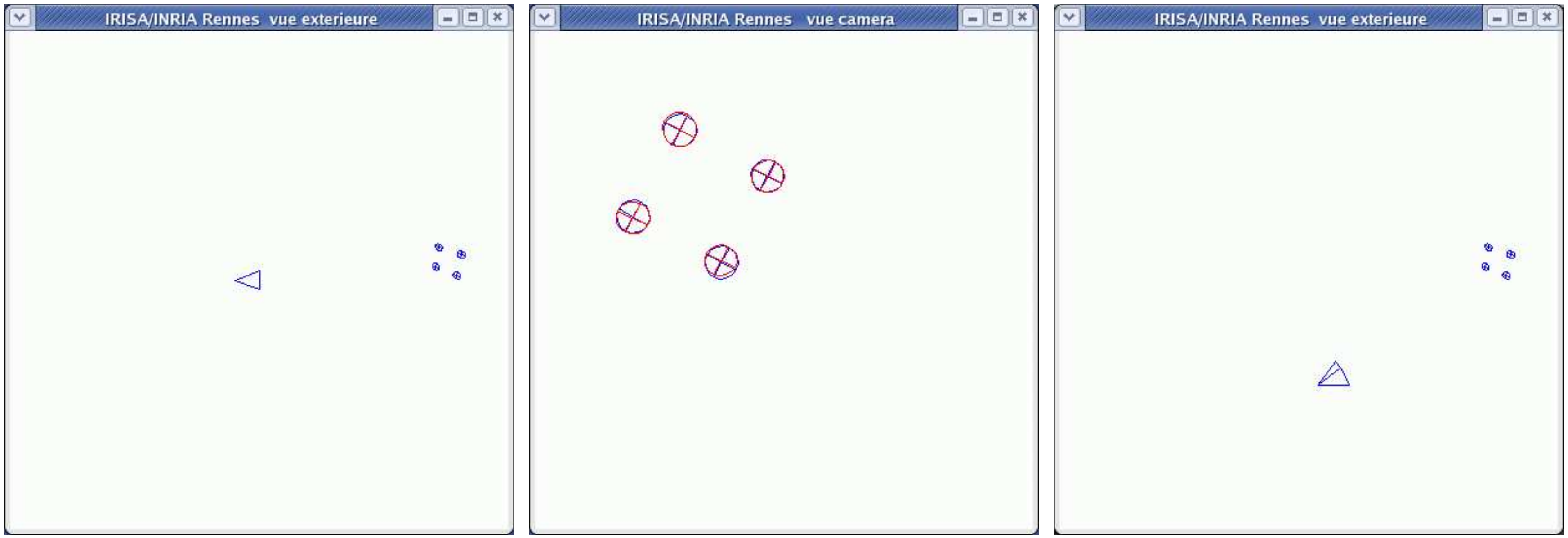
(intrinsic camera parameters already known)

3D visual features with one camera

Estimated pose $\hat{\mathbf{p}}(t) = \hat{\mathbf{p}}(\mathbf{x}(t), \mathbf{X}, x_c, y_c, f_x, f_y)$

$$\Rightarrow \quad \dot{\hat{\mathbf{p}}}(t) = \frac{\partial \hat{\mathbf{p}}}{\partial \mathbf{x}} \dot{\mathbf{x}} = \frac{\partial \hat{\mathbf{p}}}{\partial \mathbf{x}} \mathbf{L_x} \, \mathbf{v} \quad \Rightarrow \quad \mathbf{L}_{\hat{\mathbf{p}}} = \frac{\partial \hat{\mathbf{p}}}{\partial \mathbf{x}} \mathbf{L_x}$$

where $\mathbf{L_x}$ is known but $\frac{\partial \hat{\mathbf{p}}}{\partial \mathbf{x}}$ is unknown (and sometimes unstable)



3D visual features

Under the strong hypothesis that 3D estimation is perfect:

$$\frac{\partial \hat{\mathbf{p}}}{\partial \mathbf{p}} = \mathbf{I}_6 \Rightarrow \dot{\hat{\mathbf{p}}} = \dot{\mathbf{p}} = \mathbf{M}_{\mathbf{p}} \mathbf{v}$$

- rotation $\theta \mathbf{u}$ to realize

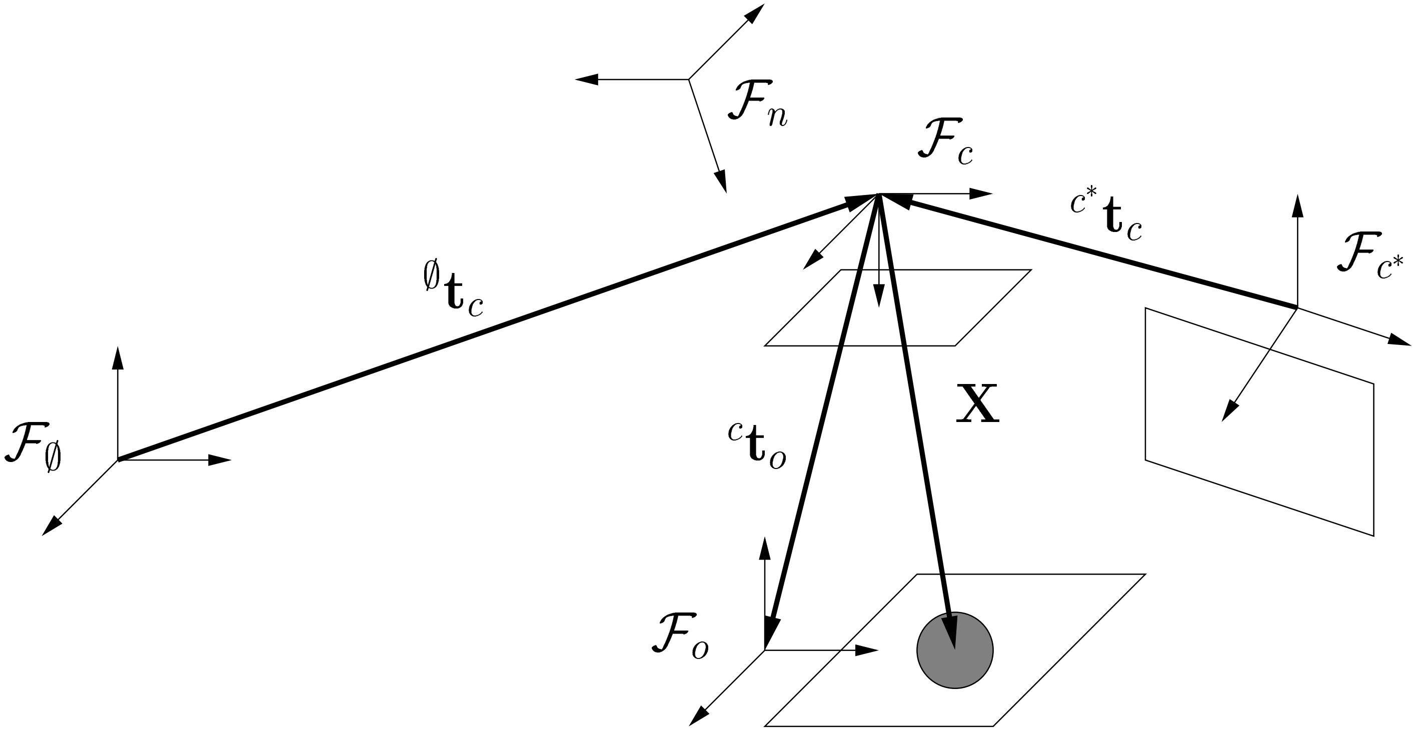
$$\mathbf{L}_{\theta \mathbf{u}} = \left[\begin{array}{cc} \mathbf{0}_3 & \mathbf{L}_\omega \end{array} \right] \text{ where } \mathbf{L}_\omega \text{ such that } \mathbf{L}_\omega \theta \mathbf{u} = \mathbf{L}_\omega^{-1} \, \theta \mathbf{u} = \theta \mathbf{u}$$

$$\mathbf{L}_\omega = \mathbf{I}_3 - \frac{\theta}{2} \, [\mathbf{u}]_\times + (1 - \frac{\text{sinc}\theta}{\text{sinc}^2 \frac{\theta}{2}}) \, [\mathbf{u}]_\times^2$$

- coordinates of a 3D point $\mathbf{X}$: $\dot{\mathbf{X}} = -\boldsymbol{\upsilon} - [\boldsymbol{\omega}]_\times \mathbf{X}$

$$\Rightarrow \mathbf{L}_{\mathbf{X}} = \left[\begin{array}{cc} -\mathbf{I}_3 & [\mathbf{X}]_\times \end{array} \right]$$

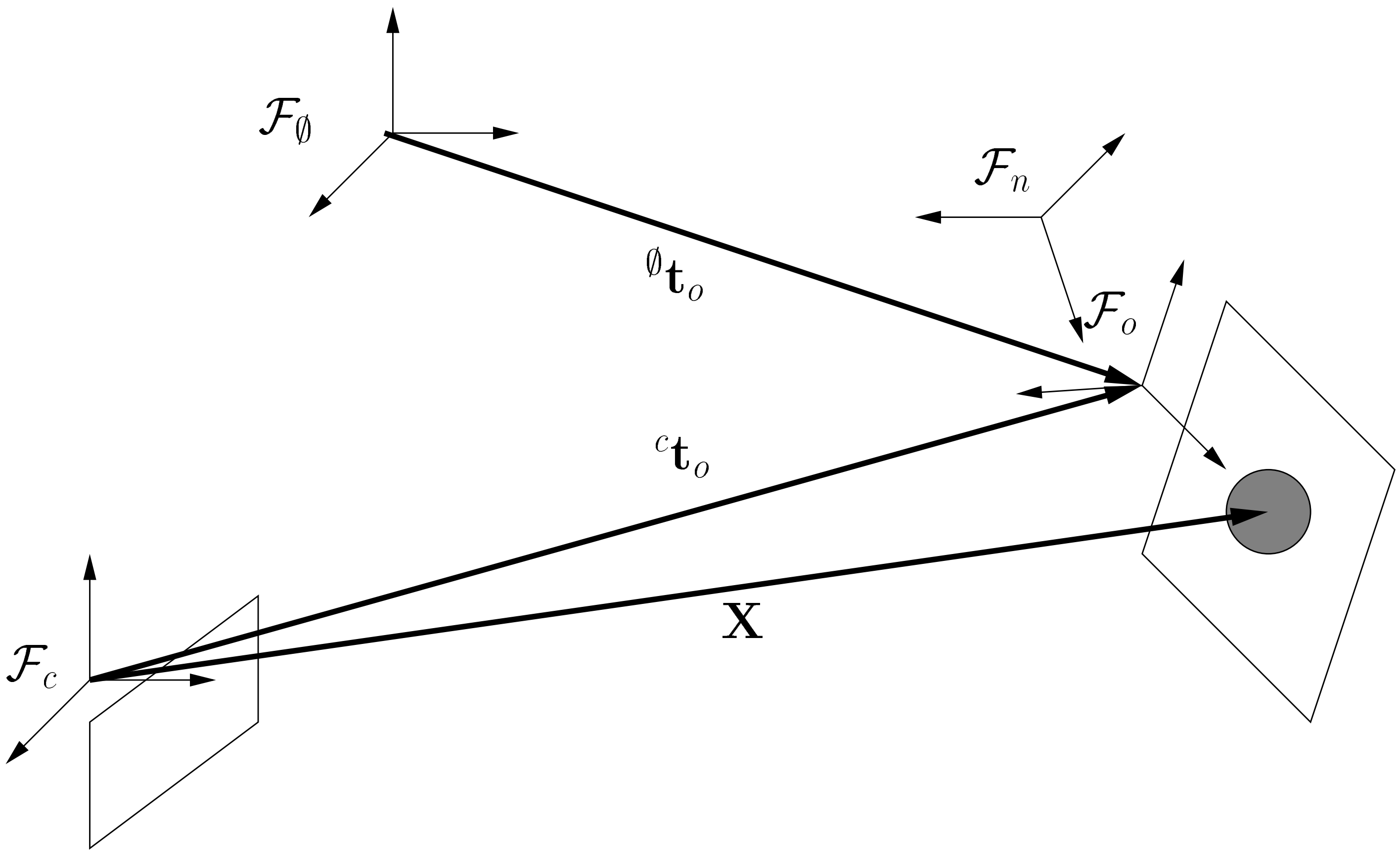
3D visual features for an eye-in-hand system



$$\mathbf{L}_{{}^c\mathbf{t}_o} = \begin{bmatrix} -\mathbf{I}_3 & [{}^{\mathbf{c}}\mathbf{t}_{\mathbf{o}}]_\times \end{bmatrix} \qquad \mathbf{L}_{{}^o\mathbf{t}_c} = \begin{bmatrix} {}^o\mathbf{R}_c & \mathbf{0}_3 \end{bmatrix}$$

$$\mathbf{L}_{{}^\emptyset\mathbf{t}_c} = \begin{bmatrix} {}^\emptyset\mathbf{R}_c & \mathbf{0}_3 \end{bmatrix} \qquad \mathbf{L}_{{}^{c^*}\mathbf{t}_c} = \begin{bmatrix} {}^{c^*}\mathbf{R}_c & \mathbf{0}_3 \end{bmatrix}$$

3D visual features for an eye-to-hand system



$$\mathbf{L}_{{}^c\mathbf{t}_o}\,{}^c\mathbf{V}_o = \left[\, -{}^c\mathbf{R}_o \;\; \mathbf{0}_3 \,\right] \qquad {}^\emptyset\dot{\mathbf{t}}_o = \left[\, \mathbf{I}_3 \;\; \mathbf{0}_3 \,\right] \; {}^\emptyset\mathbf{v}_o$$

- Modeling issues
  - ▷ Basics
  - ▷ 2D visual features
  - ▷ 3D visual features

$\Rightarrow$ Omni-directional vision sensor, vision + structured light

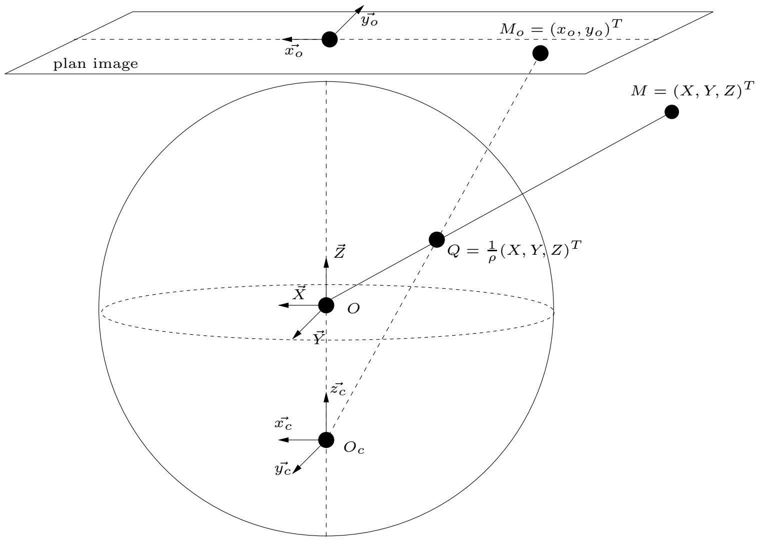
Modeling for omnidirectionnal vision [Nayar 01]

Single viewpoint systems

$$x_O = \frac{X}{\eta Z + \xi\sqrt{X^2+Y^2+Z^2}} \qquad (x = \frac{X}{Z})$$

$$y_O = \frac{Y}{\eta Z + \xi\sqrt{X^2+Y^2+Z^2}} \qquad (y = \frac{Y}{Z})$$

- $\eta = 1, \xi = 1$: parabolic mirror
- $\eta = 1, \xi = \xi_1$: planar mirror
- $\eta = 1, \xi = \xi_2$: hyperbolic mirror
- $\eta = 1, \xi = 0$: perspective projection
- $\eta = 0, \xi = 1$: spherical projection



Interaction matrix for a point

Using perspective projection:

$$\mathbf{L}_{xy} = \begin{bmatrix} -1/Z & 0 & x/Z & xy & -1-x^2 & y \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{bmatrix}$$

Using omnidirectionnal vision [Barreto 2002]:

$$\mathbf{L}_{x_o y_o} = \begin{bmatrix} -\frac{1}{\rho}\left(\frac{\eta\gamma-\xi}{\nu} - \xi x_o^2\right) & \frac{\xi x_o y_o}{\rho} & \frac{x_o\gamma}{\rho} & \eta x_o y_o & -\frac{\eta-\xi\gamma}{\nu} - \eta x_o^2 & y_o \\ \frac{\xi x_o y_o}{\rho} & -\frac{1}{\rho}\left(\frac{\eta\gamma-\xi}{\nu} - \xi y_o^2\right) & \frac{y_o\gamma}{\rho} & \frac{\eta-\xi\gamma}{\nu} + \eta y_o^2 & -\eta x_o y_o & -x_o \end{bmatrix}$$

with $\nu = \eta^2 - \xi^2$, $\rho = \sqrt{X^2+Y^2+Z^2}$ and $\gamma = \sqrt{1+\nu(x_o^2+y_o^2)}$.

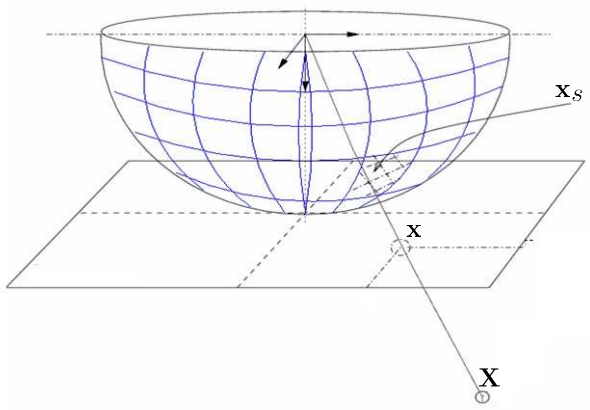
- For a parabolic mirror:

$$\eta = 1, \xi = 1, \nu = 0, \gamma = 1, \frac{\eta\gamma-\xi}{\nu} = \frac{1+x_o^2+y_o^2}{2}, \frac{\eta-\xi\gamma}{\nu} = \frac{1-x_o^2-y_o^2}{2}$$

For the image of a straight line, i.e. an ellipse, see [Mezouar, IROS 04]

Modeling for spherical projection ($\eta = 0, \xi = 1$)

- can be used from a perspective sensor or an omnidirectional sensor



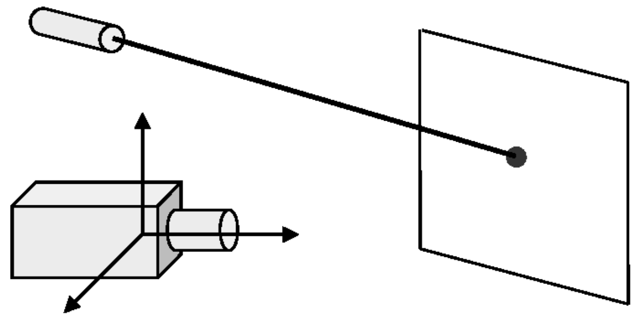
$\mathbf{x}_s = \mathbf{X}/\rho$ with $\rho = \sqrt{X^2 + Y^2 + Z^2}$

$$\mathbf{L}_{\mathbf{x}_s} = \begin{bmatrix} -\frac{1}{\rho}\left(1 - x_s^2\right) & \frac{x_s y_s}{\rho} & \frac{x_s z_s}{\rho} & 0 & -z_s & y_s \\ \frac{x_s y_s}{\rho} & -\frac{1}{\rho}\left(1 - y_s^2\right) & \frac{y_s z_s}{\rho} & z_s & 0 & -x_s \\ \frac{x_s z_s}{\rho} & \frac{y_s z_s}{\rho} & -\frac{1}{\rho}\left(1 - z_s^2\right) & -y_s & x_s & 0 \end{bmatrix}$$

Passivity property ($\|\dot{\mathbf{x}}_s\|$ independent of $\boldsymbol{\omega}$) [Hamel-Mahony 02]

Invariance property: for instance $\mathbf{L}_{a_s} = \left[\, a_x \;\; a_y \;\; a_z \;\; 0 \;\; 0 \;\; 0 \,\right]$

Modeling for coupling vision and structured light



- Structured ligth rigidly linked to the object:

  - no change at all in the modeling

- Structured ligth rigidly linked to the camera:

  - Points, straight lines, ellipses, see [Motyl 92]

  - Points revisited, see [Pagès, IROS 04]

1) Modeling issues

$\Rightarrow$ 2) Control issues

- $\triangleright$ Control of visual tasks ($m = n$)
- $\triangleright$ Classification of the visual tasks
- $\triangleright$ Hybrid tasks ($m < n$)

Control in visual servoing

Regulation of a task function: $\mathbf{e}(\mathbf{p}(t)) = \mathbf{C}\,(\mathbf{s}(\mathbf{p}(t)) - \mathbf{s}^*)$

Numerous solutions:

- P, PI, PID controller [Weiss 87]
- Non linear control law [Hashimoto 93, Reyes 98]
- Optimal control (LQ, LQG) [Papanikilopoulos 93, Hashimoto 96]
- Predictive controller [Gangloff 98]
- Robust controller H_∞ [Khadraoui 96]

Visual task function

With k visual features s, one constraints m robot dof ($m = \text{rank } \mathbf{L_s}$)

- If $m < n$, it is possible to consider a supplementary task (trajectory following, joint limits avoidance, etc.)
  $\Rightarrow$ **Hybrid tasks**

- If $m = n$, all the robot dof are controlled using the **visual task function** :

$$\mathbf{e}(\mathbf{p}(t)) = \mathbf{C}\ (\mathbf{s}(\mathbf{p}(t)) - \mathbf{s}^*)$$

where $\mathbf{C}$ is a $m \times k$ combination matrix of full rank m.

Control law

Since $\mathbf{e}(\mathbf{p}(t)) = \mathbf{C}\,(\mathbf{s}(\mathbf{p}(t)) - \mathbf{s}^*)$, we have

$$\dot{\mathbf{e}} = \mathbf{L_e}\,\mathbf{v}_q + \frac{\partial \mathbf{e}}{\partial t} \quad \text{where} \quad \begin{cases} \mathbf{v}_q = \mathbf{v}_c \text{ for eye-in-hand system} \\ \mathbf{v}_q = -\mathbf{v}_o \text{ for eye-to-hand system} \end{cases}$$

We obtain ideally for an exponential decrease of $\mathbf{e}$ $\quad (\dot{\mathbf{e}} = -\lambda\,\mathbf{e})$

$$\mathbf{v}_q = \mathbf{L_e}^{-1}\,(-\lambda\,\mathbf{e} - \frac{\partial \mathbf{e}}{\partial t}) \quad \text{with } \mathbf{L_e} = \mathbf{C}\mathbf{L_s} \text{ if } \mathbf{C} \text{ is constant}$$

Since $\mathbf{L_e}$ and $\frac{\partial \mathbf{e}}{\partial t}$ are not perfectly known, one uses

$$\mathbf{v}_q = \widehat{\mathbf{L_e}}^{-1}\,(-\lambda\,\mathbf{e} - \widehat{\frac{\partial \mathbf{e}}{\partial t}})$$

Stability analysis

Behavior of the closed-loop system :

$$\dot{\mathbf{e}} = \mathbf{L_e}\,\mathbf{v}_q + \frac{\partial \mathbf{e}}{\partial t} = -\lambda\,\mathbf{L_e}\widehat{\mathbf{L_e}}^{-1}\mathbf{e} \;-\; \mathbf{L_e}\widehat{\mathbf{L_e}}^{-1}\widehat{\frac{\partial \mathbf{e}}{\partial t}} + \frac{\partial \mathbf{e}}{\partial t}$$

If $\frac{\partial \mathbf{e}}{\partial t} = \widehat{\frac{\partial \mathbf{e}}{\partial t}} = 0$, $\|\mathbf{e}\|$ always decreases (global stability) if

$$\mathbf{L_e}\widehat{\mathbf{L_e}}^{-1} > 0$$

To suppress tracking errors and obtain the desired behavior $\dot{\mathbf{e}} = -\lambda\mathbf{e}$:

$$\widehat{\mathbf{L_e}} = \mathbf{L_e} \quad \text{and} \quad \widehat{\frac{\partial \mathbf{e}}{\partial t}} = \frac{\partial \mathbf{e}}{\partial t}$$

In practice $(m = n)$

- If $k = m$, $\mathbf{C} = \mathbb{I}_m$, $\mathbf{e} = \mathbf{s} - \mathbf{s}^*$ $\Rightarrow$ $\mathbf{L_e} = \mathbf{L_s}$, $\widehat{\mathbf{L_e}} = \widehat{\mathbf{L_s}}$

$$\mathbf{v}_q = -\lambda\, \widehat{\mathbf{L_s}}^{-1} (\mathbf{s} - \mathbf{s}^*) - \widehat{\mathbf{L_s}}^{-1} \widehat{\frac{\partial \mathbf{s}}{\partial t}} \quad \text{stable if} \quad \mathbf{L_s}\, \widehat{\mathbf{L_s}}^{-1} > 0$$

- If $k > m$, $\mathbf{C} = \widehat{\mathbf{L_s}}^{+}_{|\mathbf{s}=\mathbf{s}^*}$ (1) or $\mathbf{C} = \widehat{\mathbf{L_s}}^{+}$ (2) , $\widehat{\mathbf{L_e}} = \mathbb{I}_n$,

$$\mathbf{v}_q = -\lambda\, \mathbf{e} - \widehat{\frac{\partial \mathbf{e}}{\partial t}} \text{ stable if :}$$

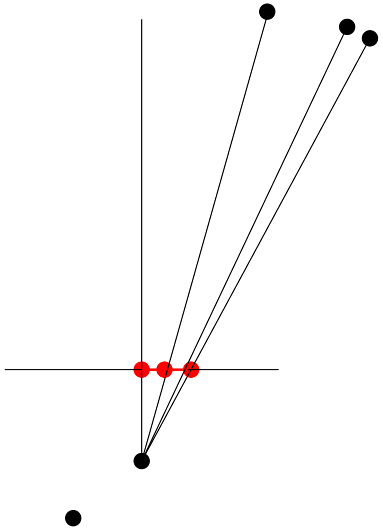
(1) $\widehat{\mathbf{L_s}}^{+}_{|\mathbf{s}=\mathbf{s}^*} \mathbf{L_s} > 0$ (only around $\mathbf{s}^*$)

(2) $\widehat{\mathbf{L_s}}^{+} \mathbf{L_s} > 0$ (only locally since $\mathbf{C}$ not constant)

If translational dof are controlled and 2D visual features are used, an estimation $\widehat{\mathbf{P}}$ or $\widehat{\mathbf{P}^*}$ is necessary to compute $\widehat{\mathbf{L_s}}$

A simple case $k = m = n = 2$

Case of a pan-tilt camera observing a point :

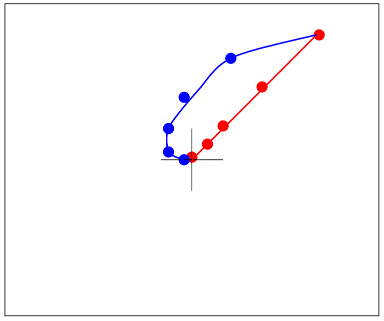


$$\mathbf{s} = (x, y) \;,\; \mathbf{s}^* = (0, 0) \;,\; \mathbf{C} = \mathbb{I}_2$$

$$\dot{\mathbf{e}} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} xy & -(1+x^2) \\ 1+y^2 & -xy \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix}$$

$$\mathbf{v}_c = -\lambda \, \widehat{\mathbf{L_s}}^{-1} \, (\mathbf{s} - \mathbf{s}^*)$$

$$\Leftrightarrow \begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix} = -\frac{\lambda}{1+x^2+y^2} \begin{bmatrix} y \\ -x \end{bmatrix}$$



If no error occurs, $\dot{\mathbf{s}} = -\lambda \, \mathbf{s}$: trajectory = straight line in the image

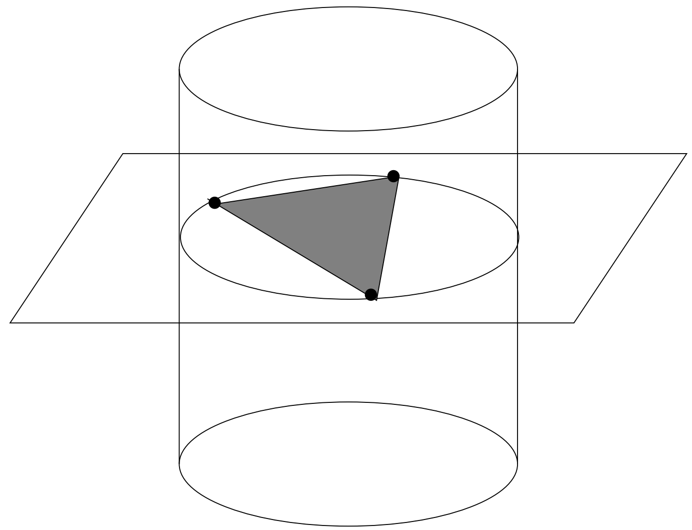
Control when $k = m = n = 6$

$$\mathbf{C} = \mathbb{I}_6 \quad \Rightarrow \quad \mathbf{v}_q = -\lambda\, \widehat{\mathbf{L_s}}^{-1}(\mathbf{s} - \mathbf{s}^*) \ \text{ stable if } \ \mathbf{L_s}\, \widehat{\mathbf{L_s}}^{-1} > 0$$

- **Impossible with only 2D visual features**

Using 3 points ($\mathbf{L_s}$ is 6×6)

- pose ambiguity (4 solutions)
- possible singularity of $\mathbf{L_s}$



Control when $k = m = n = 6$

- **Possible with 3D visual features**

For instance, if $\mathbf{s} = \begin{bmatrix} {}^{c^*}\mathbf{t}_c \\ \theta\mathbf{u} \end{bmatrix}$, $\mathbf{v}_c = \begin{bmatrix} \boldsymbol{\upsilon} \\ \boldsymbol{\omega} \end{bmatrix} = -\lambda \begin{bmatrix} {}^{c}\mathbf{R}_{c^*}\, {}^{c^*}\mathbf{t}_c \\ \theta\mathbf{u} \end{bmatrix}$

Advantages

- $\mathbf{L_s}$ block-diagonal and never singular
- Translational and rotational motions decoupled
- Camera trajectory : straight line in 3D space

Drawbacks

- No control in the image (the target may get out of the image)

Control when $k = m = n = 6$: 2 1/2 D visual servoing

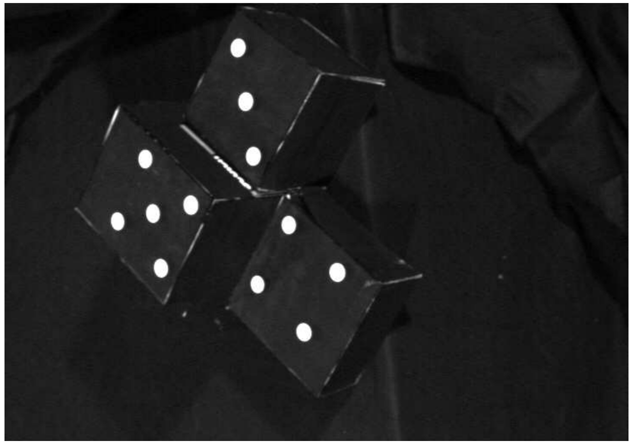
Idea : Combine 2D image data and 3D data

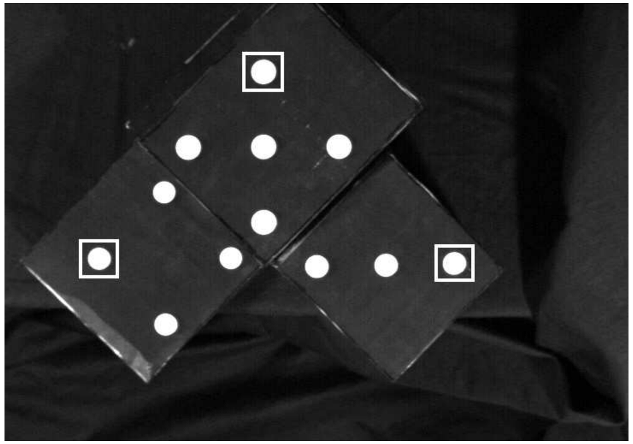
$$\mathbf{s} = \begin{bmatrix} x \\ y \\ \log Z \\ \theta u_x \\ \theta u_y \\ \theta u_z \end{bmatrix} \begin{array}{l} \left.\begin{array}{l} \text{image point} \\ \text{coordinates} \end{array}\right\} \\ \rightarrow \text{rel. depth} \\ \left.\begin{array}{c} \text{rotation} \\ \text{to} \\ \text{realize} \end{array}\right\} \end{array} \Rightarrow \begin{array}{l} \mathbf{L_s} \ \textbf{triangular} \\ \quad \text{and never singular} \end{array}$$

Advantages :

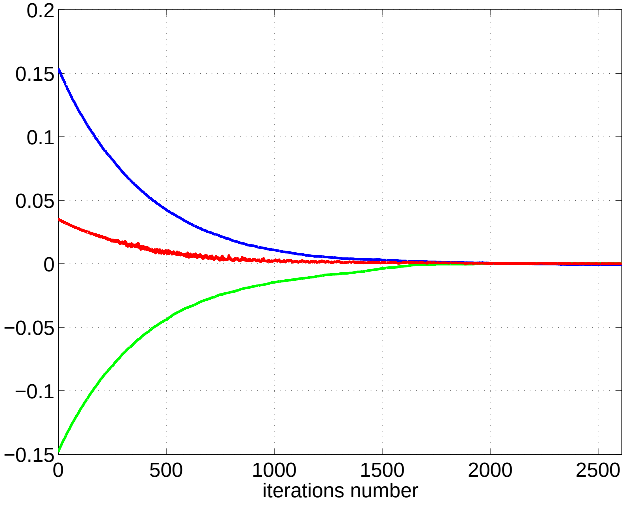
- Decoupled control scheme (image point trajectory : straight line)
- Analytical conditions for global stability possible in presence of calibration errors
- No 3D CAD model needed, only one scalar unknown

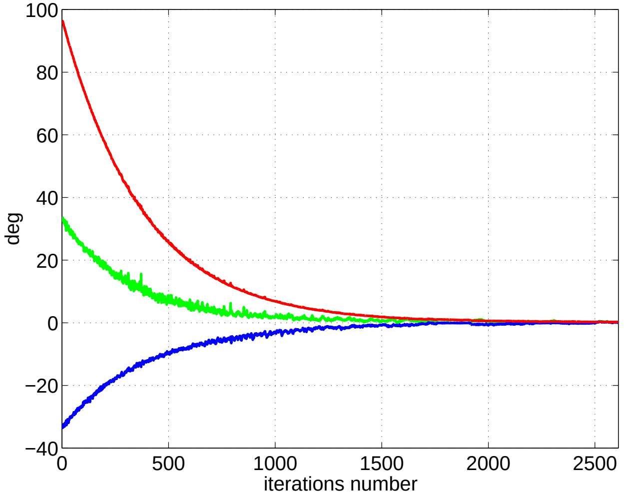
Results

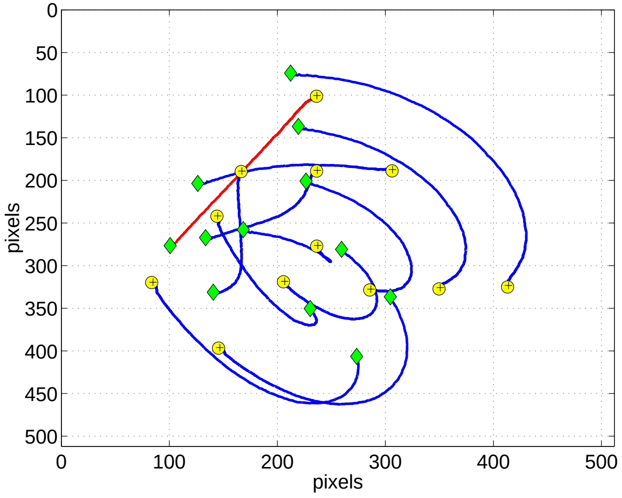




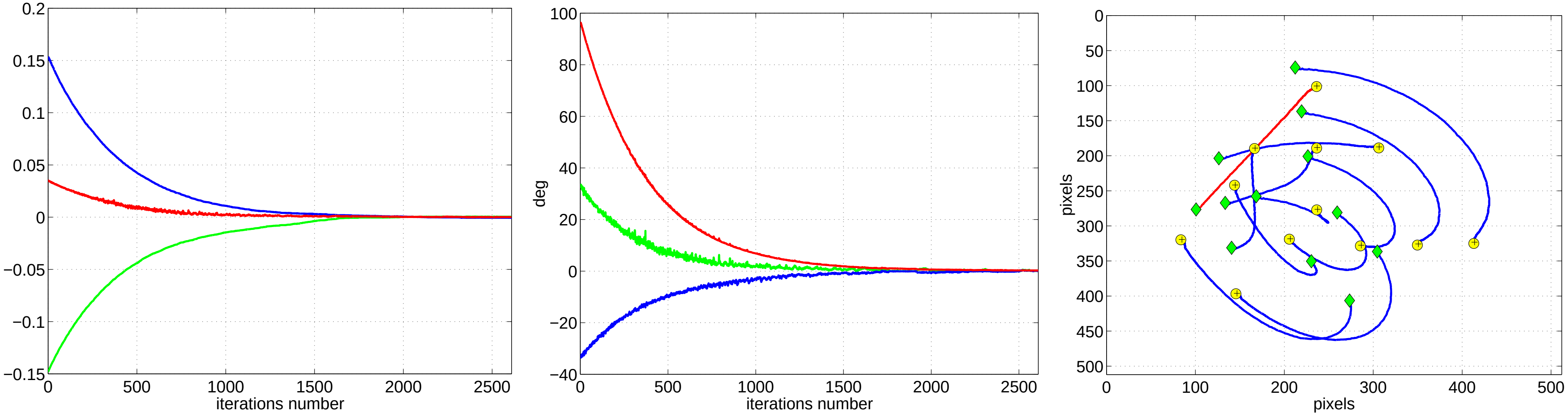
Coarse calibration



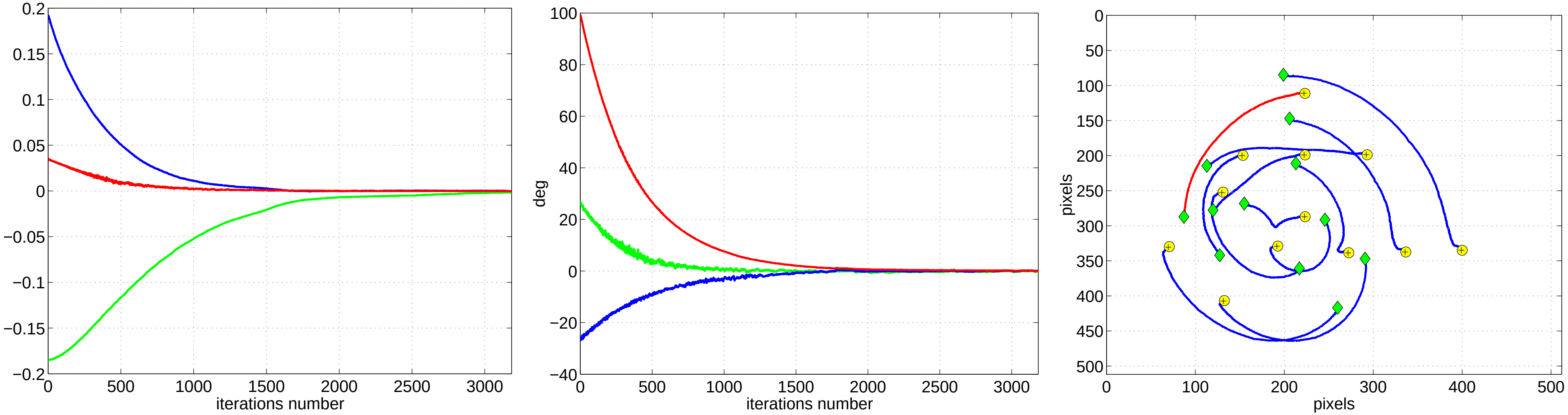




Coarse calibration



Bad calibration



Other 2 1/2 D VS scheme

$$\mathbf{s} = \begin{bmatrix} {}^{c^*}\mathbf{t}_{cx} \\ {}^{c^*}\mathbf{t}_{cy} \\ {}^{c^*}\mathbf{t}_{cz} \\ x \\ y \\ \theta \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{translation to realize} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{image point coordinates} \\ \longrightarrow \text{orientation} \end{matrix} \quad \Rightarrow \mathbf{L_s} = \begin{bmatrix} {}^{c^*}\mathbf{R}_c & \mathbf{0}_3 \\ \frac{1}{Z}\mathbf{L}_{v\omega} & \mathbf{L}_\omega \end{bmatrix}$$

Advantages:

- Camera trajectory : straight line in 3D space
- Trajectory in the image of the selected point : straight line

Drawback:

$\mathbf{L_s}$ only block-triangular

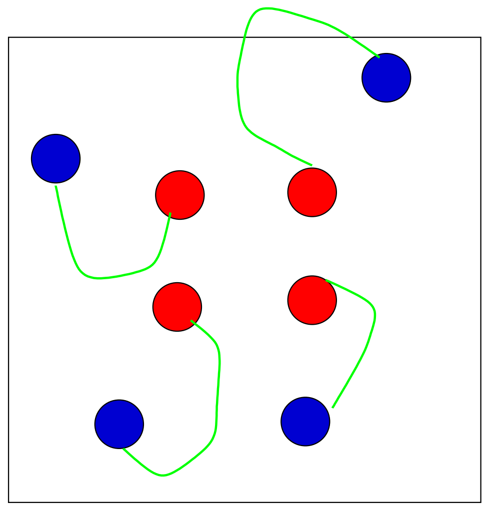
$\Rightarrow$ analytical conditions for global stability difficult to obtain

Control when $k > m, m = n = 6$ using 2D visual features

- Control law $\mathbf{v}_q = -\lambda \, \widehat{\mathbf{L}_{\mathbf{s}}}^{+}_{|\mathbf{s}=\mathbf{s}^*} (\mathbf{s} - \mathbf{s}^*)$

  stable if $\widehat{\mathbf{L}_{\mathbf{s}}}^{+}_{|\mathbf{s}=\mathbf{s}^*} \mathbf{L}_{\mathbf{s}} > 0$ (only around $\mathbf{s}^*$)

  No real control of the image trajectories

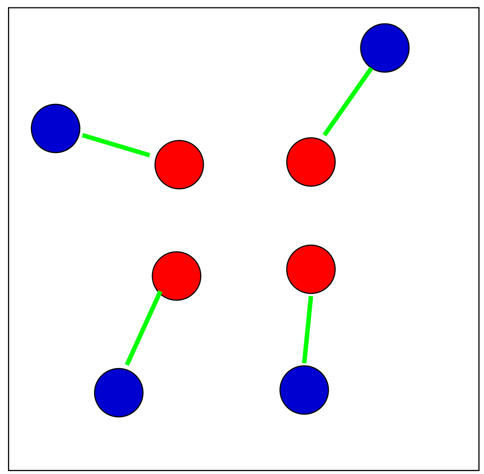


- Control law $\mathbf{v}_q = -\lambda \, \widehat{\mathbf{L}_{\mathbf{s}}}^{+} (\mathbf{s} - \mathbf{s}^*)$

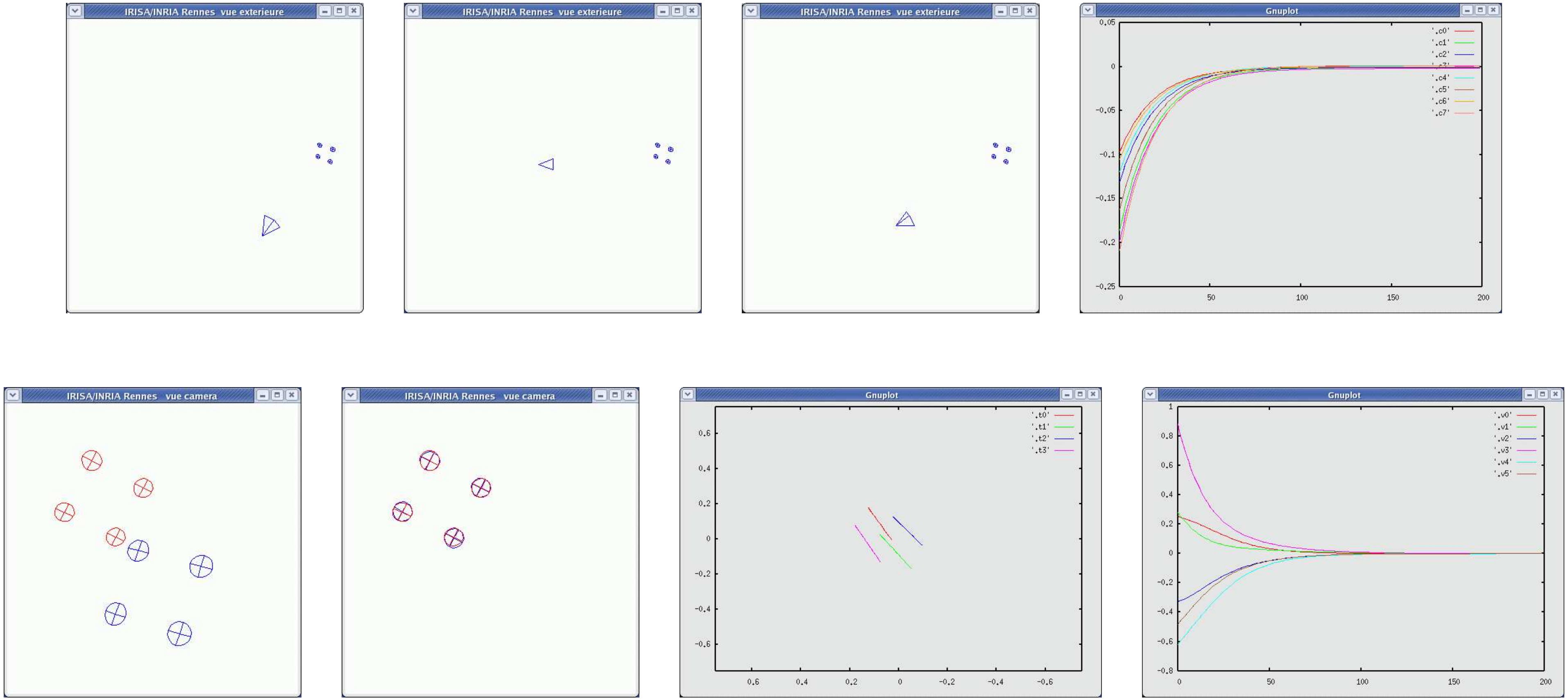
  tries to ensure $\dot{\mathbf{s}} = -\lambda \, (\mathbf{s} - \mathbf{s}^*)$

  Possible local minima

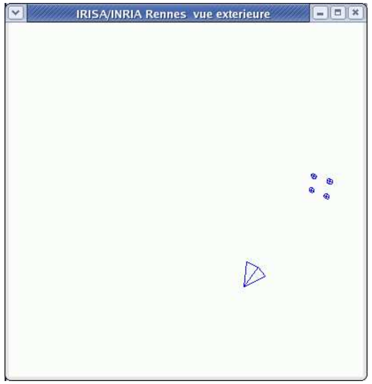
  because $\mathbf{C} \; (= \widehat{\mathbf{L}_{\mathbf{s}}}^{+})$ is not constant

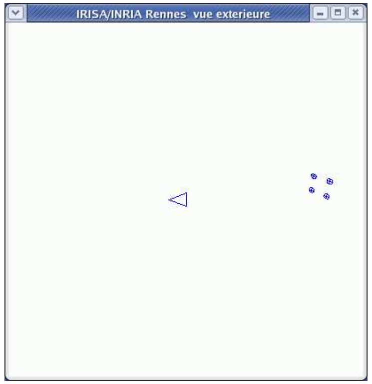


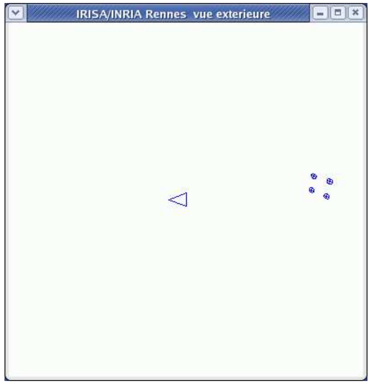
Reaching a local minimum using $\widehat{\mathbf{L_s}}^+$

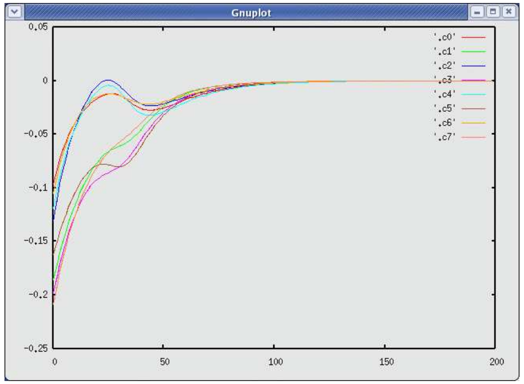


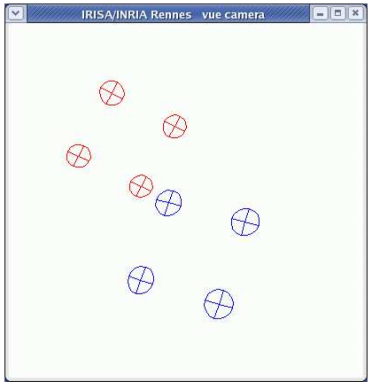
Reaching the global minimum using $\widehat{\mathbf{L}_{\mathbf{s}}}^{+}_{|\mathbf{s}=\mathbf{s}^*}$

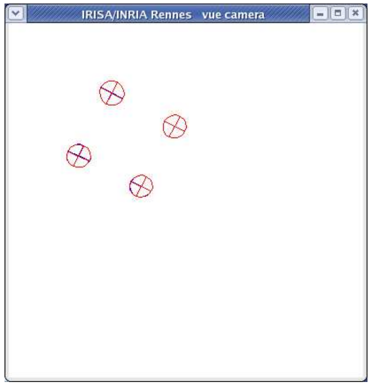


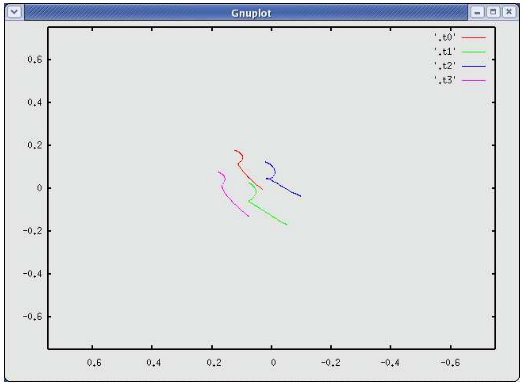


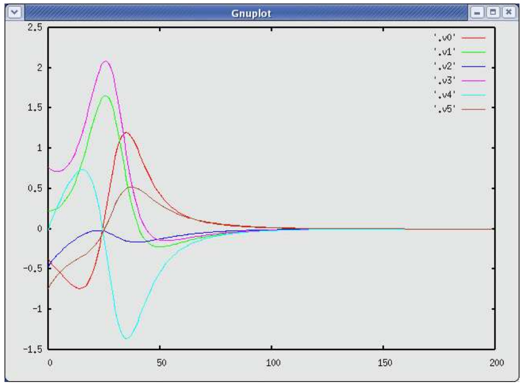




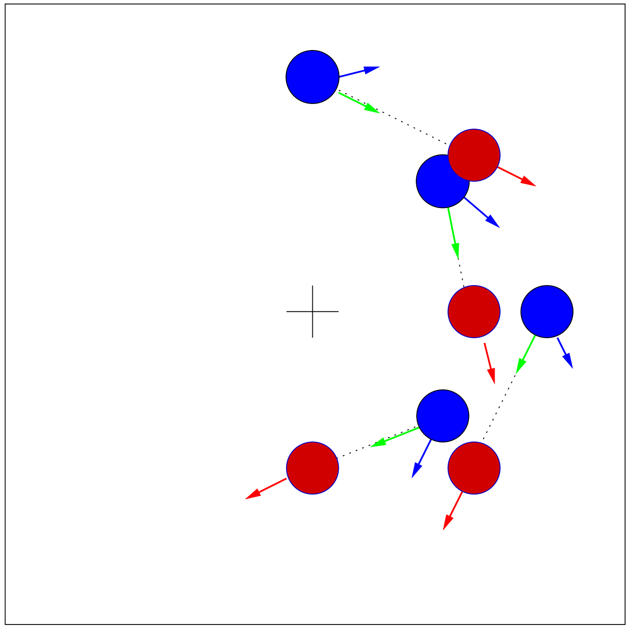








Geometrical interpretation of the behavior obtained



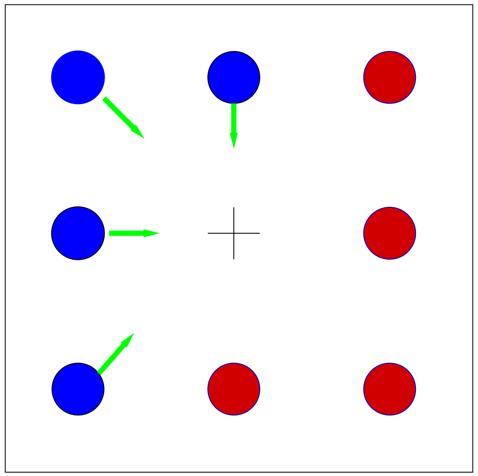
Rotation of 45^o

Reaching a singularity of $\mathbf{L_s}$

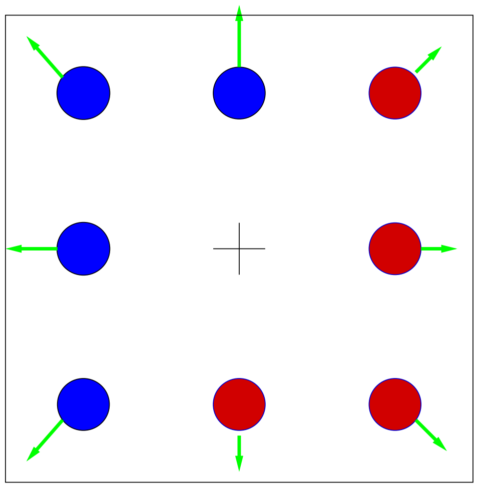
Example : rotation of 180° around the optical axis

s composed of image points coordinates

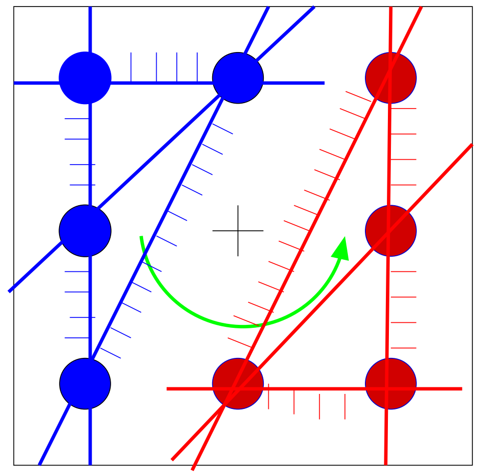
Other choice



Using $\widehat{\mathbf{L_s}}^{+}$



Using $\widehat{\mathbf{L_s}}_{|\mathbf{s}=\mathbf{s}^*}^{+}$



Using $\widehat{\mathbf{L_s}}^{+}$ or $\widehat{\mathbf{L_s}}_{|\mathbf{s}=\mathbf{s}^*}^{+}$

At singularity, rank $\mathbf{L_s}$ = 2.

Perfect behavior if s is composed of 2D straight lines parameters

(or 2D moments, or 2.5D VS, or 3D VS)

Target tracking

PI Controller

Integral term classical in Automatic Control to reduce tracking errors

If $\mathbf{I}_k$ is the estimation of $\frac{\partial \mathbf{e}}{\partial t}$ at iteration k, we have :

$$\begin{aligned} \mathbf{I}_{k+1} &= I_k + \mu\, \mathbf{e}_k \;\; \text{with } \mathbf{I}_0 = 0 \\ &= \mu \sum_{j=0}^{k} \mathbf{e}_j \end{aligned}$$

Efficient to track a target at constant velocity :

$$\mathbf{I}_{k+1} = \mathbf{I}_k \text{ if } \mathbf{e}_k = 0$$

Target tracking by estimating the target velocity

If it is possible to measure the camera velocity, we get:

$$\widehat{\frac{\partial \mathbf{e}}{\partial t}} = \hat{\dot{\mathbf{e}}} - \widehat{\mathbf{L}_{\mathbf{e}}}\, \mathbf{v}_c$$

with

$$\begin{cases} \widehat{\mathbf{L}_{\mathbf{e}}} = \mathbf{C}\widehat{\mathbf{L}_{\mathbf{s}}} \\ \\ \hat{\dot{\mathbf{e}}}_k = \frac{\mathbf{e}_k - \mathbf{e}_{k-1}}{\Delta t} = \mathbf{C}\, \frac{\mathbf{s}_k - \mathbf{s}_{k-1}}{\Delta t} \end{cases}$$

A Kalman filter may then be used.

$\Rightarrow$ Control issues

- $\triangleright$ Control of visual tasks ($m = n$)

$\Rightarrow$ Classification of the visual tasks

- $\triangleright$ Hybrid tasks ($m < n$)

Classification of the vision-based tasks

$\mathcal{S}^*$ = set of motions such that $\dot{\mathbf{s}} = 0$: $\mathcal{S}^* =$ Ker $\mathbf{L_s}$

A **virtual link** between the camera and the scene is defined by a set of compatible constraints : $\mathbf{s}(\mathbf{p}(t)) - \mathbf{s}^* = 0$

A virtual link is characterized by $\mathcal{S}^*$ since $\mathbf{s}(\mathbf{p}(t)) - \mathbf{s}^* = 0 \Rightarrow \dot{\mathbf{s}} = 0$

class of the virtual link = dimension N of $\mathcal{S}^*$.

The k constraints involved by the visual features are independent
if $k = 6 - N$.

If $k > 6 - N$, the visual features are redundant.

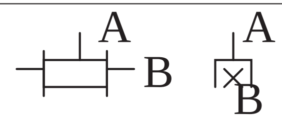
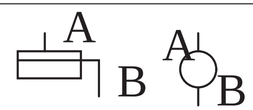
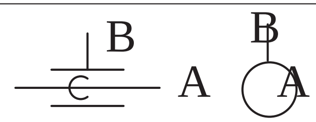
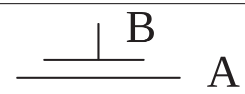
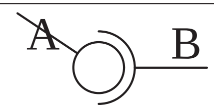
Case of a point

$$\mathbf{s} = (x, y)$$

$$\Rightarrow \mathbf{L}_{xy} = \begin{bmatrix} -1/Z & 0 & x/Z & xy & -(1+x^2) & y \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{bmatrix}$$

$$\Rightarrow \mathcal{S}^* = \begin{bmatrix} x & 0 & Z(1+x^2+y^2) & 0 \\ y & 0 & 0 & Z(1+x^2+y^2) \\ 1 & 0 & 0 & 0 \\ 0 & x & -xy & 1+x^2 \\ 0 & y & -(1+y^2) & xy \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Link of class 4

Name	Class	T	R	Geometric symbol
Rigid	0	0	0	
Prismatic	1	1	0	
Rotary	1	0	1	
Sliding pivot	2	1	1	
Plane-to-plane	3	2	1	
Bearing	3	0	3	
Linear rectlinear	4	2	2	
Linear annular	4	1	3	
Point	5	2	3	

Rigid link

$$\mathcal{S}^* = (0,0,0,0,0,0)$$

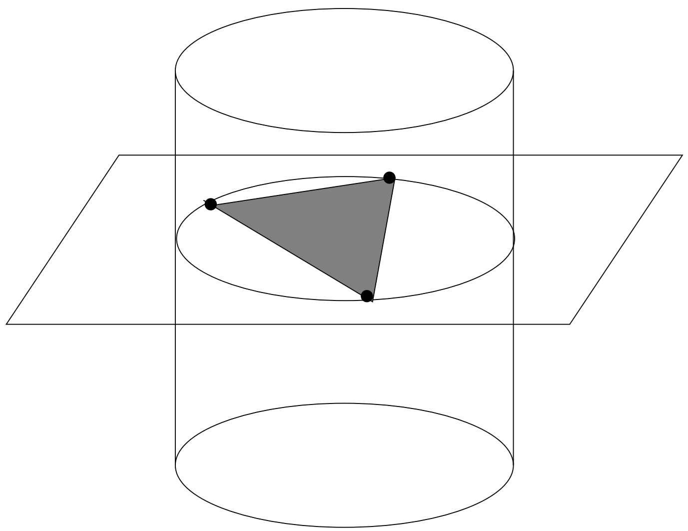
Using 3 points :

$$\mathbf{L_s} = \begin{bmatrix} -1/Z_1 & 0 & x_1/Z_1 & x_1y_1 & -(1+x_1^2) & y_1 \\ 0 & -1/Z_1 & y_1/Z_1 & 1+y_1^2 & -x_1y_1 & -x_1 \\ -1/Z_2 & 0 & x_2/Z_2 & x_2y_2 & -(1+x_2^2) & y_2 \\ 0 & -1/Z_2 & y_2/Z_2 & 1+y_2^2 & -x_2y_2 & -x_2 \\ -1/Z_3 & 0 & x_3/Z_3 & x_3y_3 & -(1+x_3^2) & y_3 \\ 0 & -1/Z_3 & y_3/z_3 & 1+y_3^2 & -x_3y_3 & -x_3 \end{bmatrix}$$

Isolated singularities exist

4 poses are solution of the **P3P** problem

Solution : Using at least 4 points



Prismatic link

$$\mathcal{S}^* = (1, 0, 0, 0, 0, 0)$$

Using 3 (horizontal) straight lines

3D straight lines : $\mathbf{h}_i(\mathbf{X}, \mathbf{P}) = \left\{ \begin{array}{l} Y - \frac{Y_i^*}{Z_i^*} Z = 0 \\ Z - Z_i^* = 0 \end{array} \right. , \; i = 1, 2, 3$

2D straight lines : $\rho_i = Y_i^* / Z_i^*, \; \theta_i = \pi/2$

$$\Rightarrow \mathbf{L}_{\rho_i \theta_i} = \begin{bmatrix} 0 & -1/Z_i^* & \rho_i / Z_i^* & (1 + \rho_i^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho_i & -1 \end{bmatrix}$$

With a 3 dof mobile robot (v_x, v_z, ω_y), 1 straight line is sufficient.

Bearing

Using a sphere with center $\mathbf{O} = (0, 0, Z_0)$

$\Rightarrow$ Image of the sphere = centered circle

$$
\begin{array}{lll}
\mathbf{L}_{x_c} & = [& -1/Z_c \quad 0 \quad 0 \quad 0 \quad -1-r^2 \quad 0 \ ] \\
\mathbf{L}_{y_c} & = [& 0 \quad -1/Z_c \quad 0 \quad 1+r^2 \quad 0 \quad 0 \ ] \\
\mathbf{L}_{\mu} & = [& 0 \quad 0 \quad 2r^2/Z_c \quad 0 \quad 0 \quad 0 \ ]
\end{array}
$$

with $Z_c = (Z_0^2 - R^2)/Z_0$ and $r^2 = R^2/(Z_0^2 - R^2)$.

$$
\Rightarrow S^* = \begin{bmatrix} 0 & -z_0 & 0 \\ z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{|\mathcal{F}_c} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{|\mathcal{F}_o}
$$

$\Rightarrow$ Control issues

- $\triangleright$ Control of visual tasks ($m = n$)
- $\triangleright$ Classification of the visual tasks

$\Rightarrow$ Hybrid tasks ($m < n$)

Visual task function

With k visual features $\mathbf{s}$, one constraints $m\ (= n - N \leq k)$ robot dof

using the **visual task function** $\quad \mathbf{e}_1(\mathbf{p}(t)) = \mathbf{C}\ (\mathbf{s}(\mathbf{p}(t)) - \mathbf{s}^*)$

where $\mathbf{C}$ is a $m \times k$ combination matrix of full rank m.

$\Rightarrow$ If $m < n$, it is possible to consider a supplementary task (trajectory following, joint limits avoidance, etc.).

Problem : How to combine both tasks ?

- $\mathbf{e}_1$: primary task
- $\mathbf{e}_2$: secondary task, expressed as a cost function to be minimized under the constraint that $\mathbf{e}_1$ is satisfied.

Global task function

A task function $\mathbf{e}$ minimizing the objective function h_s under the constraint $\mathbf{e}_1 = 0$ is given by :

$$\begin{aligned} \mathbf{e} &= \mathbf{W}^+\mathbf{e}_1 + (\mathbb{I}_n - \mathbf{W}^+\mathbf{W})\, \mathbf{g}_s^T \\ &= \mathbf{W}^+\mathbf{C}\,(\mathbf{s} - \mathbf{s}^*) + (\mathbb{I}_n - \mathbf{W}^+\mathbf{W})\, \mathbf{g}_s^T \end{aligned}$$

where :

- $\mathbf{g}_s$ = gradient of h_s ($\mathbf{g}_s = \frac{\partial h_s}{\partial \mathbf{p}}$)
- $\mathbf{W}$ is a $m \times n$ matrix of full rank m such that

  Ker $\mathbf{W}$ $=$ Ker $\mathbf{L_s}$

$$\Rightarrow (\mathbb{I}_n - \mathbf{W}^+\mathbf{W})\, \mathbf{g}_s^T \in \text{Ker}\, \mathbf{L_s}\ ,\ \forall \mathbf{g}_s^T$$

- if $m = n$, $\mathbf{W} = \mathbb{I}_n$, $\mathbf{e} = \mathbf{e}_1 = \mathbf{C}\,(\mathbf{s} - \mathbf{s}^*)$

Control law (similar as before)

Since we have

$$\dot{\mathbf{e}} = \mathbf{L_e}\,\mathbf{v}_q + \frac{\partial \mathbf{e}}{\partial t} \quad \text{where} \quad \begin{cases} \mathbf{v}_q = \mathbf{v}_c \text{ for eye-in-hand system} \\ \mathbf{v}_q = -\mathbf{v}_o \text{ for eye-to-hand system} \end{cases}$$

we obtain ideally for an exponential decrease of $\mathbf{e}$ $\quad (\dot{\mathbf{e}} = -\lambda\,\mathbf{e})$

$$\mathbf{v}_q = \mathbf{L_e}^{-1}\,(-\lambda\,\mathbf{e} - \frac{\partial \mathbf{e}}{\partial t})$$

with $\boxed{\mathbf{L_e} = \mathbf{W}^{+}\mathbf{C}\mathbf{L_s} + (\mathbb{I}_n - \mathbf{W}^{+}\mathbf{W})\,\frac{\partial \mathbf{g}_s^T}{\partial \mathbf{p}}}$ if $\mathbf{C}$ and $\mathbf{W}$ are constant

Since $\mathbf{L_e}$ and $\boxed{\frac{\partial \mathbf{e}_1}{\partial t}}$ are not perfectly known, one uses

$$\mathbf{v}_q = \widehat{\mathbf{L_e}}^{-1}\,(-\lambda\,\mathbf{e} - \widehat{\frac{\partial \mathbf{e}}{\partial t}})$$

Stability analysis (same as before)

Behavior of the closed-loop system :

$$\dot{\mathbf{e}} = \mathbf{L_e}\,\mathbf{v}_q + \frac{\partial \mathbf{e}}{\partial t} = -\lambda\,\mathbf{L_e}\widehat{\mathbf{L_e}}^{-1}\mathbf{e} \; - \; \mathbf{L_e}\widehat{\mathbf{L_e}}^{-1}\widehat{\frac{\partial \mathbf{e}}{\partial t}} + \frac{\partial \mathbf{e}}{\partial t}$$

If $\frac{\partial \mathbf{e}}{\partial t} = \widehat{\frac{\partial \mathbf{e}}{\partial t}} = 0$, $\|\mathbf{e}\|$ always decreases (global stability) if

$$\mathbf{L_e}\widehat{\mathbf{L_e}}^{-1} > 0$$

To suppress tracking errors and obtain the desired behavior $\dot{\mathbf{e}} = -\lambda\mathbf{e}$:

$$\widehat{\mathbf{L_e}}^{-1} = \mathbf{L_e} \quad \text{and} \quad \widehat{\frac{\partial \mathbf{e}}{\partial t}} = \frac{\partial \mathbf{e}}{\partial t}$$

In practice (similar as before)

- If $k = m = n$, $\boxed{\mathbf{W} = \mathbf{C} = \mathbb{I}_m}$, $\mathbf{e} = \mathbf{e}_1 = \mathbf{s} - \mathbf{s}^* \Rightarrow \mathbf{L_e} = \mathbf{L_s}, \widehat{\mathbf{L_e}} = \widehat{\mathbf{L_s}}$

$$\mathbf{v}_q = -\lambda\, \widehat{\mathbf{L_s}}^{-1} (\mathbf{s} - \mathbf{s}^*) - \widehat{\mathbf{L_s}}^{-1} \widehat{\frac{\partial \mathbf{s}}{\partial t}} \quad \text{stable if} \quad \mathbf{L_s}\, \widehat{\mathbf{L_s}}^{-1} > 0$$

- If $k > m$, $\boxed{\mathbf{C} = \mathbf{W}\widehat{\mathbf{L_s}}^{+}_{|\mathbf{s}=\mathbf{s}^*}}$, $\widehat{\mathbf{L_e}} = \mathbb{I}_n$,

$$\mathbf{v}_q = -\lambda\, \mathbf{e} - \widehat{\frac{\partial \mathbf{e}}{\partial t}} \quad \text{stable if} \quad \boxed{\mathbf{W}\widehat{\mathbf{L_s}}^{+}_{|\mathbf{s}=\mathbf{s}^*} \mathbf{L_s} \mathbf{W}^{+} > 0} \text{ (only around } \mathbf{s}^*\text{)}$$

If translational dof are controlled and 2D visual features are used, an estimation $\widehat{\mathbf{P}}$ or $\widehat{\mathbf{P}^*}$ is necessary to compute $\widehat{\mathbf{L_s}}$

Target tracking (same as before)

PI Controller

Integral term classical in Automatic Control to reduce tracking errors

If $\mathbf{I}_k$ is the estimation of $\boxed{\frac{\partial \mathbf{e}_1}{\partial t}}$ at iteration k, we have :

$$\begin{aligned} \mathbf{I}_{k+1} &= I_k + \mu\, \mathbf{e}_{1k} \ \text{ with } \mathbf{I}_0 = 0 \\ &= \mu \sum_{j=0}^{k} \mathbf{e}_{1j} \end{aligned}$$

Efficient to track a target at constant velocity :

$$\mathbf{I}_{k+1} = \mathbf{I}_k \text{ if } \mathbf{e}_{1k} = 0$$

Target tracking by estimating the target velocity

If it is possible to measure the camera velocity, we get:

$$\boxed{\widehat{\frac{\partial \mathbf{e}_1}{\partial t}} = \widehat{\dot{\mathbf{e}}_1} - \widehat{\mathbf{L}_{\mathbf{e}_1}}\, \mathbf{v}_c}$$

with

$$\begin{cases} \widehat{\mathbf{L}_{\mathbf{e}_1}} = \mathbf{C}\widehat{\mathbf{L}_{\mathbf{s}}} \\ \\ \widehat{\dot{\mathbf{e}}}_{1k} = \frac{\mathbf{e}_{1k} - \mathbf{e}_{1k-1}}{\Delta t} = \mathbf{C}\, \frac{\mathbf{s}_k - \mathbf{s}_{k-1}}{\Delta t} \end{cases}$$

A Kalman filter may then be used.