The Monogenic Framework:

A Systematic Approach to Image Processing and Computer Vision

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Coherent Framework

- Phase based image processing
- In scale-space
- Allows to derive applications in a systematic way

Phase Based Processing in nD

- Generalized quadrature filter?
- Generalized Hilbert transform?
- Generalized phase definition?
Scale-Space

- Scale relative image processing
- Efficiency
- Correctness

Systematic Derivations

- Complex IP & CV systems typically contain heuristic and systematic approaches
- Reduce the amount of heuristics
  - improve accuracy
  - reduce computational complexity
  - stability and robustness
  - correctness

Monogenic Framework

- New quadrature filter combine properties of
  - Gaussian derivatives (n-jets) and
  - Gabor filters
- The 2D phase approach allows to compute phase congruency which is
  - isotropic (no sampled orientation)
  - differential (no sampled scales)
- Relation of amplitude and phase
  - reconstruction from phase and amplitude
  - comparison of phase congruency and amplitude maxima

Monogenic Scale Space
Motivation Scale Space

Gnomonic Projection

Poisson Scale Space

2D Riesz Transform
**nD Riesz Transform**

- Generalizes the 1D analytic signal to nD
- Consists of n components
- Kernel: 
  \[ h(x) = \frac{x}{\omega_n |x|^n} \]
- Frequency response: 
  \[ H(u) = -\frac{u}{|u|} \]
- Solution by eigensystem

\[ m(x,s) = \begin{bmatrix} q(x,s) \\ p(x,s) \end{bmatrix} \]
\[ q(x,s) = (p|,s) * h(x) \]

**PDE Formulation**

1D: \( m \) corresponds to an analytic function
\[ = 2D \text{ harmonic field} \]
\[ \nabla_x \times m(x,s) = 0 \quad \text{zero curl} \]
\[ \nabla_x \cdot m(x,s) = 0 \quad \text{zero divergence} \]
if \( s > 0 \)
\[ p(x,0) = b(x) \quad \text{boundary condition} \]

**Finite Domain Solution**

- no intensity should enter or leave the image (zero Neumann boundary condition)
- effective filter kernel: reflection at the boundary
- alternative: extend the image

**Eigensystem**

- solution by eigensystem
- eigenfunctions: 
  \[ I_m(x, y) = 2 \cos(\pi x m) \cos(\pi y n) \]
- eigenvalues of the generator: 
  \[ \lambda = \pi \sqrt{m^2 + n^2} \]
- ‘frequency response’
  \[ \exp(-\pi \sqrt{m^2 + n^2} s) \]
Discrete Implementation

\[
C_{mn} = \alpha_m \sum_{x=0}^{X-1} \sum_{y=0}^{Y-1} b_{xy} \cos\left(\pi \left(x + \frac{1}{2}\right) \frac{m}{X}\right) \cos\left(\pi \left(y + \frac{1}{2}\right) \frac{n}{Y}\right)
\]

Finite Domain Implementation II

- DCT coefficients of potential
  
  \[
  C_{m,s} = \frac{C_m^p}{\exp(-\pi|m|s)}
  \]

- Gradient of potential
  
  \[
  q_{x,s} = -\nabla \psi_{x,s} = \sum_{m=0}^{X-1} \sum_{n=0}^{Y-1} C_{m,s} \begin{bmatrix}
  m \sin\left(\pi \left(x + \frac{1}{2}\right) \frac{m}{X}\right) \cos\left(\pi \left(y + \frac{1}{2}\right) \frac{n}{Y}\right) \\
  n \cos\left(\pi \left(x + \frac{1}{2}\right) \frac{m}{X}\right) \sin\left(\pi \left(y + \frac{1}{2}\right) \frac{n}{Y}\right)
  \end{bmatrix}
  \]

Algorithm Monogenic Scale Space

Spherical Quadrature Filter

- Combine radial bandpass with its Riesz transform
- Orthogonal like Gaussian derivatives / n-jets
- In quadrature like Gabor filters
**Truncated DOP Filter**

➲ truncate DOP filter at a certain energy of $l_1$

\[
l_p(x) = \frac{s_1}{2\pi ||x||^2 + \sigma_1^2}
\]

➲ ensure zero DC

\[
b_p(x) = \frac{l_1(x)}{\sum_x l_1(x) + \sum_x l_2(x)}
\]

➲ ensure same energy of components

\[
b_p(x) \Rightarrow \sqrt{\sum_x c_{bp}(x) / \sum_x b_p(x)^2 b_p(x)}
\]

**Monogenic Phase**

- attenuation: logarithm of Euclidean norm

\[
A = \log (\sqrt{q^2 + p^2}) = \frac{1}{2} \log (||q||^2 + p^2)
\]

\[
r = \frac{q}{p} \arctan \left( \frac{|q|}{p} \right)
\]
Applications of the Framework

➲ Flux gives an unbiased orientation estimate
  • regularized derivative operator ("derivator")
  • all frequency components are weighted the same
➲ Superior edge detection by phase congruency
  • differential phase congruency
  • no sampling of the orientation necessary (fast)
➲ Disparity estimation from local phase
  • no need for an a priori fixed orientation
  • optimal SNR
➲ Reconstruction from local amplitude and phase
  • extremely simple
  • fast and accurate

Orientation Estimation

➲ noisy image with different local frequencies

Edge & Line Detection by PC

Filter for PC Estimation

\[
\partial_s r = \partial_s \frac{\mathbf{q}}{||\mathbf{q}||} \arctan \left( \frac{||\mathbf{q}||}{p} \right) = \frac{\mathbf{q}}{||\mathbf{q}||} \frac{1}{1 + \frac{||\mathbf{q}||^2}{p^2}} - \frac{p \mathbf{q} - \mathbf{q} \mathbf{p}}{||\mathbf{q}||^2 + p^2} \]

\[
= \frac{bp \ast b \ast (\mathbf{c} \mathbf{b} \ast b) - (\mathbf{c} \mathbf{b} \ast b) \ast (bp \ast b)}{||\mathbf{c} \mathbf{b} \ast b||^2 + (bp \ast b)^2}
\]
Phase Congruency through Scale

Disparity Estimation

Disparity from Local Phase

- Faster than other appearance based approaches (e.g. correlation based)
- Feature / model based approaches produce more ambiguities
- Sub-pixel accuracy
Disparity Algorithm

- Compute scale pyramid
- For each scale (C2P)
  - estimate disparity
  - add to old estimate
  - upsample and CS
  - compensate disparity in next scale
- Add final estimate

Taylor Expansion of Phase

- Taylor series expansion for phase vector \((d|r)\)
  
  \[ r(x + \frac{d}{2}) = r(x) + \frac{d}{2} \nabla r(x) + O(|d|^2) \]

- Solve for normal disparity
  
  \[ d(x) \approx 2 \frac{r^T(x) - r^2(x)}{\nabla \cdot r(x) + \nabla \cdot r^2(x)} \]

- The local frequency (divergence of the phase vector) can be computed without using inverse trigonometric functions (see PC)

Advantages

- Disparity orientation \(e\) is a priori known:
  - Disparity is obtained by projection
    
    \[ d(x) = \frac{|d(x)|}{e \cdot d(x)} \]
  - Correct confidence can be computed explicitly
- Disparity orientation is unknown:
  - Estimate \(e\) from the normal disparities
  - Continue as in the first case

Synthetic Experiment
**Disparity from Phase**

**Attenuation-Phase Scale Space**

**Attenuation & Phase**

- Known: complex derivative of complex logarithm of an analytic function is analytic

\[ \partial_z \log(f(z)) = \frac{\partial_z f(z)}{f(z)} \]

- Generalizes to 2D:

\[
\begin{bmatrix}
  r \\
  A
\end{bmatrix} = \log
\begin{bmatrix}
  q \\
  p
\end{bmatrix}
\]

is a harmonic field

**Zeros in Monogenic Scale-Space**
Fundamental Relations

\[ \frac{\partial}{\partial x} r_2 - \frac{\partial}{\partial y} r_1 = 0 \quad \nabla \cdot r + A_s = 0 \quad \frac{\partial}{\partial x} A - \frac{\partial}{\partial y} r_1 = 0 \quad \frac{\partial}{\partial y} A - \frac{\partial}{\partial x} r_2 = 0 \]

\( \omega = \nabla \cdot r - A_s \)

local maxima = PC

\( r \approx h \ast A \)

reconstruction from attenuation / phase

\( b \approx \exp(A) \cos(|r|) \)

PC & Local Amplitude Maxima

phase congruency = local maxima of amplitude

Global Reconstruction

original image

reconstruction from ...

local attenuation

abs. error
Conclusion

- The monogenic scale space approach can be implemented
  - globally by DCT and
  - locally by SQFs in the spatial domain.
- PC and local frequency can be computed from implicit derivatives.

- Future topics include
  - further applications (e.g. optical flow)
  - JAVA demo / implementation
  - extension to higher dimensions (straightforward).