Descriptions Using Moments

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Moments

Family of stable binary (and grey level) shape descriptions

Can be made invariant to translation, rotation, scaling

Let $\{p_{rc}\}$ be the binary (0,1) image pixels for row r and col c where 1 pixels are the object

Moments II

Area $A = \sum_r \sum_c p_{rc}$

Center of mass

$(\hat{r}, \hat{c}) = (\frac{1}{A} \sum_r \sum_c r p_{rc}, \frac{1}{A} \sum_r \sum_c c p_{rc})$

A family of 'central' (translation invariant) moments (for any u and v):

$$m_{uv} = \sum_r \sum_c (r - \hat{r})^u (c - \hat{c})^v p_{rc}$$

Subtracting center of mass makes it translation invariant

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Scale invariant moments

If double in dimensions, then moment m_{uv} increases by $2^u 2^v$ for weightings and 4 for the number of pixels.

Similarly, area A increases by 4, and thus $A^{(u+v)/2+1}$ increases by $4 \times 2^u 2^v$

So, the ratio:

$$\mu_{uv} = \frac{m_{uv}}{A^{(u+v)/2+1}}$$

is invariant to scale.

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Rotation invariant moments

Moment invariant theory has identified methods to generate various orders of moments invariant to rotation.

6 functions ci_i with rescaling applied to get into similar numerical ranges

Area $A = \sum_r \sum_c p_{rc}$
Center of mass $(\hat{r}, \hat{c})$

Define complex uv central moment:
$c_{uv} = \sum_r \sum_c ((r - \hat{r}) + i(c - \hat{c}))^u ((r - \hat{r}) - i(c - \hat{c}))^v p_{rc}$

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Scale invariance

Get specific scale invariant moments:

$s_{11} = c_{11}/(A^2)$

$s_{20} = c_{20}/(A^2)$

$s_{21} = c_{21}/(A^{2.5})$

$s_{12} = c_{12}/(A^{2.5})$

$s_{30} = c_{30}/(A^{2.5})$

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Rotation invariant moments II

Rescaled (so values in similar range) rotation invariants:

$ci_1 = real(s_{11})$

$ci_2 = real(1000 * s_{21} * s_{12})$

$ci_3 = 10000 * real(s_{20} * s_{12} * s_{12})$

$ci_4 = 10000 * imag(s_{20} * s_{12} * s_{12})$

$ci_5 = 1000000 * real(s_{30} * s_{12} * s_{12} * s_{12})$

$ci_6 = 1000000 * imag(s_{30} * s_{12} * s_{12} * s_{12})$

Scaled Moment matlab code

```
function vec = getproperties(Image)
     area = bwarea(Image);
     perim = bwarea(bwperim(Image,4));
     compactness = perim*perim/(4*pi*area);
     s11 = complexmoment(Image,1,1) / (area^2);
     s20 = complexmoment(Image,2,0) / (area^2);
     ...
     ci1 = real(s11);
     ci2 = real(1000*s21*s12);
     ci3 = 10000*real(s20*s12*s12);
     ...
```


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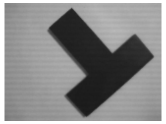
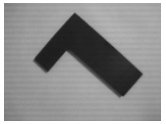
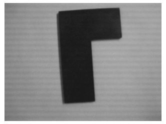
Example invariant property values

			
compactness	1.93	1.81	1.90
ci_1	0.23	0.27	0.25
ci_2	0.18	0.37	0.45
ci_3	0.08	-0.50	0.11
ci_4	-0.00	0.37	-0.64
ci_5	0.23	-0.47	0.09
ci_6	-0.00	0.07	-0.63

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Feature Vector

Standard description for many visual processes: form a vector from set of descriptions:

$\vec{x} = (compactness, ci_1, ci_2, ci_3, ci_4, ci_5, ci_6)'$

Multiple vectors if several structures or locations to describe

These vectors are then used in next processes, eg. recognition

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Lecture Overview

1. Moments: an infinite family of shape descriptions
2. A way to make them invariant to rotation, translation, and scale

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