Descriptions Using Moments

Robert B. Fisher
School of Informatics
University of Edinburgh

Moments

Family of stable binary (and grey level) shape descriptions

Can be made invariant to translation, rotation, scaling

Let $\{p_{rc}\}$ be the binary (0,1) image pixels for row r and col c where 1 pixels are the object

Moments II

Area $A = \sum_r \sum_c p_{rc}$
Center of mass
$(\hat{r}, \hat{c}) = (\frac{1}{A} \sum_r \sum_c r p_{rc}, \frac{1}{A} \sum_r \sum_c c p_{rc})$

A family of 'central' (translation invariant) moments (for any u and v):

$$m_{uv} = \sum_r \sum_c (r - \hat{r})^u (c - \hat{c})^v p_{rc}$$

Subtracting center of mass makes it translation invariant

Scale invariant moments

If double in dimensions, then moment m_{uv} increases by $2^u 2^v$ for weightings and 4 for the number of pixels.
Similarly, area A increases by 4, and thus $A^{(u+v)/2+1}$ increases by $4 \times 2^u 2^v$

So, the ratio:

$$\mu_{uv} = \frac{m_{uv}}{A^{(u+v)/2+1}}$$

is invariant to scale.

Rotation invariant moments

Moment invariant theory has identified methods to generate various orders of moments invariant to rotation.

6 functions ci_i with rescaling applied to get into similar numerical ranges

Area $A = \sum_r \sum_c p_{rc}$
Center of mass $(\hat{r}, \hat{c})$

Define complex uv central moment:
$c_{uv} = \sum_r \sum_c ((r - \hat{r}) + i(c - \hat{c}))^u ((r - \hat{r}) - i(c - \hat{c}))^v p_{rc}$

 Slide credit: Bob Fisher

Scale invariance

Get specific scale invariant moments:
$s_{11} = c_{11}/(A^2)$
$s_{20} = c_{20}/(A^2)$
$s_{21} = c_{21}/(A^{2.5})$
$s_{12} = c_{12}/(A^{2.5})$
$s_{30} = c_{30}/(A^{2.5})$

 Slide credit: Bob Fisher

Rotation invariant moments II

Rescaled (so values in similar range) rotation invariants:
$ci_1 = real(s_{11})$
$ci_2 = real(1000 * s_{21} * s_{12})$
$ci_3 = 10000 * real(s_{20} * s_{12} * s_{12})$
$ci_4 = 10000 * imag(s_{20} * s_{12} * s_{12})$
$ci_5 = 1000000 * real(s_{30} * s_{12} * s_{12} * s_{12})$
$ci_6 = 1000000 * imag(s_{30} * s_{12} * s_{12} * s_{12})$

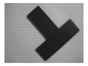
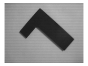
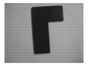
 Slide credit: Bob Fisher

Scaled Moment matlab code

```
function vec = getproperties(Image)
     area = bwarea(Image);
     perim = bwarea(bwperim(Image,4));
     compactness = perim*perim/(4*pi*area);
     s11 = complexmoment(Image,1,1) / (area^2);
     s20 = complexmoment(Image,2,0) / (area^2);
     ...
     ci1 = real(s11);
     ci2 = real(1000*s21*s12);
     ci3 = 10000*real(s20*s12*s12);
     ...
```

 Slide credit: Bob Fisher

Example invariant property values

			
compactness	1.93	1.81	1.90
ci_1	0.23	0.27	0.25
ci_2	0.18	0.37	0.45
ci_3	0.08	-0.50	0.11
ci_4	-0.00	0.37	-0.64
ci_5	0.23	-0.47	0.09
ci_6	-0.00	0.07	-0.63

Feature Vector

Standard description for many visual processes: form a vector from set of descriptions:

$$\vec{x} = (compactness, ci_1, ci_2, ci_3, ci_4, ci_5, ci_6)'$$

Multiple vectors if several structures or locations to describe

These vectors are then used in next processes, eg. recognition

Lecture Overview

1. Moments: an infinite family of shape descriptions
2. A way to make them invariant to rotation, translation, and scale