Image Geometry

Bob Fisher
School of Informatics
University of Edinburgh

Many slides in this lecture are due to other authors; they are credited at the bottom
Let’s design a camera

Idea 1: put a piece of film in front of an object
Do we get a reasonable image?
Pinhole camera

Add a barrier to block off most of the rays

• This reduces blurring
• The opening is known as the aperture
Pinhole camera model

Pinhole model:

• Captures *pencil of rays* – all rays through a single point
• The point is called **Center of Projection (focal point)**
• The image is formed on the **Image Plane**
Dimensionality reduction: from 3D to 2D

3D world

Point of observation

2D image

What is preserved?
• Straight lines, incidence

What have we lost?
• Angles, lengths
Projection properties

• Many-to-one: any points along same visual ray map to same point in image
• Points $\rightarrow$ points
  • But projection of points on focal plane is undefined
• Lines $\rightarrow$ lines (collinearity is preserved)
  • But lines through focal point (visual rays) project to a point
• Planes $\rightarrow$ planes (or half-planes)
  • But planes through focal point project to lines
Vanishing points

• Each direction in space has its own vanishing point
  • All lines going in that direction converge at that point
  • Exception: directions parallel to the image plane
Modeling projection

The coordinate system

- The optical center \((O)\) is at the origin
- The image plane is parallel to xy-plane (perpendicular to z axis)

Source: J. Ponce, S. Seitz
Modeling projection

Projection equations

- Compute intersection with image plane of ray from \( P = (x,y,z) \) to \( O \)
- Derived using similar triangles

\[
(x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z}, f \right)
\]

- We get the projection by throwing out the last coordinate:

\[
(x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)
\]

Source: J. Ponce, S. Seitz
Homogeneous coordinates

\[(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z}\right)\]

Is this a linear transformation?
- no—division by \(z\) is nonlinear

Trick: add one more coordinate:

\[
\begin{align*}
(x, y) & \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} & (x, y, z) & \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\end{align*}
\]

homogeneous image coordinates \hspace{1cm} \text{homogeneous scene coordinates}

Converting \textit{from} homogeneous coordinates

\[
\begin{align*}
\begin{bmatrix} x \\ y \\ w \end{bmatrix} & \Rightarrow \left(\frac{x}{w}, \frac{y}{w}\right) & \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} & \Rightarrow \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)
\end{align*}
\]
Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates.
Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
z/f \\
1
\end{bmatrix}
\Rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)
\]

divide by the third coordinate
Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f & 0 \\
0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} =
\begin{bmatrix}
x \\
y \\
z/f \\
1
\end{bmatrix}
\Rightarrow (f \frac{x}{z}, f \frac{y}{z})
\]

divide by the third coordinate

In practice: lots of coordinate transformations…

\[
\begin{bmatrix}
2D \\
\text{point} \\
(3x1)
\end{bmatrix}
= \begin{bmatrix}
\text{Camera to} \\
\text{pixel coord.} \\
\text{trans. matrix} \\
(3x3)
\end{bmatrix} \begin{bmatrix}
\text{Perspective} \\
\text{projection matrix} \\
(3x4)
\end{bmatrix} \begin{bmatrix}
\text{World to} \\
\text{camera coord.} \\
\text{trans. matrix} \\
(4x4)
\end{bmatrix} \begin{bmatrix}
3D \\
\text{point} \\
(4x1)
\end{bmatrix}
\]
What have we learned?

- Pinhole camera model concept
- Geometry of projection
- Mathematics of projection
- Elementary homogeneous coordinates