Image Geometry

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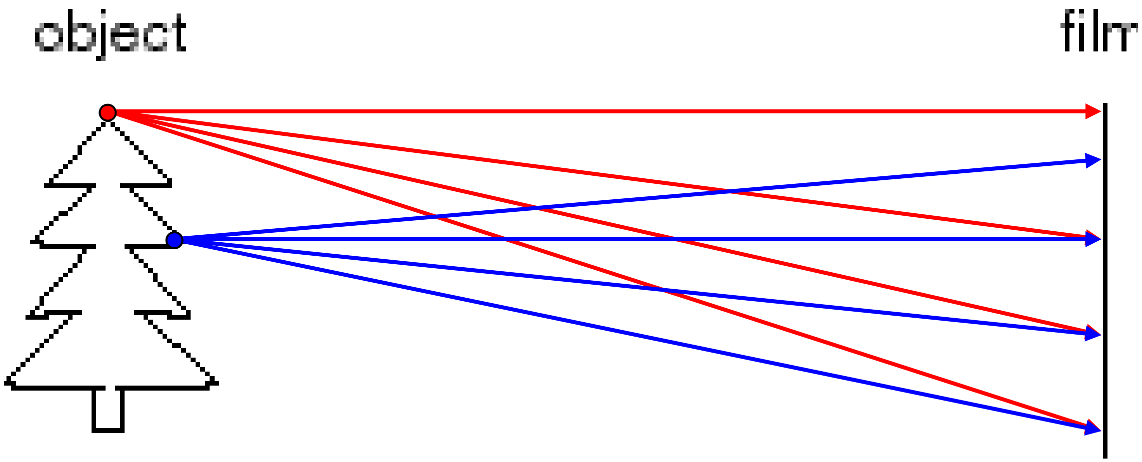
Many slides in this lecture are due to other authors; they are credited at the bottom

Let’s design a camera

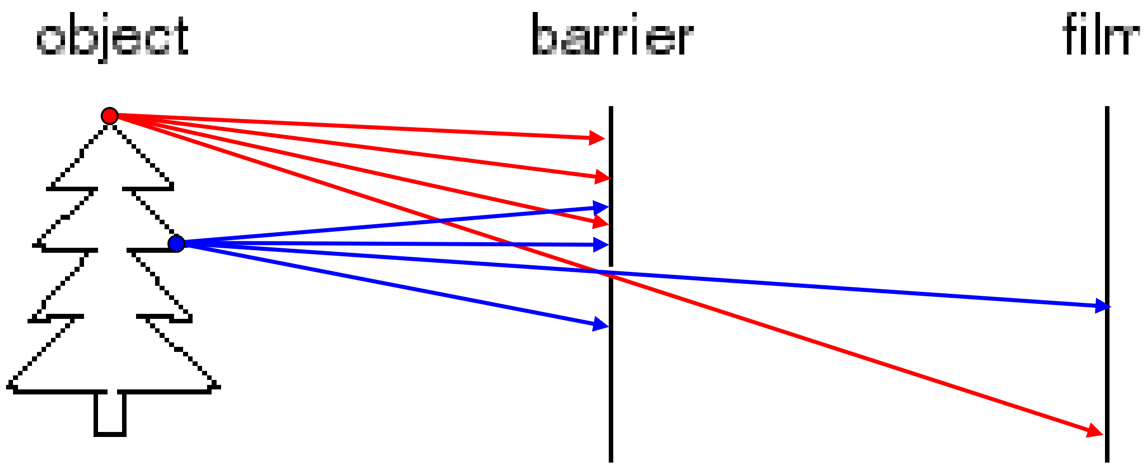


Idea 1: put a piece of film in front of an object

Do we get a reasonable image?

Slide by Steve Seitz

Pinhole camera

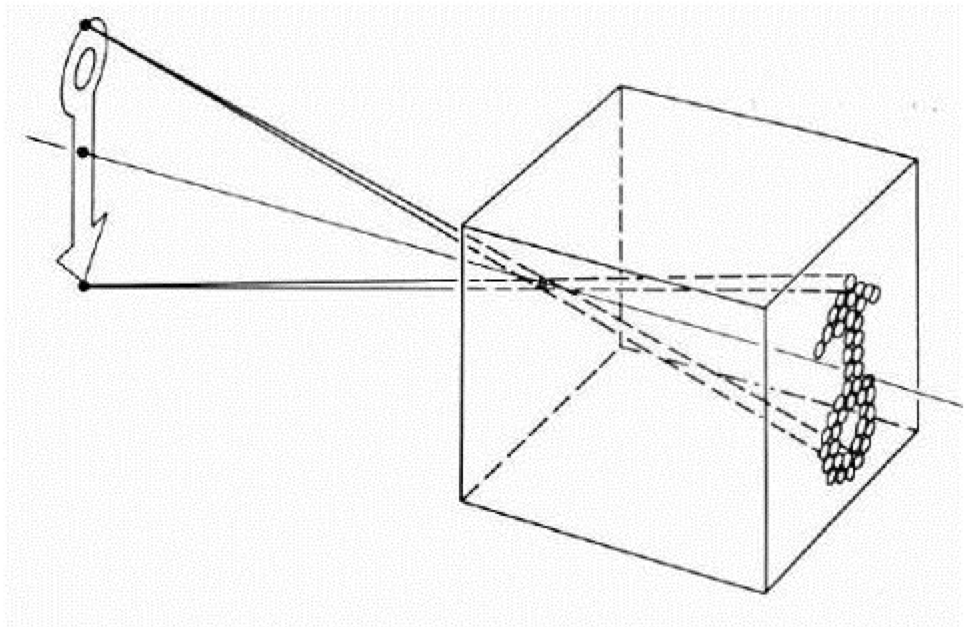


Add a barrier to block off most of the rays

- This reduces blurring
- The opening is known as the **aperture**

Slide by Steve Seitz

Pinhole camera model



Pinhole model:

- Captures **pencil of rays** – all rays through a single point
- The point is called **Center of Projection (focal point)**
- The image is formed on the **Image Plane**

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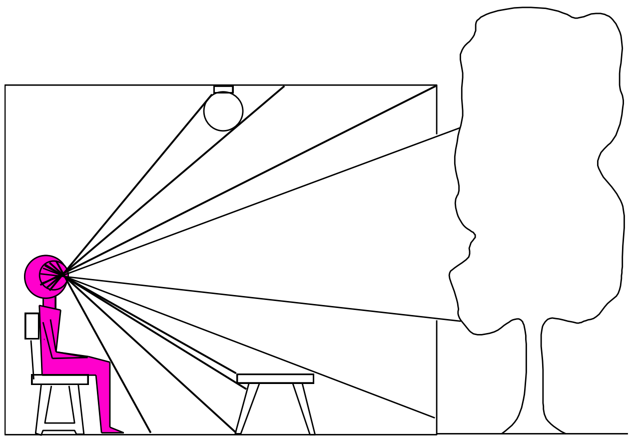
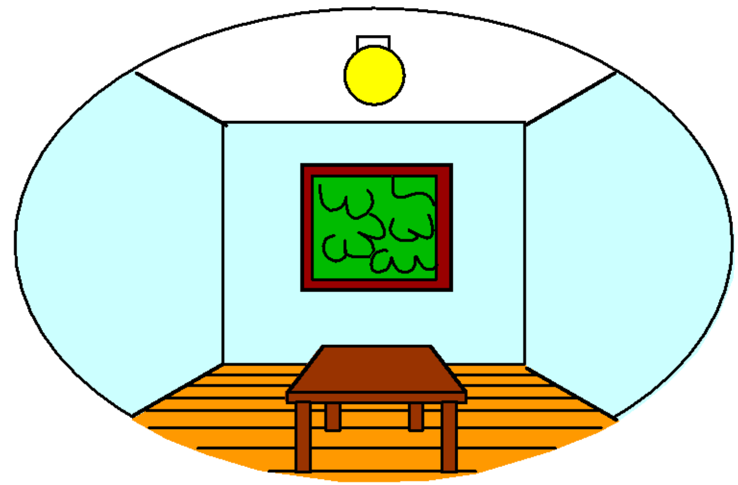
Dimensionality reduction: from 3D to 2D

3D world



Point of observation

2D image



What is preserved?

- Straight lines, incidence

What have we lost?

- Angles, lengths

Slide by A. Efros

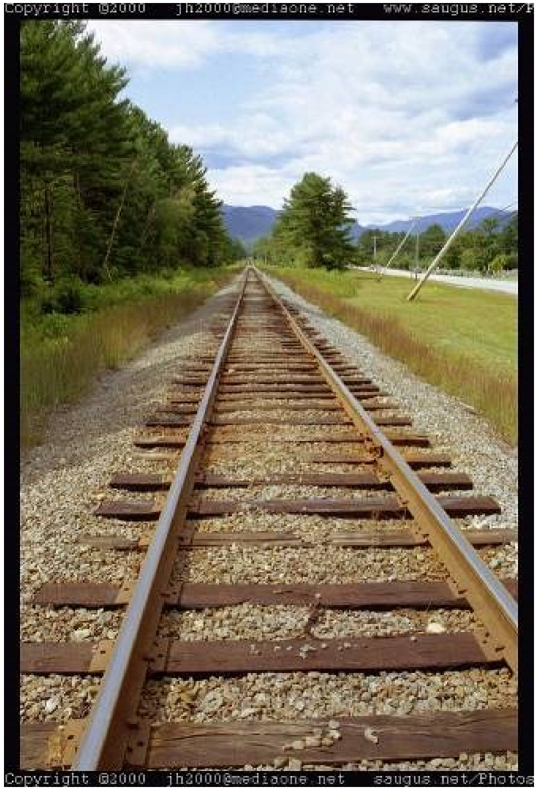
Projection properties

- Many-to-one: any points along same *visual ray* map to same point in image
- Points → points
  - But projection of points on *focal plane* is undefined
- Lines → lines (collinearity is preserved)
  - But lines through focal point (visual rays) project to a point
- Planes → planes (or half-planes)
  - But planes through focal point project to lines

slide credit: Lazebnik

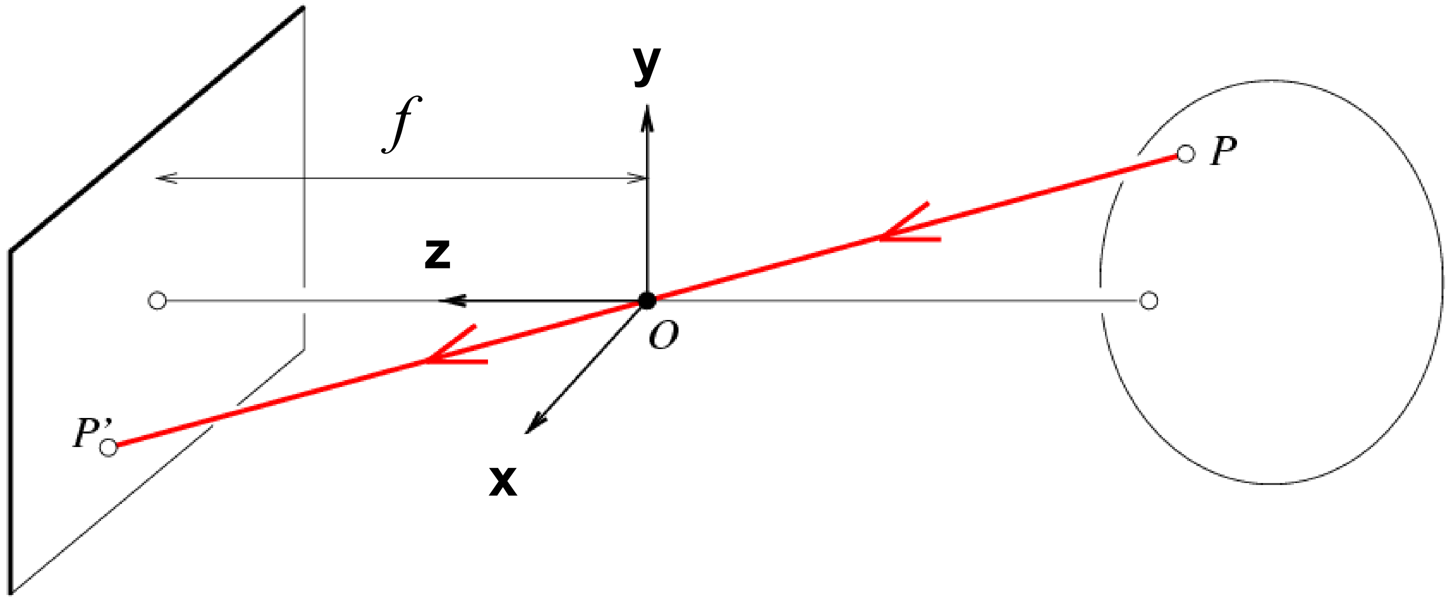
Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane



slide credit: Lazebnik

Modeling projection

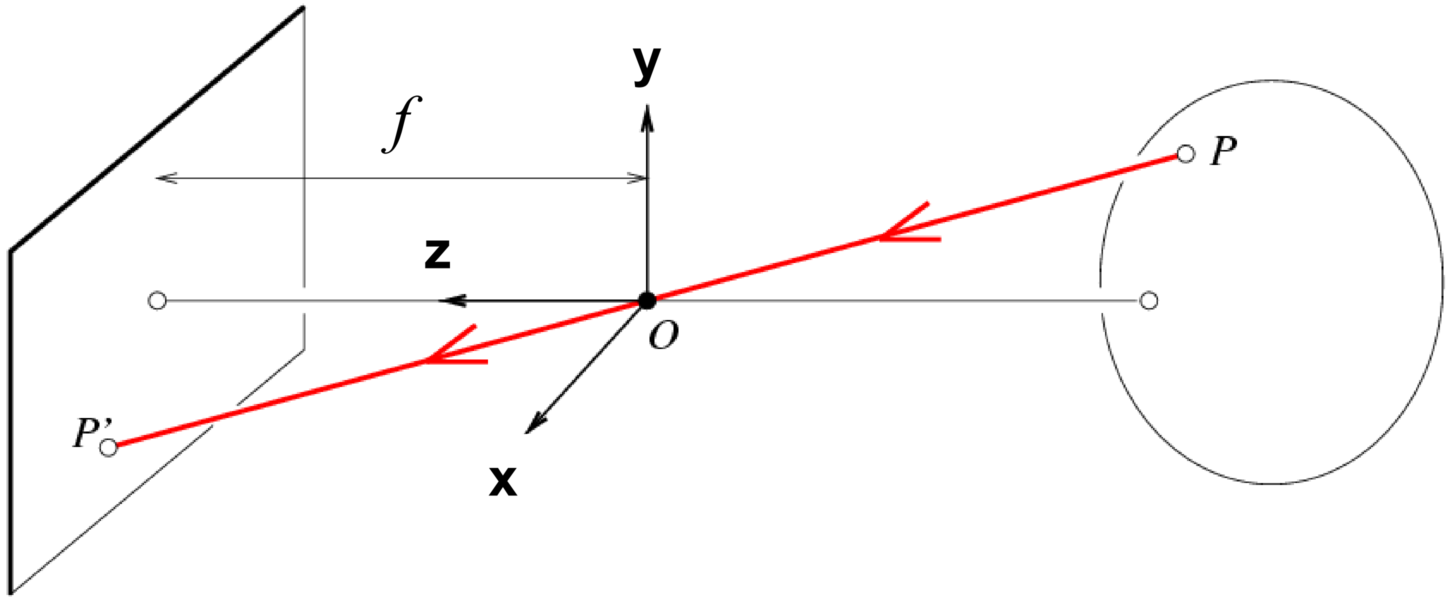


The coordinate system

- The optical center (***O***) is at the origin
- The image plane is parallel to xy-plane (perpendicular to z axis)

Source: J. Ponce, S. Seitz

Modeling projection



Projection equations

- Compute intersection with image plane of ray from ***P*** = (x,y,z) to ***O***
- Derived using similar triangles

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, f)$$

- We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Source: J. Ponce, S. Seitz

Homogeneous coordinates

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by the third coordinate

Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates

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divide by the third coordinate

In practice: lots of coordinate transformations…

$$\begin{pmatrix} \text{2D} \\ \text{point} \\ \text{(3x1)} \end{pmatrix} = \begin{pmatrix} \text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ \text{(3x3)} \end{pmatrix} \begin{pmatrix} \text{Perspective} \\ \text{projection matrix} \\ \text{(3x4)} \end{pmatrix} \begin{pmatrix} \text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ \text{(4x4)} \end{pmatrix} \begin{pmatrix} \text{3D} \\ \text{point} \\ \text{(4x1)} \end{pmatrix}$$

What have we learned?

- Pinhole camera model concept
- Geometry of projection
- Mathematics of projection
- Elementary homogeneous coordinates