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Improvement of Quadric Surfaces Estimation of Manufactured Objects.

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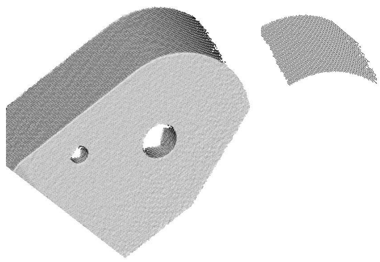
1 Introduction

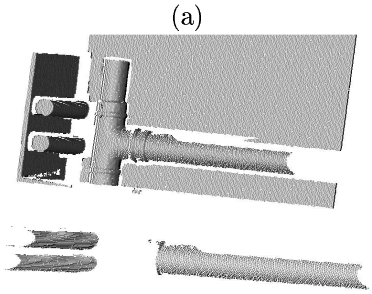
Common quadric surfaces such as cylinders, cones and spheres are found in most manufactured parts and objects. A reliable estimation of these surfaces is an essential requirement in object modelling or reverse engineering, where a faithful model is needed to be extracted from the set of range data for CAD/CAM purposes.

One obstacle to achieving this goal is the inaccuracy of shape estimates from the extracted quadric patches. This arises from the limited field of the sensor which can only cover a partial area of an object in a given view, self or external occlusion of the object and finally some surface data may be lost during the surface segmentation process either due to segmentation failure or intentionally in order to avoid unreliable data. The usable set of data points may thus represent only a small area of the surface (Figure 1) and consequently give unstable estimates of the surface shape. Furthermore the remaining available data is corrupted by measurement noise. Consequently the surface fitting fails to give a reliable estimation of the surface shape. The estimates are highly biased and may not reflect the actual type of the surface even when sophisticated techniques are applied.

The idea presented here is to compensate the poorness of information embodied in the quadric surface data by extra knowledge about the surface such as the surface type and relationships with other nearby surfaces. This additional information is either provided by the model in the case of model-based applications or deduced from a set of potential hypotheses generated, checked and verified within a perceptual organization process. E.g if preliminary estimates of a cylinder and plane lead to an angle between the plane normal and the cylinder axis close to 90^o, it is very likely that the two surfaces are orthogonal, or if the estimated shape of a quadric is an elliptic



(a)



(b)

Figure 1: (a) Object containing a cylindrical surface, only a small area of of the cylinder surface is visible. (b) A miniaturised model of a plant: because of the noise and the segmentation errors only small portions of the pipes are extracted and can be used reliably for the estimation.

cylinder with major axis and minor axis having nearly identical values it is very likely that the cylinder is circular.

The exploitation of this extra information is quite feasible since a patch is rarely captured alone in the scene but rather with close or adjacent surfaces which could be either planes or quadrics.

This paper shows how the extra information can be represented in a shape estimation process and then evaluates the estimation process against several alternatives, concluding that the extra information is both effective and easy to exploit.

2 Problem statement and related work

A quadric surface S is represented by the implicit function:

$$f(x, y, z, \vec{p}) = ax^2 + by^2 + cz^2 + 2hxy + 2gxz + 2fyz + 2ux + 2vy + 2wz + d = 0 \quad (1)$$

Given a set of N measurement points X_i we want to find the parameter vector $\vec{p} = [a, b, c, h, g, f, u, v, w, d]$ such that the function defined by (1) reflects as well as possible the actual shape of the surface. The type and shape characteristics of the surface are deduced afterwards from $\vec{p}$.

A reasonable criterion to judge the goodness of the solution is the sum of the squared Euclidean distances between each measurement point and the surface $J = \sum_{i=1}^{N} d(X_i, S)^2$. The parameter vector minimizing this criterion is the best solution in the least squares sense. Unfortunately the non-linearity of this distance measure does not lead to a nice and easy closed-form solution for the parameter vector $\vec{p}$. Various approximations of this distance have been therefore proposed in the literature to make the minimization problem easier. The most common one is using the value of the implicit function $f(x, y, z)$ known as the algebraic distance. It has been used in recovering planes and quadrics [2, 5]. Although this approximation is highly attractive because of its closed-form solution, it was subject to many criticisms since it leads to a highly biased estimation for small surfaces with low curvature. An improved approximation was suggested by expanding the implicit function into Taylor's series up to first or second degree [11, 12]. For some particular quadrics the first approximation leads to a closed-form solution. But generally the problem is a non-linear minimization which needs to be solved iteratively [7, 8, 10].

Another way for considering the Euclidean distance is to use a specific representation function for a particular case of quadric surfaces, like the circular cylinder, circular cone and sphere. With this representation the value of the function at a given point is the squared distance between this point and the surface. This representation was used in [1, 4]. In both works the solution was found with a non-linear optimization.

A common characteristic of these works is that they treated each single surface individually. When the quadric patch to be fitted covers a small amount of the surface, the fitting technique fails to give a reasonable estimation of the surface and often the estimates are highly biased. This is expected since second order functions can easily trade-off curvature and position to produce similar error measures. Thus small patches do not provide sufficient extent to distinguish between the two cases.

However if we place ourselves in an object modelling and a reverse engineering framework we usually have to fit many surfaces belonging to the same object and which are linked by some geometrical and topological relationships. By exploiting this knowledge together with the information which may be available about the quadric type and shape we hope compensate the lack of information in the quadric patch and obtain therefore a surface parameterization as accurate as possible. The geometrical relations used in this work are the relative orientation and position between a quadric surface axes or centres with respect to other surfaces (planes or quadrics), the equality of some quadric features values (e.g. cylinder and sphere having the same radius value.) and specific shape characteristics of the quadric, for instance the circularity.

3 Principle of the approach

For a given object defined by a parameter vector $\vec{p}$ containing all the parameters of the different surfaces belonging to the object, we set up the following optimization function

$$E(\vec{p}) = \vec{p}^T \mathcal{H} \vec{p} + \sum_{k=1}^{K} \lambda_k C_k(\vec{p}) \tag{2}$$

where the first term represents the minimization criterion and the second term a set of weighted constraint functions defining the relationships between the surfaces of the object, for instance planes and quadrics, as well as some characteristics of the quadrics such as circularity. More details about the formulation of the constraints are given in [13]. The parameter vector $\vec{p}$ minimizing (2) will then fit best the measurement points while satisfying the constraints. The determination of of the parameter vector is achieved with a sequential unconstrained technique [13]. The algorithm increments sequentially the set of weights and at each step (2) is minimized with the standard Levenberg-Marquardt technique and the vector $\vec{p}$ is updated. The algorithm stops when the constraints are satisfied to the desired degree or when the parameter vector remains stable for a certain number of iterations. The initial parameter $\vec{p}_o$ is determined by estimating each surface individually with a generalized eigenvalue technique [2] and then concatenating all the vectors into a single one.

4 Experiments

A series of experiments were performed on several real objects having planar and quadric surfaces. Our approach were compared with three main techniques covering a large part of the spectrum of the fitting techniques developed in the literature. These techniques are the eigenvalues solution based on the algebraic distance [2, 5], the eigenvalue technique [11] based on the approximation of the Euclidean distance and the iterative optimization technique [1, 4] based on the specific representations of a quadric: the circular cone, the circular cylinder and the sphere. These techniques are referenced respectively by AD, AED, SR and the new suggested global fitting approach by GF. The performances of the different techniques are evaluated by comparing the shape parameters of the quadrics, for instance the half angle for the cone and the radius for the cylinder and the sphere.

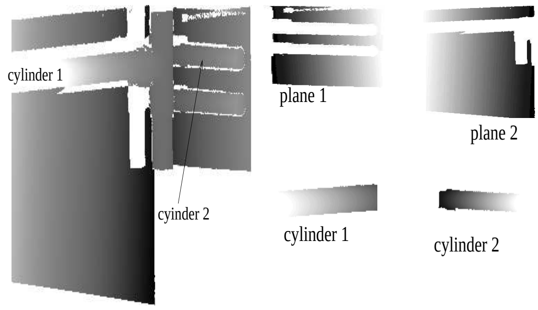


Figure 2: View of object 5 with the extracted surfaces.

AD	AED	SR	GF	true surface
ell.cylinder	ell.cylinder	cir.cylinder	cir.cylinder	cir.cylinder
$r1_{max} = 17.69$	$r1_{max} = 9.01$	$r1 = 8.08$	$r = 7.44$	$r1 = 7.50$
$r1_{min} = 12.12$	$r1_{min} = 8.13$			
$r2_{max} = 4.96$	$r2_{max} = 5.67$	$r2 = 5.23$	$r2 = 4.95$	$r1 = 5.00$
$r2_{min} = 4.28$	$r2_{min} = 5.24$			

Table 1: Estimates of the cylinder patches of object 5.

The computation time was taken into account as well. With AD, and AED the estimation time is almost instantaneous, whereas it varies from half an hour to several hours for the SR depending on the number of measurement points. For the GF technique it in the range of minutes. The different techniques procedures were implemented with Matlab on 200 MHz Sun Ultrasparc workstation.

Because of the limited space only two experiments are presented in this paper. The first object (Figure 2) is a plant model containing several pipes. The results are grouped in Table 1. The AD and the AED give elliptic cylinder estimates whereas the SR and the GF have the wright shape estimates with a better accuracy for GF, in the order of few hundredth of mm.

The second object contains a cylindrical and a conical surface (Figure 3) The results are shown in Table 2. Here again we notice that the AD and the AED give a biased shape estimates for the cylinders. The same is noticed for the cone estimate with the AD. The AED could not be applied for the cone since the gradient module is not constant along a cone surface. The SR and the GR have a faithful shape estimates with accurate values for the cylinder. The cone estimates values are less accurate however the GF has better estimate..

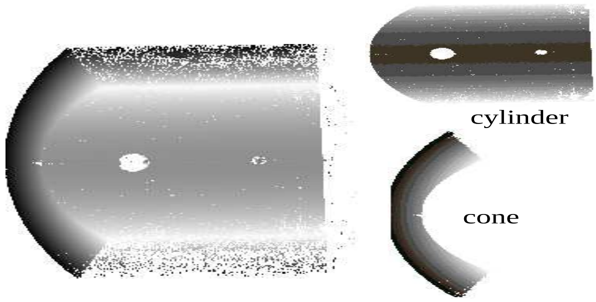


Figure 3: View of object 6 with the extracted surfaces.

AD	AED	SR	GF	true surface
ell.cylinder $r_{max} = 46.10$ $r_{min} = 33.66$	ell.cylinder $r_{max} = 57.62$ $r_{min} = 55.42$	cir.cylinder $r = 59.81$	cir.cylinder $r = 59.54$	cir.cylinder $r1 = 60$
ell.cone $\alpha_{max} = 28.86^o$ $\alpha_{min} = 25.19^o$	- - -	cir.cone $\alpha = 26.84^o$	cir.cone $\alpha = 31.80^o$	cir.cone $\alpha = 30^o$

Table 2: Estimates of the cylinder and the cone patches of object 6.

5 Conclusion

The comparison of the results issued from the different methods show that The GF methods has the best performances in term of accuracy and time consuming. The GF algorithm ensures both faithful shape estimation for the quadric surface with accurate feature value estimation. The accuracy is far away better than AD and AED and more better than the SR, however the time complexity of the latter technique (hours whereas it is a few minutes for the GF) makes the GF more advantageous and convenient for interactive applications.

The optimization technique used in the GF algorithms supposes a reasonable initialisation of the surface parameter vector. Although this condition limits the field of application of the technique, it is well satisfied in the framework of object modelling and reverse engineering. We propose to use the estimates given by the AD or when possible the AED as initialization.

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