Sub-pixel estimation

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No Institute Given

Sub-pixel estimation is the process of estimating the value of a geometric quantity to better than pixel accuracy, even though the data was originally sampled on an integer pixel quantized space.

1 Background

It is naïvely assumed that information at a scale smaller than the pixel level is lost when continuous data is sampled or quantized into pixels from *e.g.* time varying signals, images, data volumes, space-time volumes, etc. However, in fact, it may be possible to estimate geometric quantities to better than the original pixel accuracy. The underlying foundations of this estimation are:

– **Models of expected spatial variation**: discrete structures, such as edges or lines, produce characteristic patterns of data when measured, allowing fitting of a model to the data to estimate the parameters of the structure.
– **Spatial integration during sampling**: sensors typically integrate a continuous signal over a finite domain (space or time), leading to measurements whose values depend on the relative position of the sampling window and the original structure.
– **Point spread function**: knowledge of the PSF could be used, *e.g.* by deconvolution of a blurred signal, to estimate the position of the signal.

Applications commonly benefitting from subpixel estimation are 1) camera calibration and triangulation (*e.g.* in stereo and structured light depth estimation) and 2) image motion estimation for improved image stabilization and compression.

One of the earliest instances of subpixel edge detection in computer vision research was by MacVicar-Whelan and Binford [13] in 1981.

The accuracy of subpixel estimation depends on a number of factors, such as the image point spread function, noise levels and spatial frequency of the image data. A commonly quoted rule of thumb is 0.1 pixel, but lower is achievable, *e.g.* about 0.02 pixel is shown for stripe position detection in [1].

2 Theory

There are four common approaches to estimating subpixel positions:

1. **Interpolation**: An example is in subpixel edge position estimation, which is demonstrated here in one dimension in ideal form in Figure 1. One can see that $f(x)$ is a function of the edge's actual position within a pixel and the values at adjacent pixels. Here we assume that the pixel 'position' refers to the center of the pixel. Let δ be the offset of the true edge position away from the pixel center. Then, one can model the value $f(x)$ at x in terms of the values at the neighbors, assuming a step function:

$$f(x) = (\frac{1}{2} + \delta) * f(x-1) + (\frac{1}{2} - \delta) * f(x+1)$$

   from which we can solve for the subpixel edge position $x + \delta$ by:

$$\delta = \frac{2f(x) - f(x-1) - f(x+1)}{2(f(x-1) - f(x+1))}$$

   Another approach is to interpolate a continuous curve (or surface) and then find the optimal position on the reconstructed curve (*e.g.* by using correlation for curve registration).

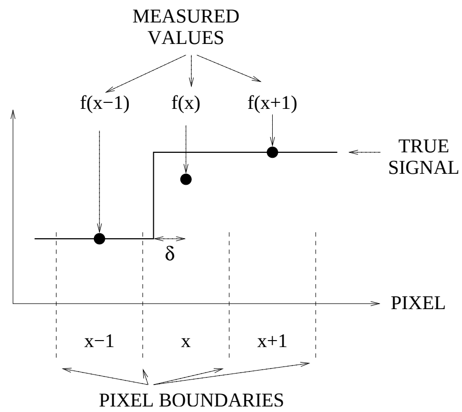

Fig. 1. The values of $f(x)$ created by integrating the continuous signal over the whole pixel.

2. **Integration**: An example is the estimation of the center point of a circular dot, such as required for control point localization in a camera calibration scheme. The assumption is that the minor deviations from many boundary pixels can be accumulated to give a more robust estimate. Suppose that $g(x, y)$ are the grey levels of a light circle on a dark background, where (x, y) are in a neighborhood N closely centered on the circle. Assume also that the mean dark background level has been subtracted from all values. Then, the center of the dot is estimated by its grey-level center of mass:

$$\hat{x} = \frac{\sum_{(x,y)\in N} xg(x, y)}{\sum_{(x,y)\in N} g(x, y)}$$

and similarly for $\hat{y}$.
3. **Taylor series approximation**: An example is the subpixel feature point position estimation in the SIFT [12] operator. Given the difference of Gaussian function $D(\boldsymbol{x})$, where $\boldsymbol{x}$ represents the two spatial and one scale dimensions, the Taylor series expansion is:

$$D(\boldsymbol{x} + \boldsymbol{\delta}) = D(\boldsymbol{x}) + \frac{\partial D(\boldsymbol{x})}{\partial \boldsymbol{x}}^{\top} \boldsymbol{\delta} + \frac{1}{2} \boldsymbol{\delta}^{\top} \frac{\partial^2 D(\boldsymbol{x})}{\partial \boldsymbol{x}^2} \boldsymbol{\delta}$$

Differentiating with respect to $\boldsymbol{\delta}$ and setting to 0 gives the subpixel (and subscale) estimate:

$$\boldsymbol{\delta} = -\frac{\partial^2 D(\boldsymbol{x})}{\partial \boldsymbol{x}^2}^{-1} \frac{\partial D(\boldsymbol{x})}{\partial \boldsymbol{x}}$$

4. **Phase Correlation**: The key principle behind phase correlation is the assumption that the pattern of data across a whole window is more distinctive than the individual pixel values. The technique is also independent of intensity, so can be used for multi-spectral or illumination varying registration. Assume that we have two image windows f_a and f_b and their discrete Fourier transforms $F_a = \mathcal{F}(f_a)$ and $F_b = \mathcal{F}(f_b)$. Compute the cross-power spectrum as $F_a F_b^*$ (by elementwise multiplication) where $*$ is the complex conjugate, normalize elementwise by $\mid F_a F_a^* \mid$ and finally apply the inverse Fourier transform:

$$T = \mathcal{F}^{-1}(\frac{F_a F_b^*}{\mid F_a F_a^* \mid})$$

The peak position in T is the desired offset. For subpixel alignment, the above method can be used to remove the integer component of the registration. Thereafter, one can estimate the subpixel peak position of the original registration, or repeat the process on an upsampled version of the image windows once the integer portion of the offset has been removed.

3 Applications

Subpixel methods have been developed to analyze:

- **shape parameters**: circle and other 'blob' shape parameters [9], ellipse parameters for improved camera calibration [20], photometric stereo [19], super-resolution [18], decomposition of mixed pixels formed by imaging two or more source types [2].
- **feature positions**: point-like signals [10], 'interest' points [12], 'edge' transitions [15], 'line' transitions [6].
- **shape matching and registration**: image registration using phase analysis [7, 16] or spatial domain matching [11], motion estimation prior to image compression [17], stereo matching [8] and disparity estimation [14], feature tracking [3], optical flow [4], image and video stabilization [5].

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