Primitive-Timing Representations of Handwriting

ANC Workshop

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Introduction

- Primitives
  - Locally fixed, arbitrary functions
  - Superimposed to create coherent movement
- Timing
  - Variation in model comes from superposition differences, from timing noise
  - Differences in character type also due to primitive timing
  - Modelling timing information allows a fixed-primitive generative model of handwriting

Piano Model

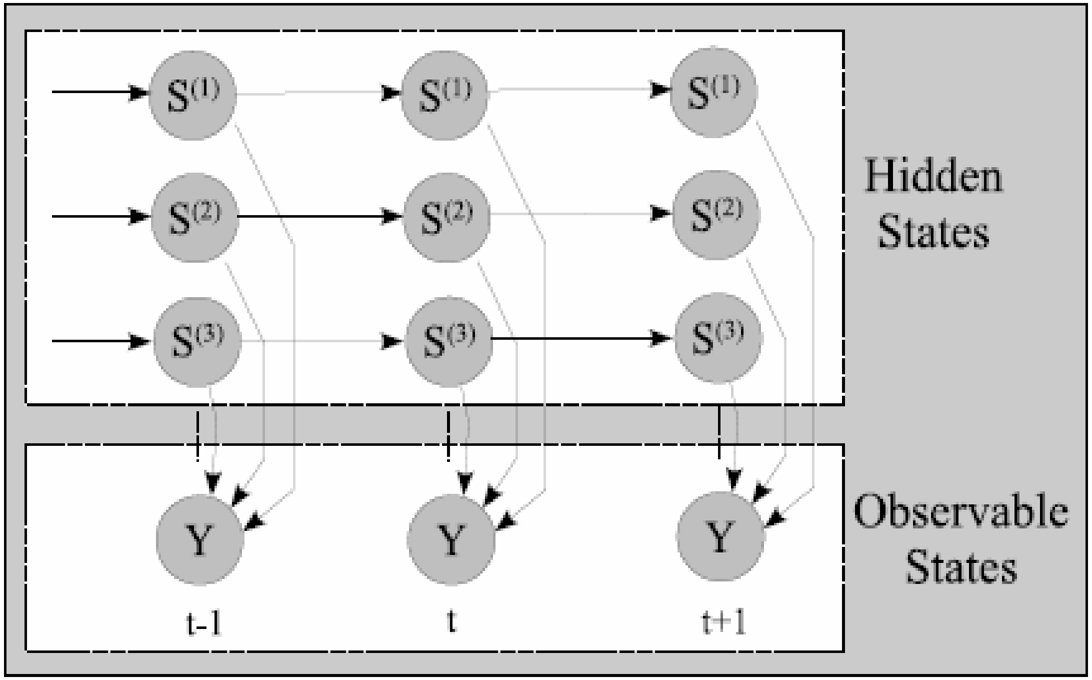
- Offset function superposition model

$$\boxed{Y(t) = \sum_{m,n} \alpha_{mn} W_m(t - \tau_{mn})}$$

- Fixed output basis functions superimposed
- Coherent movement generation relies upon timing
- Noise present at output and in primitive timing
- No pre-segmentation of characters into strokes means learning parameters requires a probabilistic framework

Factorial Hidden Markov Model

- Each factor models a primitive



M Primitives

Parameters:

P^m, p^m, W^m, C

States:

S_t^m, Y_t

$$Y_t = (\dot{x}, \dot{y}, \dot{z})$$
$$Y_t \sim \mathcal{N}(\mu_t, C)$$

$$\mu_t = \sum_{m=1}^{M} W^m S_t^m$$

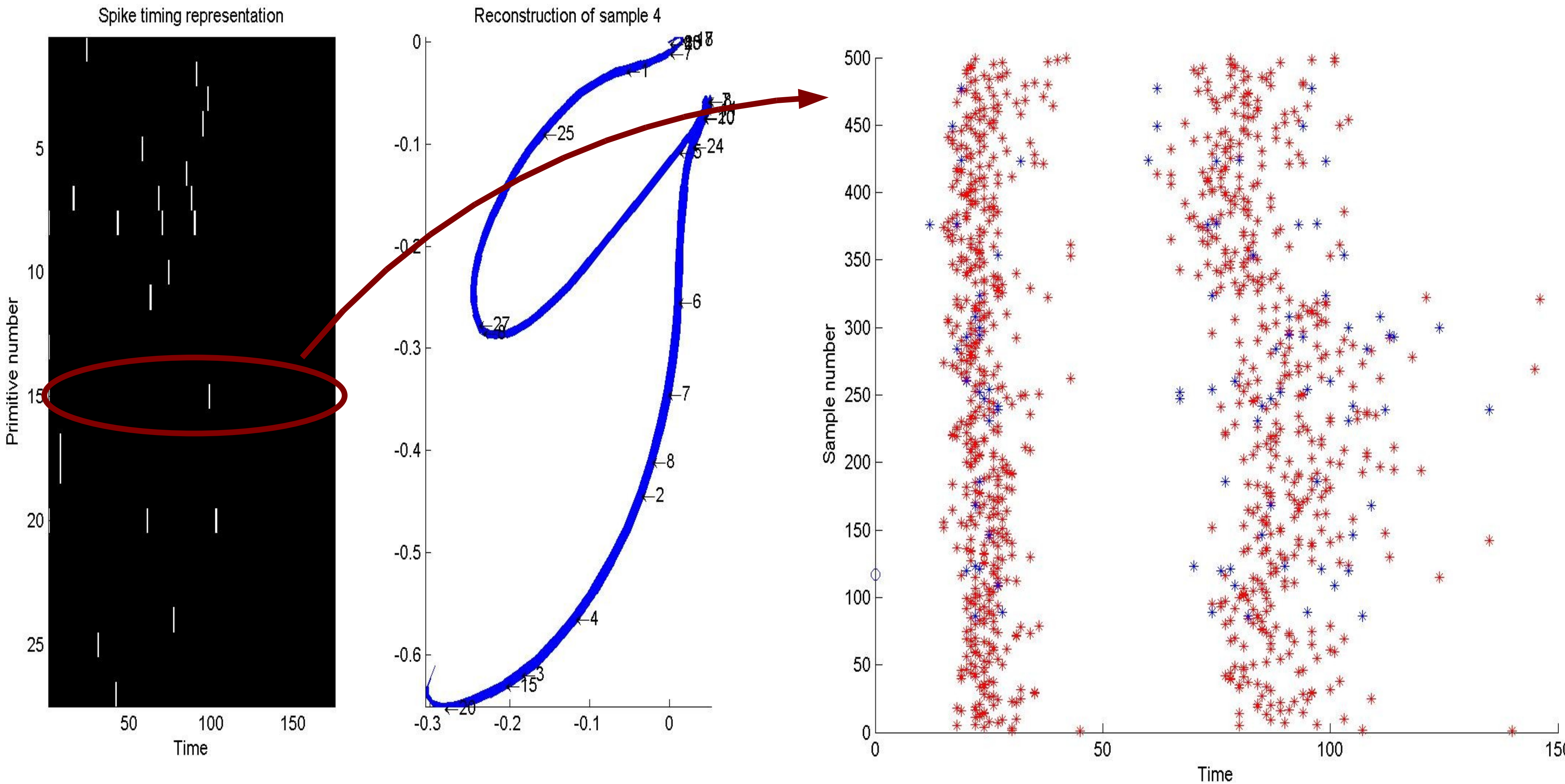
Constraints

- Prior knowledge of system is incorporated as model constraints
  - Zero state
  - Monotonicity
  - Extendible primitives
- Model inferred using EM procedure
  - Model state distributions
  - ML parameter updates

Demos

Hidden state representation

Single character | All characters



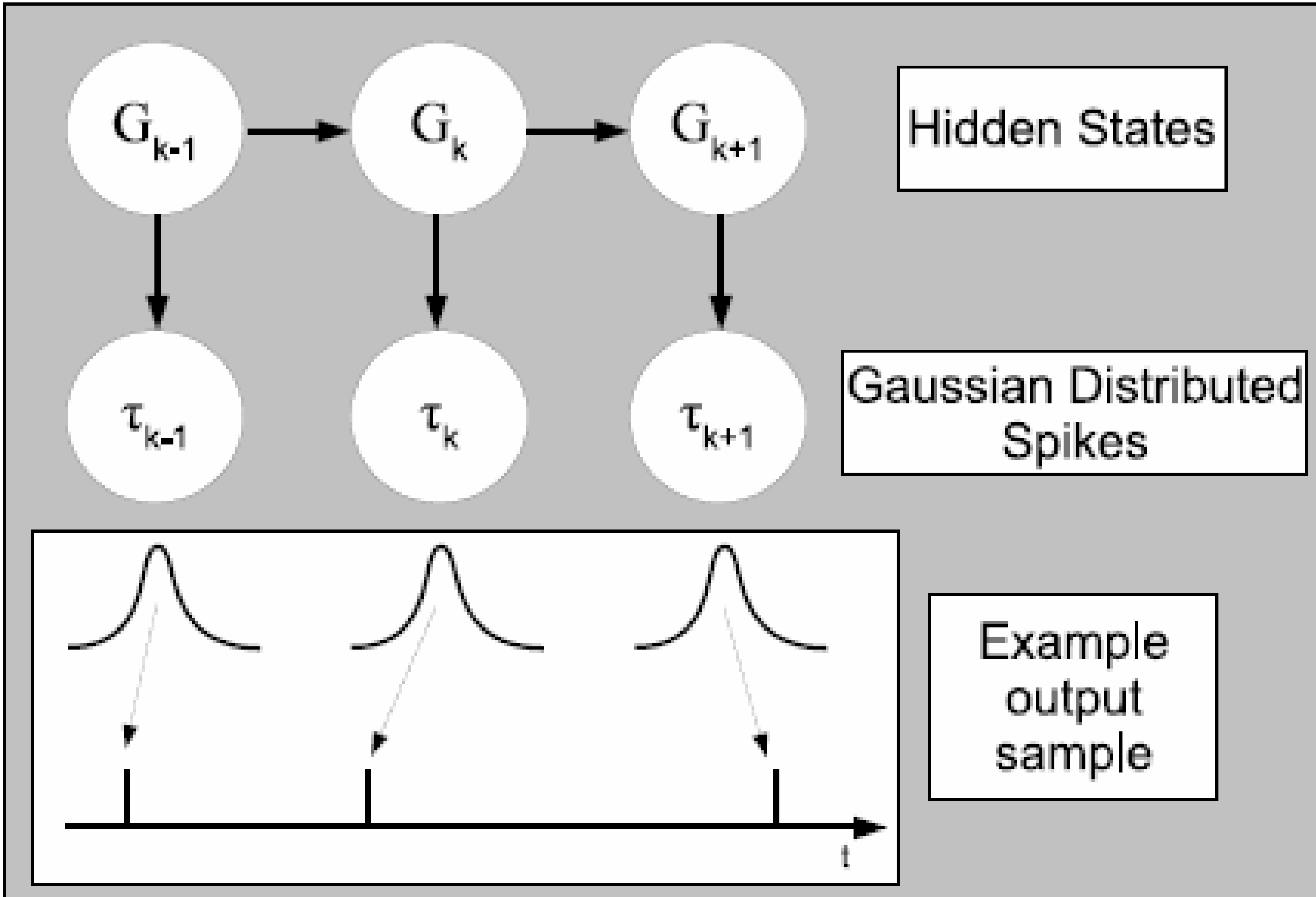
Timing Model

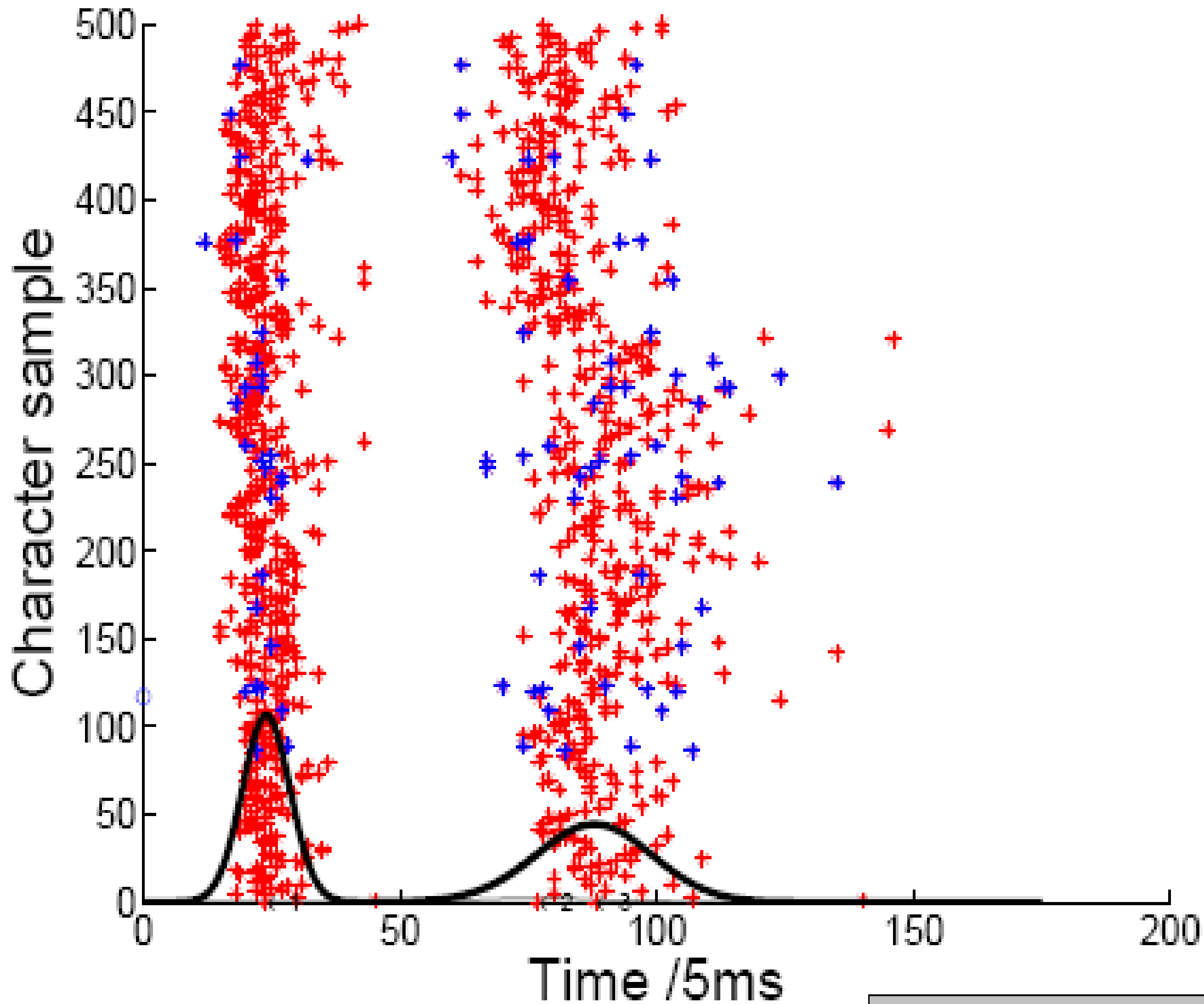
- Each primitive can be used (producing a spike) more than once, but not a fixed number of times
- Simple Gaussian representations have a problem of varying dimensionality of data
- Concatenating all the data looses the spike ordering, but allows a mixture of Gaussians model

Possible Timing Models

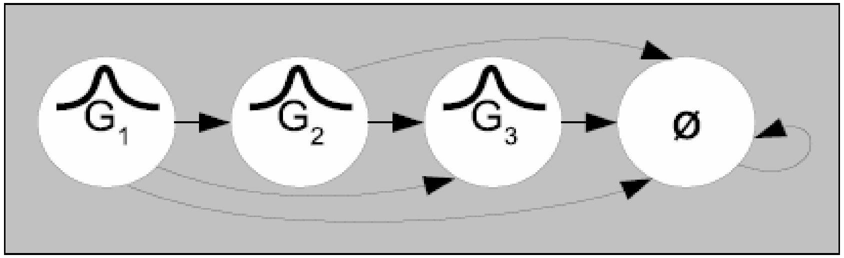
- Independent spikes model
  - i.i.d. Spikes
- Gaussian / MoG model
  - $P(s)=P(s|k).P(k)$, $P(s|k)\sim N(\mu,\sigma^2)$, k Gaussians
- Integrate and Fire model
- Structured probabilistic model
  - Short term dependencies between spikes
  - Presence of spikes can be modelled in a hidden state

HMM Timing Model





$$\tau \sim P(\tau \mid G = k) = \mathcal{N}_t(\mu_k, \sigma_k^2)$$



Learning Timing Demos

Coupling the Primitive, and Timing Models

- Learning has so far been independent for separate models
- A probabilistic timing model can produce a 'coupling prior' giving a modified onset prior for a primitive at a particular time point
- Using this spike prior whilst learning the primitive model allows the models to be coupled during learning

$$
\begin{aligned}
P(\lambda_t = 1) &= P(\text{spike} \mid t = \tau) \\
&\approx \sum_k P(\tau = t \mid G = k) P(G = k)
\end{aligned}
$$

- Timing model inference integrated into fHMM EM loop
- Original flat prior for primitive onset likelihood provides prior based on *how many times* primitive is likely to occur
- Incorporation of spike prior from timing model provides information on *where* primitive is likely to occur within character, based on generalisation across all samples

Timing demos

Conclusions / Directions

- Fixed time extended primitives can reconstruct handwriting data, where the variation is modelled with primitive timing noise
- Generative model of timing noise
- Multiple characters can be reproduced with a shared set of primitives
- Modularisation – Deficient behaviours
- Compression of posterior into spike times
- Abstraction of data into timing model
- Higher levels of abstraction – Mode, word, etc

Future direction demos