Dependent Types and Fibred Computational Effects

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(joint work with Gordon Plotkin[1] and Neil Ghani[2])

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Outline

Language design principles for combining

- dependent types ($\Pi, \Sigma, \mathsf{Id}_A(V, W), \ldots$)
- computational effects (state, I/O, probability, recursion, ...)

Our work was guided by two problems

- effectful programs in types
- assigning types to effectful programs

In the end we want to

- have a mathematically natural story
- use established tools and methods
- cover a wide range of computational effects

If time permits

- integrating dependent- and effect-typing (Idris)

Effectful programs in types

(type-dependency in the presence of effects)

Effectful programs in types

Let's assume that we have a dependent type $A(x)$, e.g.:

$$x\!:\!\mathsf{Nat} \vdash A(x) \quad \overset{\text{def}}{=} \quad \texttt{if}\ (x\ \texttt{mod}\ 2 == 0)\ \texttt{then}\ \mathsf{String}\ \texttt{else}\ \mathsf{Char}$$

Q: Should we allow $A[M/x]$ if M is an effectful program?

A1: In this work we say no

- type-checking should only depend on static information
- e.g., how would one compute $A[\mathsf{receive}(y.M)/x]$?
- we recover dependency on effectful computations via thunks

A2: In future work, we plan to also look at yes

- lifting effect operations from terms to types, e.g., $\mathsf{receive}(y.\,A)$
- similarities with ref. types and op. modalities [A.,Plotkin'15]
- type-dependency ($z\!:\!\underline{C} \vdash A(z)$) needs to be "homomorphic"

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Effectful programs in types ctd.

Aim: Types should only depend on static info about effects

Solution: CBPV/EEC style distinction between vals. and comps.

- value types $\Gamma \vdash A$ (MLTT + thunks + ...)
- computation types $\Gamma \vdash \underline{C}$ (dep. version of CBPV/EEC)
- where Γ contains only value variables $x_1 : A_1, \ldots, x_n : A_n$

Note: Other options are the monadic metalanguage and FGCBV

- but basing the work on CBPV/EEC gives a more general story
- especially for the treatment of sequential composition
- and also for integrating dependent- and effect-typing

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Assigning types to effectful programs

(i.e., typing sequential composition)

Assigning types to effectful programs

Our problem: The standard typing rule for seq. composition

$$\frac{\Gamma \vdash_{c} M : F\,A \qquad \Gamma, x{:}A \vdash_{c} N(x) : \underline{C}(x)}{\Gamma \vdash_{c} M \;\texttt{to}\; x{:}A \;\texttt{in}\; N(x) : \underline{C}(x)}$$

is not correct any more because x can appear free in

$$\underline{C}(x)$$

in the conclusion

Assigning types to effectful programs ctd.

Aim: Assigning a sensible type to sequential composition

Option 1: We could restrict the free variables in $\underline{C}$, i.e.:

$$\frac{\Gamma \vDash_{c} M : F\,A \qquad \Gamma \vdash \underline{C} \qquad \Gamma, x{:}A \vDash_{c} N : \underline{C}}{\Gamma \vDash_{c} M \ \texttt{to}\ x{:}A\ \texttt{in}\ N : \underline{C}}$$

But sometimes it is necessary for $\underline{C}$ to depend on x!

- e.g., in monadic parsing of well-typed syntax (case of functions)

$$\cdot \vDash_{c} \texttt{parseFun} : F\,(\Sigma y_1.\Sigma y_2.\mathsf{LangSyntax}(\texttt{fun}\,y_1\,y_2))$$

$$x{:}\Sigma y_1.\Sigma y_2.\mathsf{LangSyntax}(\texttt{fun}\,y_1\,y_2) \vDash_{c} \texttt{parseFunArg} : F\,(\mathsf{LangSyntax}(\texttt{fst}\,x))$$

Option 2: We could lift seq. composition to type level:

$$\Gamma \vDash_{c} M\ \texttt{to}\ x{:}A\ \texttt{in}\ N : M\ \texttt{to}\ x{:}A\ \texttt{in}\ \underline{C}$$

But then comp. types contain exactly the terms we want to type!

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Option 3: In the monadic metalanguage one could also try:

$$\frac{\Gamma \vdash M : T\,A \qquad \Gamma, x\!:\!A \vdash N : T\,B(x)}{\Gamma \vdash M \;\texttt{to}\; x\!:\!A \;\texttt{in}\; N : T\,(\Sigma x : A.B(x))}$$

But what makes this a principled solution?

Assigning types to effectful programs ctd.

Aim: Assigning a sensible type to sequential composition

Option 3: We draw inspiration from algebraic effects

- and combine it with Option 1, i.e., restricting $\underline{C}$ in seq. comp.

For example, consider the stateful program (for $x\!:\!\mathsf{Nat} \vdash_{\mathsf{c}} N : \underline{C}$)

$$M \quad \overset{\text{def}}{=} \quad \mathtt{lookup}\,(\mathtt{return}\,2, \mathtt{return}\,3)\ \mathtt{to}\ x\!:\!\mathsf{Nat}\ \mathtt{in}\ N$$

After looking up the bit, this program evaluates as either

$$N[2/x] \ \text{ at type } \ \underline{C}[2/x] \qquad \text{or} \qquad N[3/x] \ \text{ at type } \ \underline{C}[3/x]$$

Idea: M denotes an element of the coproduct of algebras

$$\underline{C}[2/x] + \underline{C}[3/x] \quad \overset{\text{def}}{=} \quad F\left(U\left(\underline{C}[2/x]\right) + U\left(\underline{C}[3/x]\right)\right)_{/\equiv}$$

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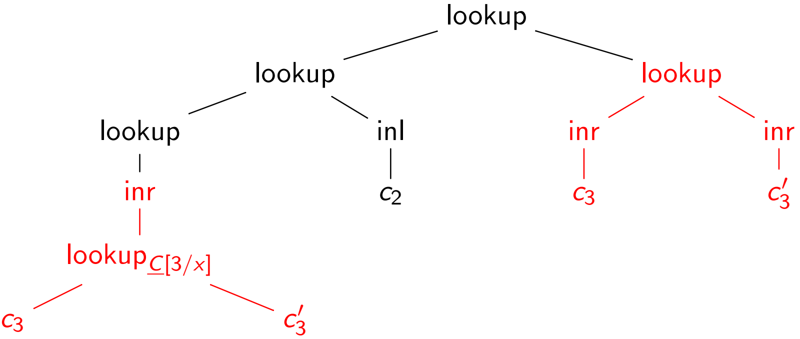
Sidenote about coproducts of algebras

Note: Elements of $\underline{C}[2/x] + \underline{C}[3/x]$ are not only $\mathsf{inl}\, c$ or $\mathsf{inr}\, c$!

- e.g., consider another computation tree in $\underline{C}[2/x] + \underline{C}[3/x]$



- where $\underline{C}[2/x] + \underline{C}[3/x] \overset{\text{def}}{=} F\Big(U(\underline{C}[2/x]) + U(\underline{C}[3/x])\Big)_{/\equiv}$
- where $c_2 \in \underline{C}[2/x]$ and $c_3, c_3' \in \underline{C}[3/x]$, and
- where the red subtrees are made equal by $\equiv$

Putting these ideas together

(a core dependently-typed calculus with comp. effects)

A computational dep.-typed language

Recall: We aim to define a dependently-typed language with

- general computational effects
- a clear distinction between values and computations
- restricting free variables in seq. composition
- using a coproducts of algebras
- a mathematically natural model theory, using standard tools

A computational dep.-typed language

Value types: MLTT's types + thunks + ...

$$A, B ::= \mathsf{Nat} \mid 1 \mid \Pi x : A.B \mid \Sigma x : A.B \mid Id_A(V, W) \mid U\,\underline{C} \mid \ldots$$

- $U\,\underline{C}$ is the type of thunked (i.e., suspended) computations

Computation types: dep.-typed version of EEC's comp. types

$$\underline{C}, \underline{D} ::= F\,A \mid \Pi x : A.\underline{C} \mid \Sigma x : A.\underline{C}$$

- $F\,A$ is the type of computations returning values of type A
- $\Pi x : A.\underline{C}$ is the type of dependent effectful functions
  - it generalises CBPV's and EEC's computational function type $A \to \underline{C}$ and product type $\underline{C} \times \underline{D}$
- $\Sigma x : A.\underline{C}$ is the generalisation of coproducts of algebras
  - it generalises EEC's computational tensor type $A \otimes \underline{C}$ and sum type $\underline{C} + \underline{D}$

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$$V, W \ ::= \ x \mid \texttt{zero} \mid \texttt{succ}\, V \mid \ \ldots \ \mid \texttt{thunk}\ M \mid \ \ldots$$

- equational theory based on MLTT with intensional id.-types
- value terms are typed using judgment $\Gamma \vdash_{\mathsf{v}} V : A$

Computation terms: dep.-typed version of CBPV/EEC c. terms

$$\begin{array}{rcll}
M, N & ::= & \texttt{force}\ V & \\
& \mid & \texttt{return}\ V & \\
& \mid & M\ \texttt{to}\ x\!:\!A\ \texttt{in}\ N & \\
& \mid & \lambda x\!:\!A.M & \\
& \mid & M V & \\
& \mid & \langle V, M \rangle & \text{(comp. } \Sigma \text{ intro.)} \\
& \mid & M\ \texttt{to}\ \langle x\!:\!A, z\!:\!\underline{C} \rangle\ \texttt{in}\ K & \text{(comp. } \Sigma \text{ elim.)}
\end{array}$$

But: These val. and comp. terms alone do not suffice, as in EEC!

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Note: We need to define K in such a way that we preserve the intended evaluation order, e.g., as in

$$\Gamma \models_{c} \langle V, M\rangle \ \texttt{to}\ \langle x{:}A, z{:}\underline{C}\rangle \ \texttt{in}\ K = K[V/x, M/z] : \underline{D}$$

Homomorphism terms: dep.-typed version of EEC's linear terms

$$\begin{array}{rcll}
K, L & ::= & z & \text{(linear comp. vars.)} \\
& \mid & K \ \texttt{to}\ x{:}A \ \texttt{in}\ M & \\
& \mid & \lambda x{:}A.K & \\
& \mid & K\,V & \\
& \mid & \langle V, K\rangle & \text{(comp-}\Sigma\text{ intro.)} \\
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Computation and homomorphism terms are typed using judgments

- $\Gamma \models_{c} M : \underline{C}$
- $\Gamma \,|\, z{:}\underline{C} \models_{h} K : \underline{D}$ (linear in z; comp. bound to z happens first)

Note: Formal presentation has more type-annotations on terms

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Homomorphism terms: dep.-typed version of EEC's linear terms

$$\begin{array}{rcll} K, L & ::= & z & \text{(linear comp. vars.)} \\ & \mid & K\ \mathtt{to}\ x\!:\!A\ \mathtt{in}\ M & \\ & \mid & \lambda x\!:\!A.K & \\ & \mid & K\,V & \\ & \mid & \langle V, K\rangle & \text{(comp-}\Sigma\text{ intro.)} \\ & \mid & K\ \mathtt{to}\ \langle x\!:\!A, z\!:\!\underline{C}\rangle\ \mathtt{in}\ L & \text{(comp-}\Sigma\text{ elim.)} \end{array}$$

Computation and homomorphism terms are typed using judgments

- $\Gamma \vDash_{\mathsf{c}} M : \underline{C}$
- $\Gamma \mid z\!:\!\underline{C} \vDash_{\mathsf{h}} K : \underline{D}$ (linear in z; comp. bound to z happens first)

Note: Formal presentation has more type-annotations on terms

A computational dep.-typed language

Typing rules: Dep.-typed versions of CBPV and EEC, e.g.:

$$\frac{\Gamma \vDash_{\mathsf{v}} V : A}{\Gamma \vDash_{\mathsf{c}} \mathtt{return}\, V : F\, A} \qquad \frac{\Gamma \vDash_{\mathsf{c}} M : F\, A \qquad \Gamma \vdash \underline{C} \qquad \Gamma, x\!:\!A \vDash_{\mathsf{c}} N : \underline{C}}{\Gamma \vDash_{\mathsf{c}} M \;\mathtt{to}\; x\!:\!A \;\mathtt{in}\; N : \underline{C}}$$

$$\ldots$$

$$\frac{\Gamma \vdash \underline{C}}{\Gamma \,|\, z\!:\!\underline{C} \vDash_{\mathsf{h}} z : \underline{C}}$$

$$\ldots$$

$$\frac{\Gamma \vDash_{\mathsf{v}} V : A \qquad \Gamma \,|\, z\!:\!\underline{C} \vDash_{\mathsf{h}} K : \underline{D}[V/x]}{\Gamma \,|\, z\!:\!\underline{C} \vDash_{\mathsf{h}} \langle V, K \rangle : \Sigma x\!:\!A.\underline{D}}$$

$$\frac{\Gamma \,|\, z_1\!:\!\underline{C} \vDash_{\mathsf{h}} K : \Sigma x\!:\!A.\underline{D}_1 \qquad \Gamma \vdash \underline{D}_2 \qquad \Gamma, x\!:\!A \,|\, z_2\!:\!\underline{D}_1 \vDash_{\mathsf{h}} L : \underline{D}_2}{\Gamma \,|\, z_1\!:\!\underline{C} \vDash_{\mathsf{h}} K \;\mathtt{to}\; \langle x\!:\!A, z_2\!:\!\underline{D}_1 \rangle \;\mathtt{in}\; L : \underline{D}_2}$$

The title fibred comp. effects comes from $\Gamma \vdash \underline{C}$ and $\Gamma \vdash \underline{D}_2$

A computational dep.-typed language

We can then account for type-dependency in seq. comp. by

$$\frac{\Gamma \vDash_c M : F\,A \qquad \dfrac{\Gamma, x\!:\!A \vDash_c N : \underline{C}(x)}{\Gamma, x\!:\!A \vDash_c \langle x, N\rangle : \Sigma y\!:\!A.\underline{C}(y)}}{\Gamma \vDash_c M \;\texttt{to}\; x\!:\!A \;\texttt{in}\; \langle x, N\rangle : \Sigma y\!:\!A.\underline{C}(y)}$$

The proposed rule for the monadic metalanguage is justified by

$$\Sigma x\!:\!A.F(B) \cong F(\Sigma x\!:\!A.B)$$

Categorical semantics

(fibrations and adjunctions)

Categorical semantics

Using fibred cat. theory, we define fibred adjunction models

- a sound and complete class of models

given by: i) a split closed comprehension category $\mathcal{P}$

$$\begin{tikzcd}
\mathcal{B}^{\to} \arrow[dr, bend right, "\mathrm{cod}"'] & & \mathcal{V} \arrow[ll, "\mathcal{P}"'] \arrow[d, bend right=60, "p"'] \arrow[d, bend left=60, "\{-\}"] \\
& & \mathcal{B} \arrow[u, "\dashv"', "1\ \dashv"]
\end{tikzcd}$$

- following Streicher and Hoffmann, we define a partial interpretation function $[\![-]\!]$ on raw syntax, that maps (if defined):
- a context Γ to and object $[\![\Gamma]\!]$ in $\mathcal{B}$
- a context Γ and a value type A to an object $[\![\Gamma; A]\!]$ in $\mathcal{V}_{[\![\Gamma]\!]}$
- a context Γ and a value term V to $[\![\Gamma; V]\!] : 1_{[\![\Gamma]\!]} \to X$ in $\mathcal{V}_{[\![\Gamma]\!]}$

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- the display maps $\pi_A = \mathcal{P}(A) : \{A\} \longrightarrow p(A)$ in $\mathcal{B}$
- induce the weakening functors $\pi_A^* : \mathcal{V}_{p(A)} \longrightarrow \mathcal{V}_{\{A\}}$
- and the value Σ- and Π-types are interpreted as adjoints

$$\Sigma_A \dashv \pi_A^* \dashv \Pi_A$$

(Σ_A is also required to be strong, i.e., support dep. elimination)

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given by: ii) a split fibration q and a split fib. adj. $F \dashv U$

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- we extend $\llbracket - \rrbracket$ so that it maps (if defined):
- a ctx. Γ and a comp. type $\underline{C}$ to an object $\llbracket \Gamma; \underline{C} \rrbracket$ in $\mathcal{C}_{\llbracket \Gamma \rrbracket}$
- a ctx. Γ and a comp. term M to $\llbracket \Gamma; M \rrbracket : 1_{\llbracket \Gamma \rrbracket} \to U(Z)$ in $\mathcal{V}_{\llbracket \Gamma \rrbracket}$
- a ctx. Γ, a comp. type $\underline{C}$ and a hom. term K to $\llbracket \Gamma; \underline{C}; K \rrbracket : \llbracket \Gamma; \underline{C} \rrbracket \to Z$ in $\mathcal{C}_{\llbracket \Gamma \rrbracket}$

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Examples of fibred adjunction models

Some sources of examples (writing fib. adj. with total cats. only):

- for a split closed comprehension cat. $\mathcal{P} : \mathcal{V} \longrightarrow \mathcal{B}^{\to}$, we have

$$\mathsf{Id}_{\mathcal{V}} \dashv \mathsf{Id}_{\mathcal{V}} : \mathcal{V} \longrightarrow \mathcal{V}$$

- for a model of EEC ($\mathcal{V}$ is CCC, $\mathcal{C}$ is $\mathcal{V}$-enriched, $\mathcal{V}$-enr. adj., etc.)

$$F_{\mathsf{EEC}} \dashv U_{\mathsf{EEC}} : \mathsf{s}(\mathcal{V}, \mathcal{C}) \longrightarrow \mathsf{s}(\mathcal{V})$$

- for a countable Lawvere theory $\mathcal{L}$ and $\mathcal{P}_{\mathsf{Fam}} : \mathsf{Fam}(\mathsf{Set}) \longrightarrow \mathsf{Set}^{\to}$

$$\widehat{F_{\mathcal{L}}} \dashv \widehat{U_{\mathcal{L}}} : \mathsf{Fam}(\mathsf{Mod}(\mathcal{L}, \mathsf{Set})) \longrightarrow \mathsf{Fam}(\mathsf{Set})$$

- for a monad $T : \mathsf{Set} \longrightarrow \mathsf{Set}$ and $\mathcal{P}_{\mathsf{Fam}} : \mathsf{Fam}(\mathsf{Set}) \longrightarrow \mathsf{Set}^{\to}$

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- for the continuations monad $R^{R^{(-)}} : \mathsf{Set} \longrightarrow \mathsf{Set}$, we have

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Examples of fibred adjunction models

More sources of examples (writing fib. adj. with total cats. only):

- these last three examples are instances of a more general result:

  for $\mathcal{P}_{\mathsf{fam}} : \mathsf{Fam}(\mathsf{Set}) \longrightarrow \mathsf{Set}^{\rightarrow}$ and $F \dashv U : \mathcal{C} \longrightarrow \mathsf{Set}$, when $\mathcal{C}$ has set-indexed products and set-indexed coproducts, we have

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- for a $\mathcal{CPO}$-enriched monad $T : \mathcal{CPO} \longrightarrow \mathcal{CPO}$ with a least algebraic operation $\Omega : 0$ and reflexive coequalizers in $\mathcal{CPO}^{T}$

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  allows us to treat general recursion as a computational effect

  $$\frac{\Gamma, x : U\underline{C} \vdash M : \underline{C}}{\Gamma \vdash \mu x : U\underline{C}.\, M : \underline{C}}$$

  (we get such monads from $\mathcal{CPO}$-enriched Law. theories with Ω)

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Algebraic effects

(primitives for programming with side-effects)

Algebraic operations and equations

Effect theories:

- we consider signatures of typed operation symbols

$$\frac{\cdot \vdash I \qquad x_i : I \vdash O \qquad I, O \text{ are pure, i.e., they do not contain } U}{\mathsf{op} : (x_i : I) \longrightarrow O}$$

- equipped with equations on derivable effect terms
- type-dependency in operation symbols simply a convenience (at least in Fam(Set)-based examples)

Example: Global store with two locations (modeled as booleans)

$$\mathsf{lookup} : (x_i : \mathsf{Bool}) \longrightarrow (\texttt{if } x_i \texttt{ then } \mathsf{String} \texttt{ else } \mathsf{Nat})$$

$$\mathsf{update} : (x_i : \Sigma x : \mathsf{Bool}.(\texttt{if } x \texttt{ then } \mathsf{String} \texttt{ else } \mathsf{Nat})) \longrightarrow 1$$

Algebraic operations:

$$\frac{\Gamma \vDash_{v} V : I \quad \Gamma \vdash \underline{C} \quad \Gamma, x : O[V/x_i] \vDash_{c} M : \underline{C}}{\Gamma \vDash_{c} \mathsf{op}^{\underline{C}}_{V}(x.M) : \underline{C}}$$

Generic effects:

$$\frac{\Gamma \vDash_{v} V : I}{\Gamma \vDash_{c} \mathsf{genop}_{V} : F\,(O[V/x_i])}$$

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What about handlers?

We ensure that K's behave like homomorphisms via

$$\Gamma \,|\, z \mathbin{:} \underline{C} \vDash_{\mathsf{h}} K : \underline{D} \quad\Longrightarrow\quad \Gamma \vDash_{\mathsf{c}} K[\mathsf{op}^{\underline{C}}_{V}(x.M)/z] = \mathsf{op}^{\underline{D}}_{V}(x.K[M/z]) : \underline{D}$$

Recall: Plotkin-Pretnar presentation of handlers is given by:

$$\Gamma \vDash_{\mathsf{c}} M \;\texttt{handled with}\; \{\mathsf{op}_x(y) \mapsto M_{\mathsf{op}}\}_{\mathsf{op}} \;\text{to}\; x \mathbin{:} A \;\texttt{in}\; M_{\mathsf{ret}} : \underline{C}$$

- semantically, $\{\mathsf{op}_x(y) \mapsto M_{\mathsf{op}}\}_{\mathsf{op}}$ defines an algebra on $U[\![\underline{C}]\!]$
- and M `handled` ... is the unique homomorphism out of $F[\![A]\!]$

Note: We have homomorphisms in the language, namely, the K's

Q: so can we accommodate?

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$$\Gamma \mid z \colon \underline{C} \vDash_{\mathsf{h}} K \; \mathtt{handled\ with} \; \{\mathsf{op}_x(y) \mapsto M_{\mathsf{op}}\}_{\mathsf{op}} \; \mathtt{to} \; x \colon A \; \mathtt{in} \; M_{\mathsf{ret}} : \underline{D}$$

A: Unfortunately not — the algebra structure only at term level

One (possible) way forward with handlers

User-defined algebra type:

(equational proof obligations about V_{op}'s omitted)

$$\frac{\Gamma \vdash A \qquad \{\Gamma, x : I, y : O[x/xi] \to A \vDash_{\mathsf{v}} V_{\mathsf{op}} : A\}_{\mathsf{op}:(x_i:I)\longrightarrow O}}{\Gamma \vdash \langle A, \{(x, y).V_{\mathsf{op}}\}_{\mathsf{op}:(x_i:I)\longrightarrow O}\rangle}$$

Introduction: $\mathtt{force}_{\langle A,\{(x,y).V_{\mathsf{op}}\}_{\mathsf{op}}\rangle} V$, where $V : A$

Elimination: (comp. term version)

(equational proof obligations about N omitted)

$$\frac{\Gamma \vDash_{\mathsf{c}} M : \langle A, \{(x, y).V_{\mathsf{op}}\}_{\mathsf{op}}\rangle \qquad \Gamma, x : A \vDash_{\mathsf{c}} N : \underline{C}}{\Gamma \vDash_{\mathsf{c}} \mathtt{run}\ M\ \mathtt{as}\ x\ \mathtt{in}\ N : \underline{C}}$$

Equations:

- $U\langle A, \{(x, y).V_{\mathsf{op}}\}_{\mathsf{op}}\rangle = A$
- $\mathtt{op}_V^{\langle A,\{(x_1,x_2).V_{\mathsf{op}}\}_{\mathsf{op}}\rangle}(x.M) = \mathtt{force}\,(V_{\mathsf{op}}[V/x_1, \lambda x.\mathtt{thunk}\,M/x_2])$
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Encoding Plotkin-Pretnar handlers:

$$M \;\texttt{handled with}\; \{\mathsf{op}_x(y) \mapsto M_{\mathsf{op}}\}_{\mathsf{op}} \;\mathsf{to}\; x\!:\!A \;\texttt{in}\; M_{\mathsf{ret}}$$

$$\stackrel{\text{def}}{=}$$

$$\texttt{force}_{\underline{C}}\left(\texttt{thunk}\left(M \;\texttt{to}\; x\!:\!A \;\texttt{in}\; \texttt{force}_{\langle U\underline{C},\ldots\texttt{thunk}\,(M_{\mathsf{op}})\ldots\rangle}\,(\texttt{thunk}\,M_{\mathsf{ret}})\right)\right)$$

$$: \underline{C}$$

Conclusions

A dependently-typed computational language with

- clear distinction between values and computations
- new and useful structure on comp. types (Σ-types)
- universes of value and comp. types (omitted)
- dep.-typed algebraic effects and handlers
- general recursion as comp. effect
- natural categorical semantics, using standard tools
- parametrised fibred computational effects and a principled account of Brady's resource-dependent effects in Idris (omitted)

Thank you for listening!

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Combining effect- and dependent-typing

(adding parameters/worlds/permissions/etc.)

Fibred parametrised comp. effects

Aim: To make our comp. types more expressive

- we extend our language with an effect-and-type system
- we build on [Atkey'09]'s parametrised notions of computation
- we take par. adjunctions as a primitive construction
- we make the effect annotations internal to our language
- we want a semantics for [Brady'13,'14]'s `Effects` DSL for Idris

We omit: Details of the accompanying denotational semantics

- based on fibred analogues of parametrised adjunctions, e.g.,

$$\begin{tikzcd} \mathcal{W} \arrow[d, "r"'] & \mathcal{V} \arrow[d, "p"'] & \int\big(\lambda X.\, \mathcal{W}_X \times \mathcal{V}_X\big) \arrow[rr, "F"] \arrow[dr, "\mathsf{fst}"'] & & \mathcal{C} \arrow[dl, "q"] \\ \mathcal{B} & \mathcal{B} & & \mathcal{B} & \end{tikzcd}$$

- in particular, we take $\mathcal{W} \overset{\text{def}}{=} \int\Big(\lambda X. \mathcal{V}_X(1_X, !_X^*(\llbracket S \rrbracket))\Big)$

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Our solution: Use fibred version of S-parametrised adjunctions

$$\frac{\Gamma \vdash A \qquad \Gamma \vDash_{\mathrm{v}} W : S}{\Gamma \vdash F_W\, A} \qquad \frac{\Gamma \vdash \underline{C} \qquad \Gamma \vDash_{\mathrm{v}} W : S}{\Gamma \vdash U_W\, \underline{C}}$$

with the resulting S-parametrised monad (`EffM` in Idris) given by

$$\Gamma \vdash T_{W_1,W_2}\, A \overset{\mathrm{def}}{=} U_{W_1}\, (F_{W_2}\, A)$$

The main changes we make to our language:

- typing judgment for comp. terms: $\Gamma \,|\, W \vDash_{\mathrm{c}} M : \underline{C}$
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Aim: We can explain [Brady'14]'s resource-dependent effects

Example: We will look at the prototypical example of:

- locking-unlocking / opening-closing / authenticating / etc.

As usual, the non-failing operations are easy to specify, e.g.,

$$\Gamma \,|\, \textbf{acquired} \vDash_c \texttt{lookup} : F_{\textbf{acquired}}\, \mathsf{String}$$

$$\Gamma \,|\, \textbf{acquired} \vDash_c \texttt{update}_V : F_{\textbf{acquired}}\, 1$$

$$\Gamma \,|\, \textbf{acquired} \vDash_c \texttt{releaseLock} : F_{\textbf{released}}\, \mathsf{Bool}$$

(in terms of generic effects, omitting the corresponding signature)

Q: However, what to do with possibly failing operations?

$$\Gamma \,|\, \textbf{released} \vDash_c \texttt{acquireLock} : F_{???}\, \mathsf{Bool}$$

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Q: What to do with possibly failing operations?

A1: If going with the monadic view, then we can try to define another (more dep.-parametrised) monad-like functor

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and specify the lock acquiring generic effect as

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- a natural generalisation of the functor part of fib. par. monads
- this is the approach that [Brady'14] took for Idris
- but no clear way of equipping it with par. adjunction structure

But: We can achieve the same with our less dep.-typed F and U!

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using the comp. Σ-types to quantify over the possible outcomes

A2b: We can then specify the lock acquiring generic effect as

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Parametrised fibred algebraic effects

Parametrised effect theories:

- we consider signatures of typed operation symbols

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Result: Such alg. ops. and gen. effs. are in 1-1 relationship

Note: Currently working on equipping W's with order/morphisms

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