

# Mixing Metaphors

## Actors as Channels and Channels as Actors

Simon Fowler, Sam Lindley, and Philip Wadler

The University of Edinburgh

`simon.fowler@ed.ac.uk`, `sam.lindley@ed.ac.uk`, `wadler@inf.ed.ac.uk`

**Abstract.** Channel- and actor-based programming languages are both used in practice, but the two are often confused. Languages such as Go provide anonymous processes which communicate using typed buffers—known as channels—while languages such as Erlang provide addressable processes each with a single incoming message queue—known as actors. The lack of a common representation makes it difficult to reason about the translations that exist in the folklore. We define a calculus  $\lambda_{\text{ch}}$  for typed asynchronous channels, and a calculus  $\lambda_{\text{act}}$  for typed actors. We show translations from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  and  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  and prove that both translations are type- and semantics-preserving.

## 1 Introduction

When comparing channels (as used by Go) and actors (as used by Erlang), one runs into an immediate mixing of metaphors. The words themselves do not refer to comparable entities!

In languages such as Go, anonymous processes pass messages via named channels, whereas in languages such as Erlang, named processes accept messages from an associated mailbox. A channel is a buffer, whereas an actor is a process. We should really be comparing named processes (actors) with anonymous processes, and buffers tied to a particular process (mailboxes) with buffers that can link any process to any process (channels). Nonetheless, we will stick with the popular names, even if it is as inapposite as comparing TV channels with TV actors.

Figure 1 compares channels with actors. On the left, three anonymous processes communicate via channels named  $a, b, c$ . On the right, three processes named  $A, B, C$  send messages to each others' associated mailboxes. A common misunderstanding is that channels are synchronous but actors are asynchronous [32], however while asynchrony *is* required by actor systems, channels may be either synchronous or asynchronous; to ease comparison, we consider asynchronous channels. A more significant difference is that each actor has a single buffer, its mailbox, which can be read only by that actor, whereas channels are free-floating buffers that can be read by any process with a reference to the channel.

Channel-based languages such as Go enjoy a firm basis in process calculi such as the  $\pi$ -calculus [31]. It is easy to type channels, either with simple types [30] or more complex systems such as session types [13, 20, 21]. Actor-based languages such as Erlang are seen by many as the "gold standard" for distributed computing due to their support for fault tolerance through supervision hierarchies [4, 6].

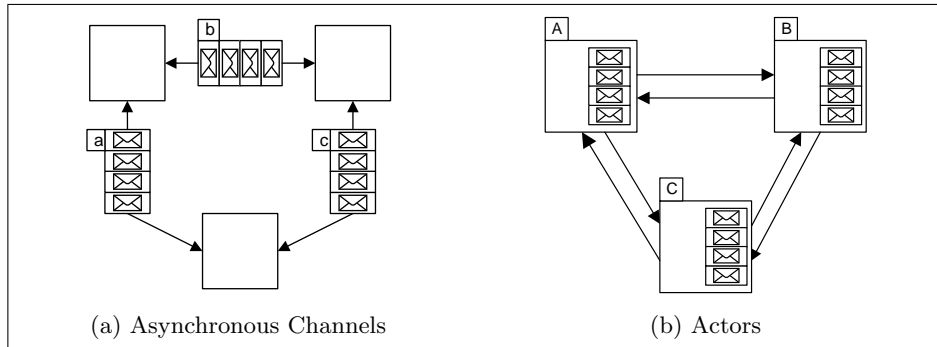


Fig. 1: Channels and Actors

Both models are popular with developers, with channel-based languages and frameworks such as Go, Concurrent ML [37], and Hopac [22]; and actor-based languages and frameworks such as Erlang, Elixir, and Akka.

There is often confusion over the differences between channels and actors. For example, two questions about this topic are:

“If I wanted to port a Go library that uses Goroutines, would Scala be a good choice because its inbox/akka framework is similar in nature to coroutines?” [24], and

“I don’t know anything about [the] actor pattern however I do know goroutines and channels in Go. How are [the] two related to each other?” [23]

The success of actor-based languages is largely due to their support for *supervision hierarchies*: processes are arranged in trees, where *supervisor* processes restart child processes should they fail. Projects such as the Go Actor Model framework (GAM) [15] emulate actor-style programming in a channel-based language in an attempt to gain some of the benefits. Hopac [22] is a channel-based library for F#, based on Concurrent ML [37]. The documentation [1] contains a comparison with actors, including an implementation of a simple actor-based communication model using Hopac-style channels, as well as an implementation of Hopac-style channels using an actor-based communication model. By comparing the two, this paper provides a formal model for the implementation technique used by GAM, and a formal model for an asynchronous variant of the translation from channels into actors as specified by the Hopac documentation.

**Putting Practice into Theory.** We seek to characterise the core features of channel- and actor-based models of concurrent programming, and distil them into minimal concurrent  $\lambda$ -calculi. In doing so, we:

- Obtain concise and expressive core calculi, which can be used as a basis to explore more advanced features such as behavioural typing, and;
- Make the existing folklore about the two models explicit, gaining formal guarantees about the correctness of translations. In turn, we give a formal grounding to implementations based on the translations, such as GAM.

Our common framework is that of a concurrent  $\lambda$ -calculus: that is, a  $\lambda$ -calculus with a standard term language equipped with primitives for communication and concurrency, as well as a language of *configurations* to model concurrent behaviour. We choose a  $\lambda$ -calculus rather than a process calculus as our starting point because we are ultimately interested in actual programming languages, and in particular functional programming languages.

While actor-based languages must be asynchronous by design, channels may be either synchronous (requiring a rendezvous between sender and receiver) or asynchronous (where sending always happens immediately). We base  $\lambda_{\text{ch}}$  on asynchronous channels since actors are naturally asynchronous, and since it is possible to emulate asynchronous channels using synchronous channels [37]. By working in the asynchronous setting, we can concentrate on the more fundamental differences between the two models.

**Outline and Contributions.** In §2 we present side-by-side implementations of a concurrent stack using channels and using actors. The main contributions of this paper are as follows.

- We define a calculus  $\lambda_{\text{ch}}$  with typed asynchronous channels (§3), and a calculus  $\lambda_{\text{act}}$  with type-parameterised actors (§4), by extending the simply-typed  $\lambda$ -calculus with communication primitives specialised to each model. We give a type system and operational semantics for each calculus, and precisely characterise the notion of progress that each calculus enjoys.
- We define a simple translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$ , prove that the translation is type-preserving, and prove that  $\lambda_{\text{ch}}$  can simulate  $\lambda_{\text{act}}$  (§5).
- We define a more involved translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ , again proving that the translation is type-preserving, and that  $\lambda_{\text{act}}$  can simulate  $\lambda_{\text{ch}}$  (§6).
- We consider an extension of  $\lambda_{\text{act}}$  with the ability to make synchronous calls; an extension of  $\lambda_{\text{ch}}$  with input-guarded nondeterministic choice; and discuss how  $\lambda_{\text{act}}$  could be extended with behavioural types (§7).

In §8 we discuss related work and §9 concludes.

## 2 Channels and Actors Side-by-Side

Let us consider the example of a concurrent stack. A concurrent stack carrying values of type  $A$  can receive a command to push a value onto the top of the stack, or to pop a value from the stack and return it to the process making the request. Assuming a standard encoding of algebraic datatypes using binary sums, we define a type  $\text{Operation}(A) = \text{Push}(A) \mid \text{Pop}(B)$  (where  $B = \text{ChanRef}(A)$  for channels, and  $\text{ActorRef}(A)$  for actors) to describe operations on the stack, and  $\text{Option}(A) = \text{Some}(A) \mid \text{None}$  to handle the possibility of popping from an empty stack.

Figure 2 shows the stack implemented using channels (Figure 2a) and using actors (Figure 2b). Each implementation uses a common core language based on the simply-typed  $\lambda$ -calculus extended with recursion, lists, and sums.

At first glance, the two stack implementations seem remarkably similar. Each:

<pre> chanStack(<i>ch</i>) <math>\triangleq</math>   rec loop(<i>st</i>).   let <i>cmd</i> <math>\leftarrow</math> take <i>ch</i> in   case <i>cmd</i> {     Push(<i>v</i>) <math>\mapsto</math> loop(<i>v</i> :: <i>st</i>)     Pop(<i>resCh</i>) <math>\mapsto</math>       case <i>st</i> {         [ ] <math>\mapsto</math> give (None) <i>resCh</i>;                 loop [ ]         <i>x</i> :: <i>xs</i> <math>\mapsto</math> give (Some(<i>x</i>)) <i>resCh</i>;                 loop <i>xs</i> }       } </pre>	<pre> actorStack <math>\triangleq</math>   rec loop(<i>st</i>).   let <i>cmd</i> <math>\leftarrow</math> receive in   case <i>cmd</i> {     Push(<i>v</i>) <math>\mapsto</math> loop(<i>v</i> :: <i>st</i>)     Pop(<i>resPid</i>) <math>\mapsto</math>       case <i>st</i> {         [ ] <math>\mapsto</math> send (None) <i>resPid</i>;                 loop [ ]         <i>x</i> :: <i>xs</i> <math>\mapsto</math> send (Some(<i>x</i>)) <i>resPid</i>;                 loop <i>xs</i> }       } </pre>
<pre> chanStackClient(<i>stackCh</i>) <math>\triangleq</math>   give (Push(5)) <i>stackCh</i>;   let <i>resCh</i> <math>\leftarrow</math> newCh in   give (Pop(<i>resCh</i>)) <i>stackCh</i>;   take <i>resCh</i> </pre>	<pre> actorStackClient(<i>stackPid</i>) <math>\triangleq</math>   send (Push(5)) <i>stackPid</i>;   let <i>selfPid</i> <math>\leftarrow</math> self in   send (Pop(<i>selfPid</i>)) <i>stackPid</i>;   receive </pre>
<pre> chainMain <math>\triangleq</math>   let <i>stackCh</i> <math>\leftarrow</math> newCh in   fork (chanStack(<i>stackCh</i>) [ ]);   chanStackClient(<i>stackCh</i>) </pre>	<pre> actorMain <math>\triangleq</math>   let <i>stackPid</i> <math>\leftarrow</math> spawn (actorStack [ ]) in   actorStackClient(<i>stackPid</i>) </pre>
(a) Channel-Based Stack	(b) Actor-Based Stack

Fig. 2: Concurrent Stacks using Channels and Actors

1. Waits for a command
2. Case splits on the command, and either:
  - Pushes a value onto the top of the stack, or;
  - Takes the value from the head of the stack and returns it in a response message
3. Loops with an updated state.

The main difference is that `chanStack` is parameterised over a channel `ch`, and retrieves a value from the channel using `take ch`. Conversely, `actorStack` retrieves a value from its mailbox using the nullary primitive `receive`.

Let us now consider functions which interact with the stacks. The `chanStackClient` function sends commands over the `stackCh` channel, and begins by pushing 5 onto the stack. Next, the function creates a channel `resCh` to be used to receive the result and sends this in a request, before retrieving the result from the result channel using `take`. In contrast, `actorStackClient` performs a similar set of steps, but sends its process ID (retrieved using `self`) in the request instead of creating a new channel; the result is then retrieved from the mailbox using `receive`.

**Type Pollution.** The differences become more prominent when we consider clients which interact with multiple stacks containing different types of values, as

<pre> <b>chanClient2</b>(<i>intStackCh</i>,               <i>stringStackCh</i>) <math>\triangleq</math> let <i>intResCh</i> <math>\leftarrow</math> <b>newCh</b> in let <i>strResCh</i> <math>\leftarrow</math> <b>newCh</b> in <b>give</b> (<b>Pop</b>(<i>intResCh</i>)) <i>intStackCh</i>; let <i>res1</i> <math>\leftarrow</math> <b>take</b> <i>intResCh</i> in <b>give</b> (<b>Pop</b>(<i>strResCh</i>)) <i>stringStackCh</i>; let <i>res2</i> <math>\leftarrow</math> <b>take</b> <i>strResCh</i> in (<i>res1</i>, <i>res2</i>)                 </pre>	<pre> <b>actorClient2</b>(<i>intStackPid</i>,               <i>stringStackPid</i>) <math>\triangleq</math> let <i>selfPid</i> <math>\leftarrow</math> <b>self</b> in <b>send</b> (<b>Pop</b>(<i>selfPid</i>)) <i>intStackPid</i>; let <i>res1</i> <math>\leftarrow</math> <b>receive</b> in <b>send</b> (<b>Pop</b>(<i>selfPid</i>)) <i>stringStackPid</i>; let <i>res2</i> <math>\leftarrow</math> <b>receive</b> in (<i>res1</i>, <i>res2</i>)                 </pre>
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Fig. 3: Clients Interacting with Multiple Stacks

shown in Figure 3. Here, `chanStackClient2` pushes 5 onto the integer stack as before. Next, the client creates new result channels for integers and strings, sends requests for the results, and creates a pair of type  $(\text{Option}(\text{Int}) \times \text{Option}(\text{String}))$ .

The `actorStackClient2` function attempts to do something similar, but cannot create separate result channels. Consequently, the actor must be able to handle messages either of type  $\text{Option}(\text{Int})$  or type  $\text{Option}(\text{String})$ , meaning that the final pair has type  $(\text{Option}(\text{Int}) + \text{Option}(\text{String})) \times (\text{Option}(\text{Int}) + \text{Option}(\text{String}))$ .

Additionally, it is necessary to modify `actorStack` to use the correct injection into the actor type when sending the result; for example an integer stack would have to send a value `inl(Some(5))` instead of simply `Some(5)`. The requirement of knowing all message types received by another actor is known as the *type pollution* problem; some implementations sidestep this through the use of subtyping [17], and support synchronisation through either selectively receiving from a mailbox [5], or through synchronisation abstractions such as futures [9].

### 3 $\lambda_{\text{ch}}$ : A Concurrent $\lambda$ -calculus for Channels

In this section we introduce  $\lambda_{\text{ch}}$ , a concurrent  $\lambda$ -calculus extended with asynchronous channels.

#### 3.1 Syntax and Typing of Terms

Figure 4 gives the syntax and typing rules of  $\lambda_{\text{ch}}$ , a lambda calculus based on fine-grain call-by-value [27]: terms are partitioned into values and computations. Fine-grain call-by-value is convenient because it makes evaluation-order completely explicit and (unlike A-normal form, for instance) is closed under reduction. Types consist of the unit type  $\mathbf{1}$ , function types  $A \rightarrow B$ , and channel reference types  $\text{ChanRef}(A)$  which can be used to communicate along a channel of type  $A$ . We let  $\alpha$  range over variables  $x$  and run time names  $a$ . We write `let  $x = V$  in  $M$`  for  $(\lambda x.M) V$  and `let  $x \leftarrow M$  in  $N$` , where  $x$  is fresh.

**Communication and Concurrency for Channels.** The `give  $V W$`  operation sends value  $V$  along channel  $W$ , while `take  $V$`  retrieves a value from a channel  $V$ .

Syntax		
Types	$A, B ::= \mathbf{1} \mid A \rightarrow B \mid \text{ChanRef}(A)$	
Variables and names	$\alpha ::= x \mid a$	
Values	$V, W ::= \alpha \mid \lambda x.M \mid ()$	
Computations	$L, M, N ::= V W$ <div style="margin-left: 20px;"> <math>\mid \text{let } x \leftarrow M \text{ in } N \mid \text{return } V</math>  <math>\mid \text{fork } M \mid \text{give } V W \mid \text{take } V \mid \text{newCh}</math> </div>	
Value typing rules <span style="float: right; border: 1px solid black; padding: 2px;"><math>\Gamma \vdash V : A</math></span>		
VAR	ABS	UNIT
$\frac{\alpha : A \in \Gamma}{\Gamma \vdash \alpha : A}$	$\frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \rightarrow B}$	$\frac{}{\Gamma \vdash () : \mathbf{1}}$
Computation typing rules <span style="float: right; border: 1px solid black; padding: 2px;"><math>\Gamma \vdash M : A</math></span>		
APP	EFFLET	RETURN
$\frac{\Gamma \vdash V : A \rightarrow B \quad \Gamma \vdash W : A}{\Gamma \vdash V W : B}$	$\frac{\Gamma \vdash M : A \quad \Gamma, x : A \vdash N : B}{\Gamma \vdash \text{let } x \leftarrow M \text{ in } N : B}$	$\frac{\Gamma \vdash V : A}{\Gamma \vdash \text{return } V : A}$
GIVE	TAKE	FORK
$\frac{\Gamma \vdash V : A \quad \Gamma \vdash W : \text{ChanRef}(A)}{\Gamma \vdash \text{give } V W : \mathbf{1}}$	$\frac{\Gamma \vdash V : \text{ChanRef}(A)}{\Gamma \vdash \text{take } V : A}$	$\frac{\Gamma \vdash M : \mathbf{1}}{\Gamma \vdash \text{fork } M : \mathbf{1}}$
NEWCH		
$\frac{}{\Gamma \vdash \text{newCh} : \text{ChanRef}(A)}$		

Fig. 4: Syntax and Typing Rules for  $\lambda_{\text{ch}}$  Terms and Values

Assuming an extension of the language with integers and arithmetic operators, we can define a function  $\text{neg}(c)$  which receives a number  $n$  along channel  $c$  and replies with the negation of  $n$  as follows:

$$\text{neg}(c) \triangleq \text{let } n \leftarrow \text{take } c \text{ in} \\ \text{let } \text{neg}N \leftarrow (-n) \text{ in give } \text{neg}N c$$

The operation  $\text{newCh}$  creates a new channel. The operation  $\text{fork } M$  spawns a new process that performs computation  $M$ . Firstly, note that  $\text{fork}$  returns the unit value; the spawned process is anonymous and therefore it is not possible to interact with it directly. Secondly, note that channel creation is completely decoupled from process creation, meaning that a process can have access to multiple channels.

### 3.2 Operational Semantics

**Configurations.** The concurrent behaviour of  $\lambda_{\text{ch}}$  is given by a nondeterministic reduction relation on *configurations*, ranged over by  $\mathcal{C}$  and  $\mathcal{D}$  (Figure 5). Configu-

Syntax of evaluation contexts and configurations	
Evaluation contexts	$E ::= [] \mid \text{let } x \leftarrow E \text{ in } M$
Configurations	$\mathcal{C}, \mathcal{D} ::= \mathcal{C} \parallel \mathcal{D} \mid (\nu a)\mathcal{C} \mid a(\vec{V}) \mid M$
Configuration contexts	$G ::= [] \mid G \parallel \mathcal{C} \mid (\nu a)G$
Typing rules for configurations <span style="float: right; border: 1px solid black; padding: 2px 5px;"><math>\Gamma; \Delta \vdash \mathcal{C}</math></span>	
$\frac{\text{PAR} \quad \Gamma; \Delta_1 \vdash \mathcal{C}_1 \quad \Gamma; \Delta_2 \vdash \mathcal{C}_2}{\Gamma; \Delta_1, \Delta_2 \vdash \mathcal{C}_1 \parallel \mathcal{C}_2}$	$\frac{\text{CHAN} \quad \Gamma, a : \text{ChanRef}(A); \Delta, a : A \vdash \mathcal{C}}{\Gamma; \Delta \vdash (\nu a)\mathcal{C}}$
$\frac{\text{BUF} \quad (\Gamma \vdash V_i : A)_i}{\Gamma; a : A \vdash a(\vec{V})}$	$\frac{\text{TERM} \quad \Gamma \vdash M : A}{\Gamma; \cdot \vdash M}$

 Fig. 5:  $\lambda_{\text{ch}}$  Configurations and Evaluation Contexts

rations consist of parallel composition ( $\mathcal{C} \parallel \mathcal{D}$ ), restrictions ( $(\nu a)\mathcal{C}$ ), computations ( $M$ ), and buffers ( $a(\vec{V})$ ).

**Evaluation Contexts.** Reduction is defined in terms of evaluation contexts  $E$ , which are simplified due to fine-grain call-by-value. We also define configuration contexts, allowing reduction modulo parallel composition and name restriction.

**Reduction.** Figure 6 shows the reduction rules for  $\lambda_{\text{ch}}$ . Reduction is defined as a deterministic reduction on terms ( $\longrightarrow_{\text{M}}$ ) and a nondeterministic reduction relation on configurations ( $\longrightarrow$ ). Reduction on configurations is defined modulo structural congruence rules which capture commutativity and associativity of parallel composition, scope extrusion, and that structural congruence extends to configuration contexts.

**Typing of Configurations.** Figure 5 includes typing rules on configurations, which ensure that buffers are well-scoped and contain values of the correct type. The judgement  $\Gamma; \Delta \vdash \mathcal{C}$  states that under environments  $\Gamma$  and  $\Delta$ ,  $\mathcal{C}$  is well-typed;  $\Gamma$  is a typing environment for terms, whereas  $\Delta$  is a linear typing environment for configurations, mapping names  $a$  to channel types  $A$ . Note that CHAN extends both  $\Gamma$  and  $\Delta$ , adding a *reference* into  $\Gamma$  and the *capability* to type a buffer into  $\Delta$ . PAR states that two configurations are typeable if they are each typeable under disjoint linear environments; a term is typeable as a process if it has type **1** under an empty linear environment and a buffer  $a(\vec{V})$  is typeable under a singleton linear environment  $a : A$  if all  $V_i \in \vec{V}$  have type  $A$ . Linearity ensures that under a name restriction  $(\nu a)\mathcal{C}$  a configuration  $\mathcal{C}$  will contain exactly one buffer with name  $a$ .

**Relation Notation.** Given a relation  $R$ , we write  $R^+$  for its transitive closure, and  $R^*$  for its reflexive, transitive closure.

Reduction on terms																									
	$ \begin{aligned} (\lambda x.M) V &\longrightarrow_M M\{V/x\} \\ \text{let } x \leftarrow \text{return } V \text{ in } M &\longrightarrow_M M\{V/x\} \\ E[M_1] &\longrightarrow_M E[M_2] \quad (\text{if } M_1 \longrightarrow_M M_2) \end{aligned} $																								
Structural congruence																									
	$ \begin{aligned} \mathcal{C} \parallel \mathcal{D} &\equiv \mathcal{D} \parallel \mathcal{C} & \mathcal{C} \parallel (\mathcal{D} \parallel \mathcal{E}) &\equiv (\mathcal{C} \parallel \mathcal{D}) \parallel \mathcal{E} & \mathcal{C} \parallel (\nu a)\mathcal{D} &\equiv (\nu a)(\mathcal{C} \parallel \mathcal{D}) \text{ if } a \notin \text{fv}(\mathcal{C}) \\ G[\mathcal{C}] &\equiv G[\mathcal{D}] \text{ if } \mathcal{C} \equiv \mathcal{D} \end{aligned} $																								
Reduction on configurations																									
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">GIVE</td> <td style="padding: 2px 5px;"><math>E[\text{give } W a] \parallel a(\vec{V})</math></td> <td style="padding: 2px 5px;"><math>\longrightarrow</math></td> <td style="padding: 2px 5px;"><math>E[\text{return } ()] \parallel a(\vec{V} \cdot W)</math></td> </tr> <tr> <td style="padding: 2px 5px;">TAKE</td> <td style="padding: 2px 5px;"><math>E[\text{take } a] \parallel a(W \cdot \vec{V})</math></td> <td style="padding: 2px 5px;"><math>\longrightarrow</math></td> <td style="padding: 2px 5px;"><math>E[\text{return } W] \parallel a(\vec{V})</math></td> </tr> <tr> <td style="padding: 2px 5px;">FORK</td> <td style="padding: 2px 5px;"><math>E[\text{fork } M]</math></td> <td style="padding: 2px 5px;"><math>\longrightarrow</math></td> <td style="padding: 2px 5px;"><math>E[\text{return } ()] \parallel M</math></td> </tr> <tr> <td style="padding: 2px 5px;">NEWCH</td> <td style="padding: 2px 5px;"><math>E[\text{newCh}]</math></td> <td style="padding: 2px 5px;"><math>\longrightarrow</math></td> <td style="padding: 2px 5px;"><math>(\nu a)(E[\text{return } a] \parallel a(\epsilon))</math> <span style="float: right;">(<math>a</math> is a fresh name)</span></td> </tr> <tr> <td style="padding: 2px 5px;">LIFTM</td> <td style="padding: 2px 5px;"><math>G[M_1]</math></td> <td style="padding: 2px 5px;"><math>\longrightarrow</math></td> <td style="padding: 2px 5px;"><math>G[M_2]</math> <span style="float: right;">(if <math>M_1 \longrightarrow_M M_2</math>)</span></td> </tr> <tr> <td style="padding: 2px 5px;">LIFT</td> <td style="padding: 2px 5px;"><math>G[\mathcal{C}_1]</math></td> <td style="padding: 2px 5px;"><math>\longrightarrow</math></td> <td style="padding: 2px 5px;"><math>G[\mathcal{C}_2]</math> <span style="float: right;">(if <math>\mathcal{C}_1 \longrightarrow \mathcal{C}_2</math>)</span></td> </tr> </table>	GIVE	$E[\text{give } W a] \parallel a(\vec{V})$	$\longrightarrow$	$E[\text{return } ()] \parallel a(\vec{V} \cdot W)$	TAKE	$E[\text{take } a] \parallel a(W \cdot \vec{V})$	$\longrightarrow$	$E[\text{return } W] \parallel a(\vec{V})$	FORK	$E[\text{fork } M]$	$\longrightarrow$	$E[\text{return } ()] \parallel M$	NEWCH	$E[\text{newCh}]$	$\longrightarrow$	$(\nu a)(E[\text{return } a] \parallel a(\epsilon))$ <span style="float: right;">(<math>a</math> is a fresh name)</span>	LIFTM	$G[M_1]$	$\longrightarrow$	$G[M_2]$ <span style="float: right;">(if <math>M_1 \longrightarrow_M M_2</math>)</span>	LIFT	$G[\mathcal{C}_1]$	$\longrightarrow$	$G[\mathcal{C}_2]$ <span style="float: right;">(if <math>\mathcal{C}_1 \longrightarrow \mathcal{C}_2</math>)</span>	
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Fig. 6: Reduction on  $\lambda_{\text{ch}}$  Terms and Configurations

**Properties of the Term Language.** Reduction on terms is standard. It preserves typing and purely-functional terms enjoy progress. We omit proofs in the body of the paper which are mainly straightforward inductions; selected full proofs can be found in Appendix B.

**Lemma 1 (Preservation ( $\lambda_{\text{ch}}$  terms)).**

If  $\Gamma \vdash M : A$  and  $M \longrightarrow_M M'$ , then  $\Gamma \vdash M' : A$ .

**Lemma 2 (Progress ( $\lambda_{\text{ch}}$  terms)).**

Assume  $\Gamma$  is either empty or only contains entries of the form  $a_i : \text{ChanRef}(A_i)$ . If  $\Gamma \vdash M : A$ , then either:

1.  $M = \text{return } V$  for some value  $V$
2.  $M$  can be written  $E[M']$ , where  $M'$  is a communication or concurrency primitive (i.e.  $\text{give } V W$ ,  $\text{take } V$ ,  $\text{fork } M$ , or  $\text{newCh}$ )
3. There exists some  $M'$  such that  $M \longrightarrow_M M'$

**Reduction on Configurations.** Concurrency and communication is entirely captured by reduction on configurations. The GIVE rule reduces  $\text{give } W a$  in parallel with a buffer  $a(\vec{V})$  by adding the value  $W$  onto the end of the buffer. The TAKE rule reduces  $\text{take } a$  in parallel with a non-empty buffer by returning the first value in the buffer. The FORK rule reduces  $\text{fork } M$  by spawning a new thread  $M$  in parallel with the parent process. The NEWCH rule reduces  $\text{newCh}$  by creating an empty buffer and returning a fresh name for that buffer.

We now state some properties of  $\lambda_{\text{ch}}$ . Firstly, typeability of configurations is preserved by structural congruence.



**Lemma 3.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\mathcal{C} \equiv \mathcal{D}$  for some configuration  $\mathcal{D}$ , then  $\Gamma; \Delta \vdash \mathcal{D}$ .*

Next, we can show that reduction preserves the typeability of configurations.

**Theorem 4 (Preservation ( $\lambda_{\text{ch}}$  configurations)).**

*If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$  then  $\Gamma; \Delta \vdash \mathcal{C}_2$ .*

### 3.3 Progress and Canonical Forms

While it is possible to prove deadlock-freedom in systems with more discerning type systems based on linear logic (such as those of Wadler [39], and Lindley and Morris [28]) or those using channel priorities (for example, the calculus of Padovani and Novara [35]), more liberal calculi such as  $\lambda_{\text{ch}}$  and  $\lambda_{\text{act}}$  allow deadlocked configurations. We thus define a form of progress which does not preclude deadlock. To help with proving a progress result, it is useful to consider the notion of a *canonical form* in order to allow us to reason about the configuration as a whole.

**Definition 5 (Canonical form ( $\lambda_{\text{ch}}$ )).**

*A configuration  $\mathcal{C}$  is in canonical form if  $\mathcal{C}$  can be written*

$$(\nu a_1) \dots (\nu a_n)(M_1 \parallel \dots \parallel M_m \parallel a_1(\vec{V}_1) \parallel \dots \parallel a_n(\vec{V}_n)).$$

The following lemma states that well-typed open configurations can be written in a form similar to canonical form, but without bindings for names already in the environment. An immediate corollary is that well-typed closed configurations can always be written in a canonical form.

**Lemma 6.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  with  $\Delta = a_1 : A_1, \dots, a_k : A_k$ , then there exists a  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}' = (\nu a_{k+1}) \dots (\nu a_n)(M_1 \parallel \dots \parallel M_m \parallel a_1(\vec{V}_1) \parallel \dots \parallel a_n(\vec{V}_n))$ .*

**Corollary 7.** *If  $;\cdot \vdash \mathcal{C}$ , then there exists some  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}'$  is in canonical form.*

Armed with a canonical form, we can now capture precisely the intuition that the only situation in which a well-typed closed configuration  $\mathcal{C}$  cannot reduce further is if all threads are either blocked or fully evaluated.

**Theorem 8 (Weak progress ( $\lambda_{\text{ch}}$  configurations)).**

*Let  $;\cdot \vdash \mathcal{C}$ ,  $\mathcal{C} \not\rightarrow$ , and let  $\mathcal{C}' = (\nu a_1) \dots (\nu a_n)(M_1 \parallel \dots \parallel M_m \parallel a_1(\vec{V}_1) \parallel \dots \parallel a_n(\vec{V}_n))$  be a canonical form of  $\mathcal{C}$ . Then every leaf of  $\mathcal{C}$  is either:*

1. A fully-reduced term of the form `return`  $V$ ;
2. A buffer  $a_i(\vec{V}_i)$ , or;
3. A term of the form  $E[\text{take } a_i]$ , where  $\vec{V}_i = \epsilon$ .

## 4 $\lambda_{\text{act}}$ : A Concurrent $\lambda$ -calculus for Actors

In this section, we introduce  $\lambda_{\text{act}}$ , a core language providing actor-like concurrency.

Actors were originally introduced by Hewitt et al. [18] as a formalism for artificial intelligence, where an actor is an entity endowed with an unforgeable address known as a process ID, and a single incoming message queue known as a mailbox. There are many variations of actor-based languages (the work of De Koster et al. [10] provides a detailed taxonomy), but the main common feature is that each provide lightweight processes which are associated with a mailbox. We follow the model of Erlang by providing an explicit `receive` operation to allow an actor to retrieve a message from its mailbox, as opposed to making an event loop implicit.

While it is common to parameterise channels over types, parameterising actors over types is more challenging, in particular due to type pollution. Type-parameterised actors were introduced by He [16] and He et al. [17], and more recently implemented in libraries such as Akka Typed [3] and Typed Actors [38].

A key difference between  $\lambda_{\text{ch}}$  and  $\lambda_{\text{act}}$  is that `receive` (unlike `take`) is a nullary operation to receive a value from the actor’s mailbox. Consequently, it is necessary to use a simple *type-and-effect system* (as inspired by Gifford and Lucassen [14]) to type terms with respect to the mailbox type of the enclosing actor.

### 4.1 Syntax and Typing of Terms

Figure 7 shows the syntax and typing rules for  $\lambda_{\text{act}}$ . As with  $\lambda_{\text{ch}}$ ,  $\alpha$  ranges over variables and names. `ActorRef(A)` is an *actor reference* or process ID, and allows messages to be sent to an actor. As for communication and concurrency primitives, `spawn M` spawns a new actor to evaluate a computation  $M$ ; `send V W` sends a value  $V$  to an actor referred to by reference  $W$ ; `receive` receives a value from the actor’s mailbox; and `self` returns an actor’s own process ID.

Function arrows  $A \rightarrow^C B$  are annotated with a type  $C$  which denotes the type of the mailbox of the actor evaluating the term. As an example, consider a function which multiplies a received number by a given value:

$$\text{recvAndMult} \triangleq \lambda n. \text{let } x \leftarrow \text{receive in } (x \times n)$$

Such a function would have type  $\text{Int} \rightarrow^{\text{Int}} \text{Int}$ , and as an example would not be typeable for an actor that could only receive strings. Again, we work in the setting of fine-grain call-by-value; the distinction between values and computations is helpful when reasoning about the metatheory. We have two typing judgements: the standard judgement on values  $\Gamma \vdash V : A$ , and a judgement  $\Gamma \mid B \vdash M : A$  which states that a term  $M$  has type  $A$  under typing context  $\Gamma$ , and can receive values of type  $B$ . The typing of `receive` and `self` depends on the type of the actor’s mailbox.

Syntax		
Types	$A, B, C ::= \mathbf{1} \mid A \rightarrow^C B \mid \text{ActorRef}(A)$	
Variables and names	$\alpha ::= x \mid a$	
Values	$V, W ::= \alpha \mid \lambda x. M \mid ()$	
Computations	$L, M, N ::= V W$ $\mid \text{let } x \leftarrow M \text{ in } N \mid \text{return } V$ $\mid \text{spawn } M \mid \text{send } V W \mid \text{receive} \mid \text{self}$	
Value typing rules <span style="float: right; border: 1px solid black; padding: 2px;"><math>\Gamma \vdash V : A</math></span>		
VAR	ABS	UNIT
$\frac{\alpha : A \in \Gamma}{\Gamma \vdash \alpha : A}$	$\frac{\Gamma, x : A \mid C \vdash M : B}{\Gamma \vdash \lambda x. M : A \rightarrow^C B}$	$\frac{}{\Gamma \vdash () : \mathbf{1}}$
Computation typing rules <span style="float: right; border: 1px solid black; padding: 2px;"><math>\Gamma \mid B \vdash M : A</math></span>		
APP	EFFLET	
$\frac{\Gamma \vdash V : A \rightarrow^C B \quad \Gamma \vdash W : A}{\Gamma \mid C \vdash V W : B}$	$\frac{\Gamma \mid C \vdash M : A \quad \Gamma, x : A \mid C \vdash N : B}{\Gamma \mid C \vdash \text{let } x \leftarrow M \text{ in } N : B}$	
EFFRETURN	SEND	RECV
$\frac{\Gamma \vdash V : A}{\Gamma \mid C \vdash \text{return } V : A}$	$\frac{\Gamma \vdash V : A \quad \Gamma \vdash W : \text{ActorRef}(A)}{\Gamma \mid C \vdash \text{send } V W : \mathbf{1}}$	$\frac{}{\Gamma \mid C \vdash \text{receive} : C}$
SPAWN	SELF	
$\frac{\Gamma \mid A \vdash M : \mathbf{1}}{\Gamma \mid C \vdash \text{spawn } M : \text{ActorRef}(A)}$	$\frac{}{\Gamma \mid A \vdash \text{self} : \text{ActorRef}(A)}$	

 Fig. 7:  $\lambda_{\text{act}}$  Typing Rules

## 4.2 Operational Semantics

Figure 8 shows the syntax of  $\lambda_{\text{act}}$  evaluation contexts, as well as the syntax and typing rules of  $\lambda_{\text{act}}$  configurations. Evaluation contexts for terms and configurations are similar to  $\lambda_{\text{ch}}$ . The primary difference from  $\lambda_{\text{ch}}$  is the actor configuration  $\langle a, M, \vec{V} \rangle$ , which can be read as “an actor with name  $a$  evaluating term  $M$ , with a mailbox consisting of values  $\vec{V}$ ”. Whereas a term  $M$  is itself a configuration in  $\lambda_{\text{ch}}$ , a term in  $\lambda_{\text{act}}$  *must* be evaluated as part of an actor configuration. The typing rules for  $\lambda_{\text{act}}$  configurations ensure that all values contained in an actor mailbox are well-typed with respect to the mailbox type, and that a configuration  $C$  under a name restriction  $(\nu a)C$  contains an actor with name  $a$ .

Figure 9 shows the reduction rules for  $\lambda_{\text{act}}$ . Again, reduction on terms preserves typing, and the functional fragment of  $\lambda_{\text{act}}$  enjoys progress.

### Lemma 9 (Preservation ( $\lambda_{\text{act}}$ terms)).

If  $\Gamma \vdash M : A$  and  $M \longrightarrow_M M'$ , then  $\Gamma \vdash M' : A$ .

Syntax of evaluation contexts and configurations	
Evaluation contexts	$E ::= [] \mid \text{let } x \Leftarrow E \text{ in } M$
Configurations	$\mathcal{C}, \mathcal{D}, \mathcal{E} ::= \mathcal{C} \parallel \mathcal{D} \mid (\nu a)\mathcal{C} \mid \langle a, M, \vec{V} \rangle$
Configuration contexts	$G ::= [] \mid G \parallel \mathcal{C} \mid (\nu a)G$
Typing rules for configurations <span style="float: right; border: 1px solid black; padding: 2px 5px;"><math>\Gamma; \Delta \vdash \mathcal{C}</math></span>	
<small>PAR</small>	<small>PID</small>
$\frac{\Gamma; \Delta_1 \vdash \mathcal{C}_1 \quad \Gamma; \Delta_2 \vdash \mathcal{C}_2}{\Gamma; \Delta_1, \Delta_2 \vdash \mathcal{C}_1 \parallel \mathcal{C}_2}$	$\frac{\Gamma, a : \text{ActorRef}(A); \Delta, a : A \vdash \mathcal{C}}{\Gamma; \Delta \vdash (\nu a)\mathcal{C}}$
<small>ACTOR</small>	
$\frac{\Gamma, a : \text{ActorRef}(A) \mid A \vdash M : \mathbf{1} \quad (\Gamma, a : \text{ActorRef}(A) \vdash V_i : A)_i}{\Gamma, a : \text{ActorRef}(A); a : A \vdash \langle a, M, \vec{V} \rangle}$	

Fig. 8:  $\lambda_{\text{act}}$  Evaluation Contexts and Configurations**Lemma 10 (Progress ( $\lambda_{\text{act}}$  terms)).**

Assume  $\Gamma$  is either empty or only contains entries of the form  $a_i : \text{ActorRef}(A_i)$ .  
If  $\Gamma \mid B \vdash M : A$ , then either:

1.  $M = \text{return } V$  for some value  $V$
2.  $M$  can be written as  $E[M']$ , where  $M'$  is a communication or concurrency primitive (i.e. `spawn`  $N$ , `send`  $V W$ , `receive`, or `self`)
3. There exists some  $M'$  such that  $M \rightarrow_M M'$

**Reduction on Configurations.** While  $\lambda_{\text{ch}}$  makes use of separate constructs to create new processes and channels,  $\lambda_{\text{act}}$  uses a single construct `spawn`  $M$  to spawn a new actor with an empty mailbox to evaluate term  $M$ . Communication happens directly between actors instead of through an intermediate entity: as a result of evaluating `send`  $V a$ , the value  $V$  will be appended directly to the end of the mailbox of actor  $a$ . `SENDSelf` allows reflexive sending; an alternative would be to decouple mailboxes from the definition of actors, but this complicates both the configuration typing rules and the intuition. `Self` returns the name of the current process, and `RECEIVE` retrieves the head value of a non-empty mailbox.

As before, typing is preserved modulo structural congruence and under reduction.

**Lemma 11.** If  $\Gamma; \Delta \vdash \mathcal{C}$  and there exists a  $\mathcal{D}$  such that  $\mathcal{C} \equiv \mathcal{D}$ , then  $\Gamma; \Delta \vdash \mathcal{D}$ .

**Theorem 12 (Preservation ( $\lambda_{\text{act}}$  configurations)).**

If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \rightarrow \mathcal{C}_2$ , then  $\Gamma; \Delta \vdash \mathcal{C}_2$ .

**4.3 Progress and Canonical Forms**

Again, we cannot guarantee deadlock-freedom for  $\lambda_{\text{act}}$ . Instead, we characterise the exact form of progress that  $\lambda_{\text{act}}$  enjoys: a well-typed configuration can always

Reduction on terms	
	$(\lambda x.M) V \rightarrow_M M\{V/x\}$ $\text{let } x \leftarrow \text{return } V \text{ in } M \rightarrow_M M\{V/x\}$ $E[M] \rightarrow_M E[M'] \quad \text{if } M \rightarrow_M M'$
Structural congruence	
$\mathcal{C} \parallel \mathcal{D} \equiv \mathcal{D} \parallel \mathcal{C} \quad \mathcal{C} \parallel (\mathcal{D} \parallel \mathcal{E}) \equiv (\mathcal{C} \parallel \mathcal{D}) \parallel \mathcal{E} \quad \mathcal{C} \parallel (\nu a)\mathcal{D} \equiv (\nu a)(\mathcal{C} \parallel \mathcal{D}) \text{ if } a \notin \text{fv}(\mathcal{C})$ $G[\mathcal{C}] \equiv G[\mathcal{D}] \text{ if } \mathcal{C} \equiv \mathcal{D}$	
Reduction on configurations	
SPAWN	$\langle a, E[\text{spawn } M], \vec{V} \rangle \rightarrow (\nu b)(\langle a, E[\text{return } b], \vec{V} \rangle \parallel \langle b, M, \epsilon \rangle) \quad (b \text{ is fresh})$
SEND	$\langle a, E[\text{send } b V'], \vec{V} \rangle \parallel \langle b, M, \vec{W} \rangle \rightarrow \langle a, E[\text{return } ()], \vec{V} \rangle \parallel \langle b, M, \vec{W} \cdot V' \rangle$
SENDSSELF	$\langle a, E[\text{send } a V'], \vec{V} \rangle \rightarrow \langle a, E[\text{return } ()], \vec{V} \cdot V' \rangle$
SELF	$\langle a, E[\text{self}], \vec{V} \rangle \rightarrow \langle a, E[\text{return } a], \vec{V} \rangle$
RECEIVE	$\langle a, E[\text{receive}], W \cdot \vec{V} \rangle \rightarrow \langle a, E[\text{return } W], \vec{V} \rangle$
LIFT	$G[\mathcal{C}_1] \rightarrow G[\mathcal{C}_2] \quad (\text{if } \mathcal{C}_1 \rightarrow \mathcal{C}_2)$
LIFTM	$\langle a, M_1, \vec{V} \rangle \rightarrow \langle a, M_2, \vec{V} \rangle \quad (\text{if } M_1 \rightarrow_M M_2)$

Fig. 9: Reduction Rules and Equivalences for Terms and Configurations

reduce unless all leaves of the configuration typing judgement are actors which have either fully evaluated their terms, or are blocked waiting for a message from an empty mailbox. Defining a canonical form again aids us in reasoning about progress.

**Definition 13 (Canonical form ( $\lambda_{\text{act}}$ )).** A  $\lambda_{\text{act}}$  configuration  $\mathcal{C}$  is in canonical form if  $\mathcal{C}$  can be written  $(\nu a_1) \dots (\nu a_n)(\langle a_1, M_1, \vec{V}_1 \rangle \parallel \dots \parallel \langle a_n, M_n, \vec{V}_n \rangle)$ .

**Lemma 14.** If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\Delta = a_1 : A_1, \dots, a_k : A_k$ , then there exists  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}' = (\nu a_{k+1}) \dots (\nu a_n)(\langle a_1, M_1, \vec{V}_1 \rangle \parallel \dots \parallel \langle a_n, M_n, \vec{V}_n \rangle)$ .

As before, it follows as a corollary of Lemma 14 that closed configurations can be written in canonical form, and with canonical forms defined, we can classify the notion of progress enjoyed by  $\lambda_{\text{act}}$ .

**Corollary 15.** If  $\cdot; \cdot \vdash \mathcal{C}$ , then there exists some  $\mathcal{C}' \equiv \mathcal{C}$  such that  $\mathcal{C}'$  is in canonical form.

**Theorem 16 (Weak progress ( $\lambda_{\text{act}}$  configurations)).**

Let  $\cdot; \cdot \vdash \mathcal{C}$ ,  $\mathcal{C} \not\rightarrow$ , and let  $\mathcal{C}' = (\nu a_1) \dots (\nu a_n)(\langle a_1, M_1, \vec{V}_1 \rangle \parallel \dots \parallel \langle a_n, M_n, \vec{V}_n \rangle)$  be a canonical form of  $\mathcal{C}$ . Each actor with name  $a_i$  is either of the form:

1.  $\langle a_i, \text{return } W, \vec{V}_i \rangle$  for some value  $W$ , or;
2.  $\langle a_i, E[\text{receive}], \epsilon \rangle$ .

## 5 From $\lambda_{\text{act}}$ to $\lambda_{\text{ch}}$

With both calculi in place, we are now ready to look at the translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$ . The translation strategy is straightforward, but not previously formalised. The key idea is to emulate a mailbox using a channel, and to pass the channel as an argument to each function. The translation on terms is parameterised over the variable referring to the channel, which is used to implement context-dependent operations (i.e. `receive` and `self`).

As an example, consider `recvAndDouble`, which is a specialisation of the `recvAndMult` function which doubles the number received from the mailbox.

$$\text{recvAndDouble} \triangleq \text{let } x \leftarrow \text{receive in } (x \times 2)$$

A possible configuration would be an actor evaluating `recvAndDouble`, with some name  $a$  and mailbox with values  $\vec{V}$ , under a name restriction for  $a$ .

$$(\nu a)((a, \text{recvAndDouble}, \vec{V}))$$

The translation on terms takes a channel name  $ch$  as a parameter. As a result of the translation, we have that:

$$\llbracket \text{recvAndDouble} \rrbracket ch = \text{let } x \leftarrow \text{take } ch \text{ in } (x \times 2)$$

with the corresponding configuration:

$$(\nu a)(a(\llbracket \vec{V} \rrbracket) \parallel \llbracket \text{recvAndDouble} \rrbracket a)$$

The values from the mailbox are translated pointwise and form the contents of a buffer with name  $a$ . The translation of `recvAndDouble` is provided with the name  $a$  which is used to emulate `receive`.

### 5.1 Translation ( $\lambda_{\text{act}}$ to $\lambda_{\text{ch}}$ )

Figure 10 shows the formal translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$ . Of particular note is the translation on terms:  $\llbracket - \rrbracket ch$  translates a  $\lambda_{\text{act}}$  term into an  $\lambda_{\text{ch}}$  term using a channel with name  $ch$  to emulate a mailbox.

An actor reference is simply represented as a channel reference in  $\lambda_{\text{ch}}$ ; we emulate sending a message to another actor by writing to the channel emulating the recipient's mailbox. Key to translating  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  is the translation of function arrows  $A \rightarrow^C B$ ; the effect annotation  $C$  is replaced by a second parameter  $\text{ChanRef}(C)$ , which is used to emulate the mailbox of the actor. Values are translated to themselves, with the exception of  $\lambda$  abstractions, which are translated to take an additional parameter denoting the channel used to emulate operations on a mailbox. Given a parameter  $ch$ , the translation function for terms emulates `receive` by taking a value from  $ch$ , and emulates `self` by returning  $ch$ .

Although the translation is straightforwardly defined, it is a *global* translation [11], since all functions must be modified in order to take the channel emulating the mailbox as an additional parameter.

Translation on types	
$\llbracket \text{ActorRef}(A) \rrbracket = \text{ChanRef}(\llbracket A \rrbracket)$	
$\llbracket \mathbf{1} \rrbracket = \mathbf{1}$	
$\llbracket A \rightarrow^C B \rrbracket = \llbracket A \rrbracket \rightarrow \text{ChanRef}(\llbracket C \rrbracket) \rightarrow \llbracket B \rrbracket$	
Translation on values	
$\llbracket x \rrbracket = x$	$\llbracket a \rrbracket = a$
$\llbracket \lambda x.M \rrbracket = \lambda x.\lambda ch.(\llbracket M \rrbracket ch)$	$\llbracket () \rrbracket = ()$
Translation on computation terms	
$\llbracket \text{let } x \leftarrow M \text{ in } N \rrbracket ch = \text{let } x \leftarrow (\llbracket M \rrbracket ch) \text{ in } \llbracket N \rrbracket ch$	
$\llbracket V W \rrbracket ch = (\llbracket V \rrbracket)(\llbracket W \rrbracket) ch$	$\llbracket \text{spawn } M \rrbracket ch = \text{let } chMb \leftarrow \text{newCh in}$
$\llbracket \text{return } V \rrbracket ch = \text{return } \llbracket V \rrbracket$	$\text{fork}(\llbracket M \rrbracket chMb);$
$\llbracket \text{self} \rrbracket ch = \text{return } ch$	$\text{return } chMb$
$\llbracket \text{receive} \rrbracket ch = \text{take } ch$	$\llbracket \text{send } V W \rrbracket ch = \text{give}(\llbracket V \rrbracket)(\llbracket W \rrbracket)$
Translation on configurations	
$\llbracket \mathcal{C}_1 \parallel \mathcal{C}_2 \rrbracket = \llbracket \mathcal{C}_1 \rrbracket \parallel \llbracket \mathcal{C}_2 \rrbracket$	
$\llbracket (\nu a)\mathcal{C} \rrbracket = (\nu a)\llbracket \mathcal{C} \rrbracket$	
$\llbracket \langle a, M, \vec{V} \rangle \rrbracket = a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket M \rrbracket a)$	

 Fig. 10: Translation from  $\lambda_{act}$  into  $\lambda_{ch}$ 

## 5.2 Properties of the Translation

The translation on terms and values preserves typing. We extend the translation function pointwise to typing environments:

$$\llbracket \alpha_1 : A_1, \dots, \alpha_n : A_n \rrbracket = \alpha_1 : \llbracket A_1 \rrbracket, \dots, \alpha_n : \llbracket A_n \rrbracket$$

**Lemma 17** ( $\llbracket - \rrbracket$  preserves typing (terms and values)).

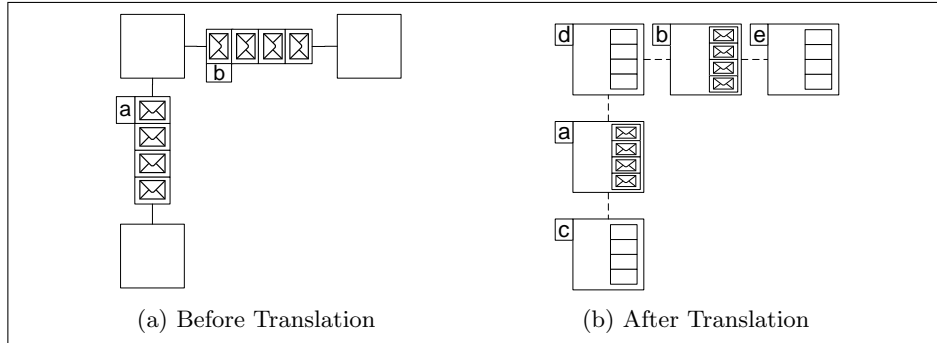
1. If  $\Gamma \vdash V : A$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket$  in  $\lambda_{ch}$ .
2. If  $\Gamma \mid B \vdash M : A$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket, \alpha : \text{ChanRef}(\llbracket B \rrbracket) \vdash \llbracket M \rrbracket \alpha : \llbracket A \rrbracket$  in  $\lambda_{ch}$ .

To state a semantics preservation result, we also define a translation on configurations; the translations on parallel composition and name restrictions are homomorphic. An actor configuration  $\langle a, M, \vec{V} \rangle$  is translated as a buffer  $a(\llbracket \vec{V} \rrbracket)$ , (writing  $\llbracket \vec{V} \rrbracket = \llbracket V_0 \rrbracket \cdot \dots \cdot \llbracket V_n \rrbracket$  for each  $V_i \in \vec{V}$ ), composed in parallel with the translation of  $M$ , using  $a$  as the mailbox channel.

We can now see that the translation preserves typeability of configurations.

**Theorem 18** ( $\llbracket - \rrbracket$  preserves typeability (configurations)).

If  $\Gamma; \Delta \vdash \mathcal{C}$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket; \llbracket \Delta \rrbracket \vdash \llbracket \mathcal{C} \rrbracket$  in  $\lambda_{ch}$ .

Fig. 11: Translation Strategy:  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ 

We describe semantics preservation in terms of a simulation theorem: should a configuration  $\mathcal{C}_1$  reduce to a configuration  $\mathcal{C}_2$  in  $\lambda_{\text{act}}$ , then there exists some configuration  $\mathcal{D}$  in  $\lambda_{\text{ch}}$  such that  $\llbracket \mathcal{C}_1 \rrbracket$  reduces in zero or more steps to  $\mathcal{D}$ , with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ . To establish the result, we begin by showing that  $\lambda_{\text{act}}$  term reduction can be simulated in  $\lambda_{\text{ch}}$ .

**Lemma 19 (Simulation of  $\lambda_{\text{act}}$  term reduction in  $\lambda_{\text{ch}}$ ).**

*If  $\Gamma \vdash M_1 : A$  and  $M_1 \rightarrow_M M_2$  in  $\lambda_{\text{act}}$ , then given some  $\alpha$ ,  $\llbracket M_1 \rrbracket \alpha \rightarrow_M^* \llbracket M_2 \rrbracket \alpha$  in  $\lambda_{\text{ch}}$ .*

Finally, we can see that the translation preserves structural congruences, and that  $\lambda_{\text{ch}}$  configurations can simulate reductions in  $\lambda_{\text{act}}$ .

**Lemma 20.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  and  $\mathcal{C} \equiv \mathcal{D}$ , then  $\llbracket \mathcal{C} \rrbracket \equiv \llbracket \mathcal{D} \rrbracket$ .*

**Theorem 21 (Simulation of  $\lambda_{\text{act}}$  configurations in  $\lambda_{\text{ch}}$ ).**

*If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \rightarrow \mathcal{C}_2$ , then there exists some  $\mathcal{D}$  such that  $\llbracket \mathcal{C}_1 \rrbracket \rightarrow^* \mathcal{D}$ , with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ .*

## 6 From $\lambda_{\text{ch}}$ to $\lambda_{\text{act}}$

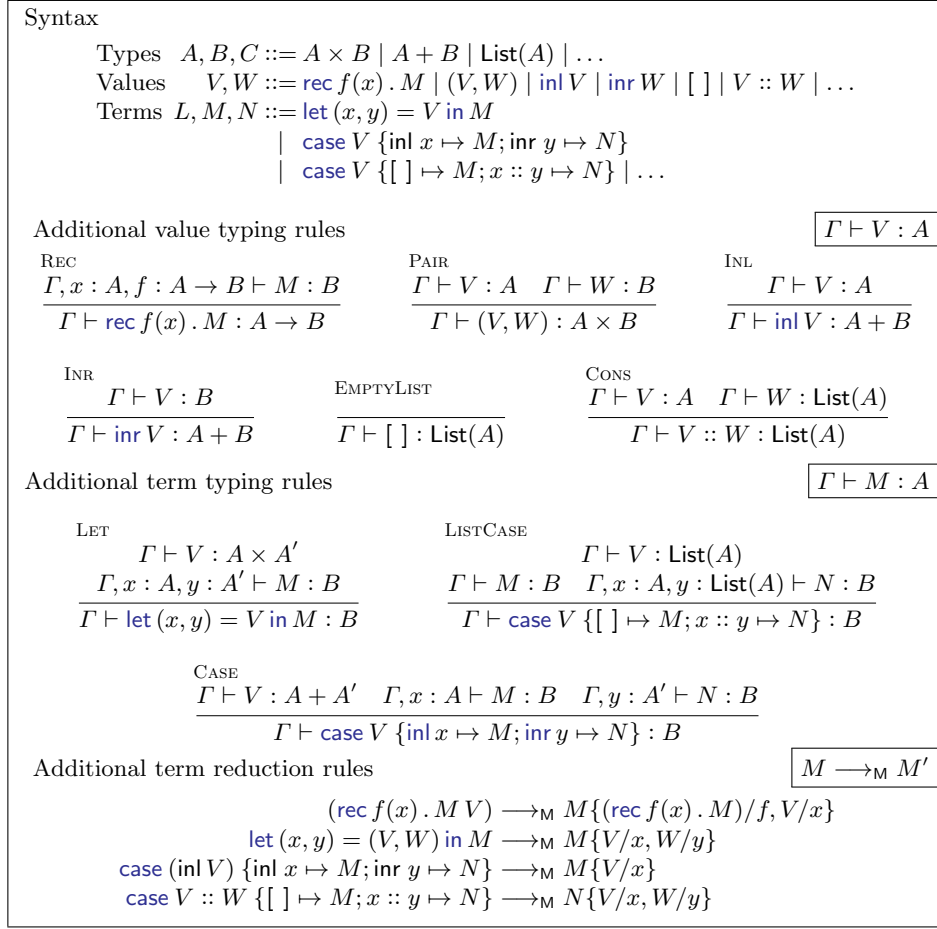
The translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  emulates an actor mailbox using a channel, using it to implement operations which normally rely on the context of the actor. Although a global translation, the translation is straightforward due to the limited form of communication supported by mailboxes.

Translating from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  is more challenging. Each channel in a system may have a different type; each process may have access to multiple channels; and (crucially) channels may be freely passed between processes.

### 6.1 Translation Strategy ( $\lambda_{\text{ch}}$ into $\lambda_{\text{act}}$ )

To translate typed actors into typed channels (shown in Figure 11), we emulate each channel using an actor process, which is crucial in retaining the mobility of channel endpoints.




 Fig. 12: Extensions to Core Languages to Allow Translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ 

Channel types describe the typing of a *communication medium* between communicating processes, where processes are unaware of the identity of other communicating parties, and the types of messages that another party may receive. Unfortunately, the same does not hold for actors. Consequently, we require that before translating into actors, that *every channel has the same type*. Although this may seem restrictive, it is both possible and safe to transform a  $\lambda_{\text{ch}}$  program with multiple channel types into a  $\lambda_{\text{ch}}$  program with a single channel type.

As an example, suppose we have a program which contains channels carrying values of types `Int`, `String`, and `Bool`. It is possible to construct a variant type  $\langle \ell_1 : \text{Int}, \ell_2 : \text{String}, \ell_3 : \text{Bool} \rangle$  which can be assigned to all channels in the system. Then, supposing we wanted to send a 5 along a channel which previously had type `ChanRef(Int)`, we would instead send a value  $\langle \ell_1 = 5 \rangle$ .

Appendix A provides more details, and proves that the transformation is safe.

<pre> <b>drain</b> <math>\triangleq</math> <b>rec</b> <math>f(x)</math>. <b>let</b> (<math>vals, pids</math>) = <math>x</math> <b>in</b> <b>case</b> <math>vals</math> {   [ ] <math>\mapsto</math> <b>return</b> (<math>vals, pids</math>)   <math>v :: vs \mapsto</math>     <b>case</b> <math>pids</math> {       [ ] <math>\mapsto</math> <b>return</b> (<math>vals, pids</math>)       <math>pid :: pids \mapsto</math> <b>send</b> <math>v</math> <math>pid</math>;                         <math>f(vs, pids)</math>     } } </pre>	<pre> <b>body</b> <math>\triangleq</math> <b>rec</b> <math>g(state)</math> . <b>let</b> <math>recvVal \leftarrow</math> <b>receive</b> <b>in</b> <b>let</b> (<math>vals, pids</math>) = <math>state</math> <b>in</b> <b>case</b> <math>recvVal</math> {   <b>inl</b> <math>v \mapsto</math> <b>let</b> <math>vals' \leftarrow vals ++ [v]</math> <b>in</b>                 <b>let</b> <math>state' \leftarrow</math> <b>drain</b> (<math>vals', pids</math>) <b>in</b>                 <math>g(state')</math>   <b>inr</b> <math>pid \mapsto</math> <b>let</b> <math>pids' \leftarrow pids ++ [pid]</math> <b>in</b>                  <b>let</b> <math>state' \leftarrow</math> <b>drain</b> (<math>vals, pids'</math>) <b>in</b>                  <math>g(state')</math> } </pre>
---	--

Fig. 13: Metalevel Definitions Required for the Translation from  $\lambda_{ch}$  into  $\lambda_{act}$ 

## 6.2 Extensions to Core Language

To emulate channels using actors, we require several more term-level language constructs: sums, products, recursive functions, and lists. Recursive functions are used to implement an event loop, and lists are used to maintain a buffer at the term level in addition to the meta-level. Products are used to emulate the state of a channel, in particular to record both a list of values in the buffer and a list of pending requests. Sum types allow the disambiguation of the two types of messages sent to an actor: one to queue a message (emulating **give**) and one to dequeue a message and return it to the actor making the request (emulating **take**). Additionally, sums can be used to encode monomorphic variant types.

Figure 12 shows the extensions required to the core term language and reduction rules; we omit the reduction rules for case analysis on the right injection of a sum and the empty list. The typing rules are shown for  $\lambda_{ch}$  but can be easily adapted for  $\lambda_{act}$ , and it is straightforward to verify that the extended languages still enjoy progress and preservation.

## 6.3 Translation

We write  $\lambda_{ch}$  judgements of the form  $\{B\} \Gamma \vdash M : A$  for a term where all channels have type  $B$ , and similarly for value and configuration typing judgements. Under such a judgement, we can write **Chan** instead of **ChanRef**( $B$ ).

**Metalevel Definitions.** The majority of the translation lies within the translation of **newCh**, which makes use of the meta-level definitions shown in Figure 13. The **body** function emulates a channel. Firstly, the actor receives a message  $recvVal$  from its mailbox, which is either of the form **inl**  $V$  to store a message  $V$ , or **inr**  $W$  to request that a value is dequeued and sent to the actor with ID  $W$ . We assume a standard implementation of the list concatenation function ( $++$ ). If the message is **inl**  $V$ , then  $V$  is appended to the tail of the list of values stored in the channel, and the new state is passed as an argument to **drain**. If the message is **inr**  $W$ , then the process ID  $W$  is appended to the end of the list of processes waiting for a value. The **drain** function satisfies all requests that can be satisfied, returning an updated channel state.

<p>Translation on types</p> $\begin{aligned} \llbracket \text{Chan} \rrbracket C &= \text{ActorRef}(\llbracket C \rrbracket C + \text{ActorRef}(\llbracket C \rrbracket C)) \\ \llbracket A \rightarrow B \rrbracket C &= (\llbracket A \rrbracket C) \rightarrow^{\llbracket C \rrbracket C} (\llbracket B \rrbracket C) \end{aligned}$ <p>Translation on communication and concurrency primitives</p> $\begin{aligned} \llbracket \text{fork } M \rrbracket &= \text{let } x \leftarrow \text{spawn } \llbracket M \rrbracket \text{ in return } () \\ \llbracket \text{give } V W \rrbracket &= \text{send } (\text{inl } \llbracket V \rrbracket) \llbracket W \rrbracket \\ \llbracket \text{take } V \rrbracket &= \text{let } \text{selfPid} \leftarrow \text{self} \text{ in } \quad \text{selfPid is a fresh variable} \\ &\quad \text{send } (\text{inr } \text{selfPid}) \llbracket V \rrbracket; \\ &\quad \text{receive} \\ \llbracket \text{newCh} \rrbracket &= \text{spawn } (\text{body } ([ ], [ ])) \end{aligned}$ <p>Translation on configurations</p> $\begin{aligned} \llbracket C_1 \parallel C_2 \rrbracket &= \llbracket C_1 \rrbracket \parallel \llbracket C_2 \rrbracket \\ \llbracket (\nu a)C \rrbracket &= (\nu a) \llbracket C \rrbracket \\ \llbracket M \rrbracket &= (\nu a) \langle a, \llbracket M \rrbracket, \epsilon \rangle \quad a \text{ is a fresh name} \\ \llbracket a(\vec{V}) \rrbracket &= \langle a, \text{body } (\llbracket \vec{V} \rrbracket), [ ], \epsilon \rangle \\ &\quad \text{where } \llbracket \vec{V} \rrbracket = \llbracket V_0 \rrbracket :: \dots :: \llbracket V_n \rrbracket :: [ ] \end{aligned}$
--

 Fig. 14: Translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ 

**Translation on Types.** Figure 14 shows the translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ . The translation function on types  $\llbracket - \rrbracket C$  takes the type of all channels  $C$  as its argument, and is used to annotate all function arrows and to assign a parameter to ActorRef types. The (omitted) translations on sums, products, and lists are homomorphic. The translation of Chan is  $\text{ActorRef}(\llbracket C \rrbracket C + \text{ActorRef}(\llbracket C \rrbracket C))$ , meaning an actor which can receive a request to either store a value of type  $\llbracket C \rrbracket C$ , or to dequeue a value and send it to a process ID of type  $\text{ActorRef}(\llbracket C \rrbracket C)$ .

**Translation on Communication and Concurrency Primitives.** We omit the translation on values and functional terms, which are homomorphisms. Processes in  $\lambda_{\text{ch}}$  are anonymous, whereas all actors in  $\lambda_{\text{act}}$  are addressable; to emulate fork, we therefore discard the reference returned by spawn. The translation of give wraps the translated value to be sent in the left injection of a sum type, and sends to the translated channel name  $\llbracket W \rrbracket$ .

To emulate take, self is firstly used to retrieve the process ID of the actor. Next, the process ID is wrapped in the right injection and sent to the actor emulating the channel, and the actor waits for the response message.

Finally, the translation of newCh spawns a new actor to execute body.

**Translation on Configurations.** The translation function  $\llbracket - \rrbracket$  is homomorphic on parallel composition and name restriction. Unlike  $\lambda_{\text{ch}}$ , a term cannot exist outwith an enclosing actor context in  $\lambda_{\text{act}}$ . Consequently, the translation of a process evaluating term  $M$  is an actor with some fresh name  $a$  and an empty mailbox evaluating  $\llbracket M \rrbracket$ , enclosed in a name restriction.

The translation of a  $\lambda_{\text{ch}}$  buffer requires a *term-level* list to be constructed from a *meta-level* sequence; the mailbox is required for requests to queue and dequeue values. Moreover, the translation of a buffer is an actor with an empty mailbox which evaluates `body` with a state containing the (term-level) list of values, and an empty request queue.

In contrast to the global transformation in the previous section, although the translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$ , is much more verbose, it is (once all channels have the same type) a *local transformation* [11].

#### 6.4 Properties of the Translation

We firstly define translations on typing environments. Since all channels in the source language of the translation have the same type, we can assume that each entry in the codomain of  $\Delta$  is the same type  $A$ . Importantly, each entry in the translated environment refers to the name of a *channel*, and thus has the same type as the translation of `Chan`.

**Definition 22 (Translation of typing environments).**

1. If  $\Gamma = \alpha_1:A_1, \dots, \alpha_n:A_n$ , define  $(\llbracket \Gamma \rrbracket B) = \alpha_1 : (\llbracket A_1 \rrbracket B), \dots, \alpha_n : (\llbracket A_n \rrbracket B)$ .
2. Given a  $\Delta = a_1 : A, \dots, a_n : A$ , define  $(\llbracket \Delta \rrbracket A) = a_1 : ((\llbracket A \rrbracket A) + \text{ActorRef}((\llbracket A \rrbracket A))), \dots, a_n : ((\llbracket A \rrbracket A) + \text{ActorRef}((\llbracket A \rrbracket A)))$ .

We can now begin to state our final set of results. The translation on terms preserves typing.

**Lemma 23 ( $(\llbracket - \rrbracket)$  preserves typing (terms and values)).**

1. If  $\{B\} \Gamma \vdash V:A$ , then  $(\llbracket \Gamma \rrbracket B) \vdash (\llbracket V \rrbracket):(\llbracket A \rrbracket B)$ .
2. If  $\{B\} \Gamma \vdash M:A$ , then  $(\llbracket \Gamma \rrbracket B) \mid (\llbracket B \rrbracket B) \vdash (\llbracket M \rrbracket):(\llbracket A \rrbracket B)$ .

The translation on configurations also preserves typeability. We write  $\Gamma \simeq \Delta$  if for each  $a : A \in \Delta$ , we have that  $a : \text{ChanRef}(A) \in \Gamma$ ; for closed configurations this is ensured by `CHAN`. This is necessary since the typing rules for  $\lambda_{\text{act}}$  require that the local actor name is present in the term environment to ensure preservation in the presence of `self`, but there is no such restriction in  $\lambda_{\text{ch}}$ .

**Theorem 24 ( $(\llbracket - \rrbracket)$  preserves typeability (configurations)).**

If  $\{A\} \Gamma; \Delta \vdash C$  with  $\Gamma \simeq \Delta$ , then  $(\llbracket \Gamma \rrbracket A); (\llbracket \Delta \rrbracket A) \vdash (\llbracket C \rrbracket)$ .

It is clear that reduction on translated  $\lambda_{\text{ch}}$  terms can simulate reduction in  $\lambda_{\text{act}}$ ; in fact, we obtain a tighter (lockstep) simulation result than the translation from  $\lambda_{\text{act}}$  into  $\lambda_{\text{ch}}$  since  $\beta$ -reduction only requires one reduction instead of two.

**Lemma 25.**

If  $\{B\} \Gamma \vdash M_1 : A$  and  $M_1 \longrightarrow_M M_2$ , then  $(\llbracket M_1 \rrbracket) \longrightarrow_M (\llbracket M_2 \rrbracket)$ .

Finally, we show that  $\lambda_{\text{act}}$  can simulate  $\lambda_{\text{ch}}$ .

**Lemma 26.** If  $\Gamma; \Delta \vdash C$  and  $C \equiv \mathcal{D}$ , then  $(\llbracket C \rrbracket) \equiv (\llbracket \mathcal{D} \rrbracket)$ .

**Theorem 27 (Simulation ( $\lambda_{\text{act}}$  configurations in  $\lambda_{\text{ch}}$ )).**

If  $\{A\} \Gamma; \Delta \vdash C_1$ , and  $C_1 \longrightarrow C_2$ , then there exists some  $\mathcal{D}$  such that  $(\llbracket C_1 \rrbracket) \longrightarrow^* \mathcal{D}$  with  $\mathcal{D} \equiv (\llbracket C_2 \rrbracket)$ .

Additional types, terms, and configuration reduction rule	
Types ::= ActorRef(A, B)   ...	Terms ::= wait V   ...
$\langle a, E[\text{wait } b], \vec{V} \rangle \parallel \langle b, \text{return } V', \vec{W} \rangle \longrightarrow \langle a, E[\text{return } V'], \vec{V} \rangle \parallel \langle b, \text{return } V', \vec{W} \rangle$ $(\nu a)(\langle a, \text{return } V, \vec{V} \rangle) \parallel \mathcal{C} \equiv \mathcal{C}$	
Modified typing rules for terms	
$\boxed{\Gamma \mid A, B \vdash M : A}$	
$\frac{\text{SYNC-SPAWN} \quad \Gamma \mid A, B \vdash M : B}{\Gamma \mid C, C' \vdash \text{spawn } M : \text{ActorRef}(A, B)}$	$\frac{\text{SYNC-WAIT} \quad \Gamma \vdash V : \text{ActorRef}(A, B)}{\Gamma \mid C, C' \vdash \text{wait } V : B}$
$\frac{\text{SYNC-SELF}}{\Gamma \mid A, B \vdash \text{self} : \text{ActorRef}(A, B)}$	
Modified typing rules for configurations	
$\boxed{\Gamma; \Delta \vdash \mathcal{C}}$	
$\frac{\text{SYNC-ACTOR} \quad \begin{array}{l} \Gamma, a : \text{ActorRef}(A, B) \vdash M : B \\ (\Gamma, a : \text{ActorRef}(A, B) \vdash V_i : A)_i \end{array}}{\Gamma, a : \text{ActorRef}(A, B); a : (A, B) \vdash \langle a, M, \vec{V} \rangle}$	$\frac{\text{SYNC-NU} \quad \Gamma, a : \text{ActorRef}(A, B); \Delta, a : (A, B) \vdash \mathcal{C}}{\Gamma; \Delta \vdash (\nu a)\mathcal{C}}$

 Fig. 15: Extensions to Add Synchronisation to  $\lambda_{\text{act}}$ 

## 7 Extensions and Future Work

In this section, we discuss common extensions to channel- and actor-based languages. Firstly, we discuss synchronisation, which is ubiquitous in practical implementations of actor-inspired languages. Adding synchronisation simplifies the translation from channels to actors, and relaxes the restriction that all channels must have the same type. Secondly, we discuss how to nondeterministically choose a message from a collection of possible sources. Thirdly, we discuss what the translations tell us about the nature of behavioural typing disciplines for actors. Establishing exactly how the latter two extensions fit into our framework is the subject of ongoing and future work.

### 7.1 Synchronisation

While communicating with an actor via asynchronous message passing suffices for many purposes, the approach can become cumbersome when implementing “call-response” style interactions. Practical implementations such as Erlang and Akka implement some way of synchronising on a result: Erlang achieves this by generating a unique reference to send along with a request, *selectively receiving* from the mailbox to await a response tagged with the same unique reference. Another method of synchronisation embraced by the Active Object community [9,

<p>Modified translation on types</p> $\begin{aligned} \llbracket \text{ChanRef}(A) \rrbracket &= \text{ActorRef}(\llbracket A \rrbracket + \text{ActorRef}(\llbracket A \rrbracket, \llbracket A \rrbracket), \mathbf{1}) \\ \llbracket A \rightarrow B \rrbracket &= \llbracket A \rrbracket \rightarrow^{C, \mathbf{1}} \llbracket B \rrbracket \end{aligned}$ <p>Modified translation of <b>take</b></p> $\begin{aligned} \llbracket \text{take } V \rrbracket &= \text{let } requestorPid \leftarrow \text{spawn} ( \\ &\quad \text{let } selfPid \leftarrow \text{self} \text{ in} \\ &\quad \text{send } (\text{inr } selfPid) \llbracket V \rrbracket ; \\ &\quad \text{receive}) \text{ in} \\ &\quad \text{wait } requestorPid \end{aligned}$
--

Fig. 16: Translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  Using Synchrony

25, 26] as well as the Akka framework is to generate a *future variable* which is populated with the result of the call.

Figure 15 details an extension of  $\lambda_{\text{act}}$  with a synchronisation primitive, **wait**. In this extension, we replace the unary type constructor for process IDs with a binary type constructor  $\text{ActorRef}(A, B)$ , where  $A$  is the type of messages that the process can receive from its mailbox, and  $B$  is the type of value to which the process will eventually evaluate. We assume that the remainder of the primitives are modified to take the additional effect type into account. A variation of the **wait** primitive is implemented as part of the Links [8] concurrency model to address the type pollution problem.

We now adapt the previous translation from  $\lambda_{\text{ch}}$  to  $\lambda_{\text{act}}$ , making use of **wait** to avoid the need for the coalescing transformation. Figure 16 shows the modified translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  with **wait**. Channel references are translated into actor references which can either receive a value of type  $A$ , or a process which can receive a value of type  $A$  and will eventually evaluate to a value of type  $A$ . Note that the unbound annotation  $C, 1$  on function arrows reflects that the mailboxes can be of *any* type, since the mailboxes are unused in the actors emulating threads.

The key idea behind the modified translation is to spawn a fresh actor which makes the request to the channel and blocks waiting for the response. Once the actor spawned to make the request has received the result, the result can be retrieved synchronously using **wait** *without* reading from the mailbox.

The previous soundness theorems adapt to the new setting.

**Theorem 28.** *If  $\Gamma; \Delta \vdash \mathcal{C}$  with  $\Gamma \asymp \Delta$ , then  $\llbracket \Gamma \rrbracket; \llbracket \Delta \rrbracket \vdash \llbracket \mathcal{C} \rrbracket$ .*

**Theorem 29.** *If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then there exists some  $\mathcal{D}$  such that  $\llbracket \mathcal{C} \rrbracket \longrightarrow^* \mathcal{D}$  with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ .*

Translation in the other direction requires named threads and a **join** construct in  $\lambda_{\text{ch}}$ , but is otherwise unsurprising.

$$\begin{array}{c}
 \frac{\Gamma \vdash V : \text{ChanRef}(A) \quad \Gamma \vdash W : \text{ChanRef}(B)}{\Gamma \vdash \text{choose } V W : A + B} \\
 \\
 E[\text{choose } a b] \parallel a(W_1 \cdot \vec{V}_1) \parallel b(\vec{V}_2) \longrightarrow E[\text{return } (\text{inl } W_1)] \parallel a(\vec{V}_1) \parallel b(\vec{V}_2) \\
 E[\text{choose } a b] \parallel a(\vec{V}_1) \parallel b(W_2 \cdot \vec{V}_2) \longrightarrow E[\text{return } (\text{inr } W_2)] \parallel a(\vec{V}_1) \parallel b(\vec{V}_2)
 \end{array}$$

 Fig. 17: Additional Typing and Evaluation Rules for  $\lambda_{\text{ch}}$  with Choice

## 7.2 Choice

The calculus  $\lambda_{\text{ch}}$  only supports blocking receive from channels. A more powerful mechanism when dealing with channels is *selective communication*, where given two channels  $a$  and  $b$ , a value is taken nondeterministically from  $a$  or  $b$ . An important use case is receiving a value when either channel could be empty.

Here we have only considered the most basic case of selective choice over two channels. More generally, it can be extended to arbitrary regular data types [36]. Since Concurrent ML [37] embraces rendezvous-based synchronous communication, it provides *generalised selective communication* where a process can synchronise on a mixture of input or output communication events. Similarly, the join patterns of Erlang [12] and the selective receive operation on mailboxes of Erlang also provide general abstractions for selective communication.

As we are working in the asynchronous setting where a *give* operation can reduce immediately, we consider only input-guarded choice. Input-guarded choice can be added straightforwardly to  $\lambda_{\text{ch}}$ , as shown in Figure 17. Emulating such a construct satisfactorily in  $\lambda_{\text{act}}$  is nontrivial, because data must be received from a *local* message queue as opposed to a separate entity. One approach could be to use the work of Chaudhuri [7] which shows how to implement generalised choice using synchronous message passing, but implementing this in  $\lambda_{\text{ch}}$  may be difficult due to the asynchrony of *give*. We leave a more thorough investigation to future work.

## 7.3 Behavioural Types

Behavioural types allow the type of an object (e.g. a channel) to evolve as a program executes. A widely studied behavioural typing discipline is that of *session types* [20, 21] which supports channel types that are sufficiently rich to describe *communication protocols* between participants. As an example, the session type for a channel which sends two integers and receives their sum could be defined as follows:

$$! \text{Int}. ! \text{Int}. ? \text{Int}. \text{end}$$

where  $!A.S$  is the type of a channel which sends a value of type  $A$  before continuing with behaviour  $S$ . Session types are particularly suited to channels, whereas current work on session-typed actors concentrates on runtime monitoring [33].

A natural question to ask is whether one can combine the benefits of actors and of session types. Indeed, this was one of our original motivations for wanting to better understand the relationship between actors and channels in the first place. A session-typed channel may support both sending and receiving (at different points in the protocol it encodes). But communication with another processes mailbox is one-way. We have studied several variants of  $\lambda_{\text{act}}$  with *polarised* session types which capture such one-way communication, but they seem too weak to simulate session-typed channels. In future, we would like to find a natural extension of  $\lambda_{\text{act}}$  with behavioural types that admits a similar simulation result to the ones in this paper.

## 8 Related Work

Our formulation of concurrent  $\lambda$ -calculi is inspired by  $\lambda(\text{fut})$  [34], a concurrent  $\lambda$ -calculus with threads and future variables, as well as reference cells and an atomic exchange construct. In the presence of a list construct, futures are sufficient to encode asynchronous channels. In  $\lambda_{\text{ch}}$ , we concentrate on asynchronous channels as primitive entities to better understand the correspondence with actors. Channel-based concurrent  $\lambda$ -calculi have been used as a formalism for the design of channel-based languages with session types, richer type systems which are expressive enough to encode protocols such as SMTP [13, 28].

Channel-based programming languages are inspired by CSP [19] and the  $\pi$ -calculus [31]; the name restriction and parallel composition operators in  $\lambda_{\text{ch}}$  and  $\lambda_{\text{act}}$  are directly inspired by analogous constructs in the  $\pi$ -calculus. Concurrent ML [37] extends Standard ML with a rich set of concurrency constructs centred around synchronous channels, which again, can emulate asynchronous channels. A core notion in Concurrent ML is nondeterministically synchronising on multiple synchronous events, such as sending or receiving messages; relating such a construct to an actor calculus is nontrivial, and remains an open problem.

The actor model was designed by Hewitt et al. [18] and examined in the context of distributed systems by Agha [2]. Agha describes an operational semantics on systems of actors, with a denotational interpretation of actor behaviours. In the object-oriented setting, the actor model inspires *active objects* [26]: objects supporting asynchronous method calls which return responses using futures. De Boer et al. [9] describe a language and proof system for active objects with cooperatively scheduled threads within each object. Core ABS [25] is a specification language based on active objects. Using futures for synchronisation sidesteps the type pollution problem inherent in call-response patterns with actors, although our translations work in the absence of synchronisation. By working in the functional setting, we have smaller calculi.

Links [8] is a programming language designed for developing web applications which includes an implementation of typed message-passing concurrency built on an effect type system. The design of  $\lambda_{\text{act}}$  was inspired by Links.

Hopac [22] is a channel-based concurrency library for the F# programming language, based on Concurrent ML. The Hopac documentation includes a discussion



of CML-style synchronous channels and actors[1], providing an implementation of actor-style concurrency primitives using channels, and an implementation of channel-style concurrency primitives using actors. The implementation of channels using actors uses shared-memory concurrency in the form of ML-style references in order to implement the **take** function, whereas our translation achieves this using message passing. Additionally, our translation is formalised and we prove that the translations are type- and semantics-preserving.

## 9 Conclusion

Inspired by languages such as Go which take channels as core constructs for communication, and languages such as Erlang which are based on the actor model of concurrency, we have presented translations back and forth between a concurrent  $\lambda$ -calculus  $\lambda_{\text{ch}}$  with channel-based communication constructs and a concurrent  $\lambda$ -calculus  $\lambda_{\text{act}}$  with actor-based communication constructs. We have proved that  $\lambda_{\text{act}}$  can simulate  $\lambda_{\text{ch}}$  and vice-versa.

The translation from  $\lambda_{\text{act}}$  to  $\lambda_{\text{ch}}$  is straightforward, whereas the translation from  $\lambda_{\text{ch}}$  to  $\lambda_{\text{act}}$  requires considerably more effort. The discrepancy is illustrated in Figure 18. Any process can send and receive messages along a channel (Figure 18a), whereas, although any process can send a message to a mailbox, only one process can receive from that mailbox (Figure 18b). Viewed this way, it is apparent that the restrictions imposed on the communication behaviour of actors are exactly those captured by Merro and Sangiorgi’s localised  $\pi$ -calculus [29].

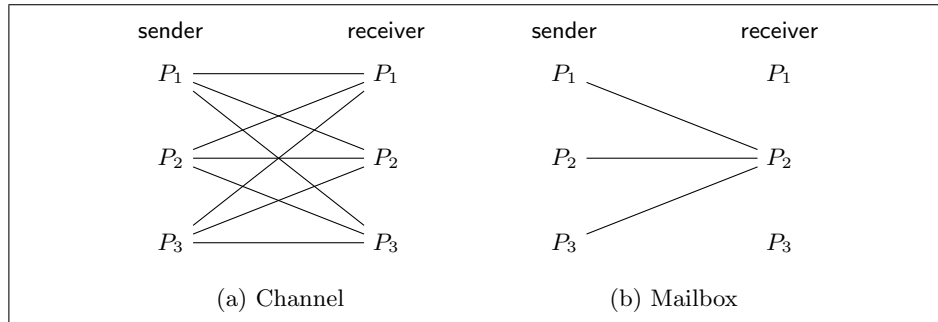


Fig. 18: Mailboxes as Pinned Channels

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## A Coalescing Transformation

Our translation from  $\lambda_{\text{ch}}$  into  $\lambda_{\text{act}}$  relies on the assumption that all channels have the same type, which is rarely the case in practice. Here, we sketch a sample type-directed transformation which we call *coalescing*, which transforms an arbitrary  $\lambda_{\text{ch}}$  program into an  $\lambda_{\text{ch}}$  program which has only one type of channel. To encode such a translation, we use an extension of the language with equirecursive types.

The transformation works by encapsulating each type of message in a variant type, and ensuring that **give** and **take** use the correct variant injections. Although the translation necessarily loses type information, thus introducing partiality, we can show that terms and configurations that are the result of the coalescing transformation never reduce to an error.

We say that a type  $A$  is a *base carried type* in a configuration  $\Gamma; \Delta \vdash \mathcal{C}$  if there exists some subterm  $\Gamma \vdash V : \text{ChanRef}(A)$ , where  $A$  is *not* of the form  $\text{ChanRef}(B)$ .

In order to perform the coalescing transformation, we require an environment  $\sigma$  which maps each base carried type  $A$  to a unique token  $\ell$ , which we use as an injection into a variant type.

We write  $\sigma \sim \Gamma; \Delta \vdash \mathcal{C}$  if  $\sigma$  contains a bijective mapping  $A \mapsto \ell$  for each base carried type in  $\Gamma; \Delta \vdash \mathcal{C}$ . We extend the relation analogously to judgements on values and computation terms.

Next, we define the notion of a *coalesced channel type*, which can be used to ensure that all channels in the system have the same type.

### Definition 30 (Coalesced channel type).

Given a token environment  $\sigma = A_0 \mapsto \ell_0, \dots, A_n \mapsto \ell_n$ , we define the coalesced channel type  $\text{cct}(\sigma)$  as

$$\text{cct}(\sigma) = \mu X. \langle \ell_0 : \underline{A_0}, \dots, \ell_n : \underline{A_n}, \ell_c : \text{ChanRef}(X) \rangle$$

where

$$\begin{array}{l} \underline{\mathbf{1}} = \mathbf{1} \\ \underline{A \rightarrow B} = \underline{A} \rightarrow \underline{B} \\ \underline{A \times B} = \underline{A} \times \underline{B} \end{array} \qquad \begin{array}{l} \underline{A + B} = \underline{A} + \underline{B} \\ \underline{\text{List}(A)} = \text{List}(\underline{A}) \\ \underline{\text{ChanRef}(A)} = \text{ChanRef}(X) \end{array}$$

which is the single channel type which can receive values of all possible types sent in the system.

Note that the definition of a base carried type excludes the possibility of a type of the form  $\text{ChanRef}(A)$  appearing in  $\sigma$ . To handle the case of sending channels, we require  $\text{cct}(\sigma)$  to be a recursive type; a distinguished token  $\ell_c$  denotes the variant case for sending a channel over a channel.

Retrieving a token from the token environment  $\sigma$  is defined by the following inference rules. Note that  $\text{ChanRef}(A)$  maps to the distinguished token  $\ell_c$ .

$$\frac{(A \mapsto \ell) \in \sigma}{\sigma(A) = \ell} \qquad \frac{}{\sigma(\text{ChanRef}(A)) = \ell_c}$$

With the coalesced channel type defined, we can define a function mapping types to coalesced types.

**Definition 31 (Type coalescing).**

$$\begin{array}{ll} |\mathbf{1}|^\sigma = \mathbf{1} & |A + B|^\sigma = |A|^\sigma + |B|^\sigma \\ |A \rightarrow B|^\sigma = |A|^\sigma \rightarrow |B|^\sigma & |\text{List}(A)|^\sigma = \text{List}(|A|^\sigma) \\ |A \times B|^\sigma = |A|^\sigma \times |B|^\sigma & |\text{ChanRef}(A)|^\sigma = \text{ChanRef}(\text{cct}(\sigma)) \end{array}$$

We then extend  $|-|^\sigma$  to typing environments  $\Gamma$ , taking into account that we must annotate channel names.

**Definition 32 (Type coalescing on environments).**

1. For  $\Gamma$ :
  - (a)  $|\emptyset|^\sigma = \emptyset$
  - (b)  $|x : A, \Gamma|^\sigma = x : |A|^\sigma, |\Gamma|^\sigma$ .
  - (c)  $|a : \text{ChanRef}(A), \Gamma|^\sigma = a_A : \text{ChanRef}(\text{cct}(\sigma)), |\Gamma|^\sigma$ .
2. For  $\Delta$ :
  - (a)  $|\emptyset|^\sigma = \emptyset$
  - (b)  $|a : A, \Delta|^\sigma = a_A : \text{cct}(\sigma), |\Delta|^\sigma$

Figure 19 describes the coalescing pass from  $\lambda_{\text{ch}}$  with multiple channel types into  $\lambda_{\text{ch}}$  with a single channel type. Judgements are of the shape  $\{\sigma\} \Gamma \vdash V : A \rightsquigarrow V'$  for values;  $\{\sigma\} \Gamma \vdash M : A \rightsquigarrow M'$  for computations; and  $\{\sigma\} \Gamma \vdash \mathcal{C} \rightsquigarrow \mathcal{C}'$  for configurations, where  $\sigma$  is a bijective mapping from types to tokens, and primed values are the results of the coalescing pass. We omit the rules for values and functional terms, which are homomorphisms.

Of particular note are the rules for **give** and **take**. The coalesced version of **give** ensures that the correct token is used to inject into the variant type. The translation of **take** retrieves a value from the channel, and pattern matches to retrieve a value of the correct type from the variant. As we have less type information, we have to account for the possibility that pattern matching fails by introducing an error term, which we define at the top-level of the term:

$$\text{let error} = (\text{rec } f(x). f x) \text{ in } \dots$$

The translation on configurations ensures that all existing values contained within a buffer are wrapped in the appropriate variant injection.

The coalescing step necessarily loses typing information on channel types. To aid us in stating an error-freedom result, we annotate channel names  $a$  with their original type; for example, a channel with name  $a$  carrying values of type  $A$  would be translated as  $a_A$ . It is important to note that annotations are irrelevant to reduction, i.e.:

Coalescing of channel names	$\{\sigma\} \Gamma \vdash V \rightsquigarrow V'$
$\frac{\text{REF} \quad a : \text{ChanRef}(A) \in \Gamma}{\{\sigma\} \Gamma \vdash a : \text{ChanRef}(A) \rightsquigarrow a_A}$	
Coalescing of communication and concurrency primitives	$\{\sigma\} \Gamma \vdash M \rightsquigarrow M'$
$\frac{\text{GIVE} \quad \begin{array}{l} \{\sigma\} \Gamma \vdash V : A \rightsquigarrow V' \\ \{\sigma\} \Gamma \vdash W : \text{ChanRef}(A) \rightsquigarrow W' \quad \sigma(A) = \ell \end{array}}{\{\sigma\} \Gamma \vdash \text{give } V W : \mathbf{1} \rightsquigarrow \text{give } \langle \ell = V' \rangle W'}$	
$\frac{\text{TAKE} \quad \begin{array}{l} \{\sigma\} \Gamma \vdash V : \text{ChanRef}(A) \rightsquigarrow V' \quad \sigma(A) = \ell_j \\ \{\sigma\} \Gamma \vdash \text{take } V : A \rightsquigarrow \text{let } x \leftarrow \text{take } V' \text{ in} \\ \quad \text{case } x \{ \\ \quad \langle \ell_0 = y \rangle \dots \langle \ell_{j-1} = y \rangle \mapsto \text{error} \\ \quad \langle \ell_j = y \rangle \mapsto y \\ \quad \langle \ell_{j+1} = y \rangle \dots \langle \ell_n = y \rangle \mapsto \text{error} \} \end{array}}{\{\sigma\} \Gamma \vdash \text{take } V : A \rightsquigarrow \text{let } x \leftarrow \text{take } V' \text{ in} \\ \quad \text{case } x \{ \\ \quad \langle \ell_0 = y \rangle \dots \langle \ell_{j-1} = y \rangle \mapsto \text{error} \\ \quad \langle \ell_j = y \rangle \mapsto y \\ \quad \langle \ell_{j+1} = y \rangle \dots \langle \ell_n = y \rangle \mapsto \text{error} \}}$	
$\frac{\text{NEWCH}}{\{\sigma\} \Gamma \vdash \text{newCh} : \text{ChanRef}(A) \rightsquigarrow \text{newCh}_A} \quad \frac{\text{FORK}}{\{\sigma\} \Gamma \vdash \text{fork } M : \mathbf{1} \rightsquigarrow \text{fork } M'}$	
Coalescing of configurations	$\{\sigma\} \Gamma \vdash \mathcal{C} \rightsquigarrow \mathcal{C}'$
$\frac{\text{PAR} \quad \begin{array}{l} \{\sigma\} \Gamma; \Delta \vdash \mathcal{C}_1 \rightsquigarrow \mathcal{C}'_1 \quad \{\sigma\} \Gamma; \Delta \vdash \mathcal{C}_2 \rightsquigarrow \mathcal{C}'_2 \\ \{\sigma\} \Gamma; \Delta \vdash \mathcal{C}_1 \parallel \mathcal{C}_2 \rightsquigarrow \mathcal{C}'_1 \parallel \mathcal{C}'_2 \end{array}}{\{\sigma\} \Gamma; \Delta \vdash \mathcal{C}_1 \parallel \mathcal{C}_2 \rightsquigarrow \mathcal{C}'_1 \parallel \mathcal{C}'_2}$	
$\frac{\text{CHAN} \quad \begin{array}{l} \{\sigma\} \Gamma, a : \text{ChanRef}(A); \Delta, a : A \vdash \mathcal{C} \rightsquigarrow \mathcal{C}' \\ \{\sigma\} \Gamma; \Delta \vdash (\nu a) \mathcal{C} \rightsquigarrow (\nu a_A) \mathcal{C}' \end{array}}{\{\sigma\} \Gamma, a : \text{ChanRef}(A); \Delta, a : A \vdash \mathcal{C} \rightsquigarrow \mathcal{C}'} \quad \frac{\text{TERM} \quad \begin{array}{l} \{\sigma\} \Gamma \vdash M : A \rightsquigarrow M' \\ \{\sigma\} \Gamma; \cdot \vdash M \rightsquigarrow M' \end{array}}{\{\sigma\} \Gamma; \cdot \vdash M \rightsquigarrow M'}$	
$\frac{\text{BUF} \quad \begin{array}{l} (\{A, \sigma\} \Gamma \vdash V_i : A \rightsquigarrow V'_i)_i \\ \{\sigma\} \Gamma; a : A \vdash a(\vec{V}) \rightsquigarrow a_A(\vec{V}'_i) \end{array}}{\{\sigma\} \Gamma; a : A \vdash a(\vec{V}) \rightsquigarrow a_A(\vec{V}'_i)}$	
Coalescing of buffer values	$\{A, \sigma\} \Gamma \vdash V : A \rightsquigarrow V'$
$\frac{\begin{array}{l} \{\sigma\} \Gamma \vdash V : A \rightsquigarrow V' \quad \sigma(A) = \ell \\ \{A, \sigma\} \Gamma \vdash V : A \rightsquigarrow \langle \ell = V \rangle \end{array}}{\{A, \sigma\} \Gamma \vdash V : A \rightsquigarrow \langle \ell = V \rangle}$	

Fig. 19: Type-directed coalescing pass

$$E[\text{give } a_A W] \parallel a_B(\vec{V}) \longrightarrow E[\text{return } ()] \parallel a_B(\vec{V} \cdot W)$$

As previously discussed, the coalescing pass means that channel types are less specific, with the pass introducing partiality in the form of an error term, **error**. However, since we began with a type-safe program in  $\lambda_{\text{ch}}$ , we can show that programs that have been coalesced from well-typed  $\lambda_{\text{ch}}$  configurations never reduce to an error term.

**Definition 33 (Error configuration).**

A configuration  $\mathcal{C}$  is an error configuration if  $\mathcal{C} \equiv G[\text{error}]$  for some configuration context  $G$ .

**Definition 34 (Error-free configuration).**

A configuration  $\mathcal{C}$  is error-free if it is not an error configuration.

We can straightforwardly see that the initial result of a coalescing pass is error-free:

**Lemma 35.** *If  $\sigma \sim \Gamma \vdash \mathcal{C} \rightsquigarrow \mathcal{C}'$ , then  $\mathcal{C}'$  is error-free.*

*Proof.* By induction on the derivation of  $\Gamma \vdash \mathcal{C} \rightsquigarrow \mathcal{C}'$ . □

Next, we show that error-freeness is preserved under reduction. To do so, we make essential use of the fact that the coalescing pass annotates each channel with its original type.

**Proposition 36 (Error-freeness (coalesced  $\lambda_{\text{ch}}$ )).**

If  $\Gamma; \Delta \vdash \mathcal{C} \rightsquigarrow \mathcal{C}'_1$ , and  $\mathcal{C}'_1 \longrightarrow^* \mathcal{C}'_2$ , then  $\mathcal{C}'_2$  is error free.

*Proof.* By preservation in  $\lambda_{\text{ch}}$ , we have that  $\Gamma; \Psi \vdash \mathcal{C}'_2$ , and by Lemma 35, we can assume that  $\mathcal{C}'_1$  is error-free.

We show that an error term can never arise. Suppose that  $\mathcal{C}'_2$  was an error configuration, meaning that  $\mathcal{C}'_2 \equiv G[\text{error}]$  for some configuration context  $G$ . As we have decreed that the **error** term does not appear in user programs, we know that **error** must have arisen from the refinement pass. By observation of the refinement rules, we see that **error** is introduced only in the refinement rule for TAKE.

Stepping backwards through the reduction sequence introduced by the TAKE rule, we have that:

$$\text{case } \langle \ell_k = W \rangle \{ \\ \langle \ell_0 = y \rangle \dots \langle \ell_{j-1} = y \rangle \mapsto \text{error} \\ \langle \ell_j = y \rangle \mapsto y \\ \langle \ell_{j+1} = y \rangle \dots \langle \ell_n = y \rangle \mapsto \text{error} \\ \}$$

for some  $k \neq j$ .

Stepping back further, we have that:

```

let  $x \Leftarrow \text{take } a_B$  in
case  $x$  {
   $\langle \ell_0 = y \rangle \dots \langle \ell_{j-1} = y \rangle \mapsto \text{error}$ 
   $\langle \ell_j = y \rangle \mapsto y$ 
   $\langle \ell_{j+1} = y \rangle \dots \langle \ell_n = y \rangle \mapsto \text{error}$ 
}

```

Now, inspecting the premises of the refinement rule for TAKE, we have that  $\Gamma; \Psi \vdash a_B : \text{ChanRef}(A) \rightsquigarrow V'$  and  $\sigma(A) = \ell_j$ . Examining the refinement rule for NAME, we have that  $\Gamma; \Psi \vdash a : \text{ChanRef}(A) \rightsquigarrow a_A$ , thus we have that  $B = A$ .

However, we have that  $\sigma(A) = \ell_j$  and  $\sigma(A) = \ell_k$  but we know that  $k \neq j$ , thus leading to a contradiction since  $\sigma$  is bijective.  $\square$

Since annotations are irrelevant to reduction, it follows that  $\mathcal{C}'$  has identical reduction behaviour with all annotations erased.



## B Selected full proofs

### B.1 $\lambda_{\text{ch}}$ Preservation

**Lemma 37 (Replacement).** *If  $\Gamma \vdash E[M] : A$ ,  $\Gamma \vdash M : B$ , and  $\Gamma \vdash N : B$ , then  $\Gamma \vdash E[N] : A$ .*

*Proof.* By induction on the structure of  $E$ . □

#### Theorem 4 (Preservation ( $\lambda_{\text{ch}}$ configurations))

*If  $\Gamma; \Delta \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then  $\Gamma; \Delta \vdash \mathcal{C}_2$ .*

*Proof.* By induction on the derivation of  $\mathcal{C} \longrightarrow \mathcal{C}'$ . We use Lemma 37 implicitly throughout.

##### Case GIVE

From the assumption  $\Gamma; \Delta \vdash E[\text{give } W a] \parallel a(\vec{V})$ , we have that  $\Gamma; \cdot \vdash E[\text{give } W a]$  and  $\Gamma; a : A \vdash a(\vec{V})$ . Consequently, we know that  $\Delta = a : A$ .

From this, we know that  $\Gamma \vdash \text{give } W a : \mathbf{1}$  and thus  $\Gamma \vdash W : A$  and  $\Gamma \vdash a : \text{ChanRef}(A)$ . We also have that  $\Gamma; a : A \vdash a(\vec{V})$ , thus  $\Gamma \vdash V_i : A$  for all  $V_i \in \vec{V}$ . By UNIT we can show  $\Gamma; \cdot \vdash E[\text{return } ()]$  and by BUF we can show  $\Gamma; a : A \vdash a(\vec{V} \cdot W)$ ; recomposing, we arrive at  $\Gamma; \Delta \vdash E[\text{return } ()] \parallel a(\vec{V} W)$  as required.

##### Case TAKE

From the assumption  $\Gamma; \Delta \vdash E[\text{take } a] \parallel a(W \cdot \vec{V})$ , we have that  $\Gamma; \cdot \vdash E[\text{take } a]$  and that  $\Gamma; a : A \vdash a(W \cdot \vec{V})$ . Consequently, we know that  $\Delta = a : A$ .

From this, we know that  $\Gamma \vdash \text{take } a : A$ , and thus  $\Gamma \vdash a : \text{ChanRef}(A)$ . Similarly, we have that  $\Gamma; a : \text{ChanRef}(A) \vdash a(W \cdot \vec{V})$ , and thus  $\Gamma \vdash W : A$ .

Consequently, we can show that  $\Gamma; \cdot \vdash E[\text{return } W]$  and  $\Gamma; a : A \vdash a(\vec{V})$ ; recomposing, we arrive at  $\Gamma; \Delta \vdash E[\text{return } W] \parallel a(\vec{V})$  as required.

##### Case NEWCH

By BUF we can type  $\Gamma; a : A \vdash a(\epsilon)$ , and since  $\Gamma \vdash \text{newCh} : \text{ChanRef}(A)$ , it is also possible to show  $\Gamma, a : \text{ChanRef}(A) \vdash a : \text{ChanRef}(A)$ , thus  $\Gamma, a : \text{ChanRef}(A) \vdash E[\text{return } a]$ .

Recomposing by PAR we have  $\Gamma, a : \text{ChanRef}(A); a : A \vdash E[\text{return } a] \parallel a(\epsilon)$ , and by CHAN we have  $\Gamma; \cdot \vdash (\nu a)(E[\text{return } a] \parallel a(\epsilon))$  as required.

##### Case FORK

From the assumption  $\Gamma; \Delta \vdash E[\text{fork } M]$ , we have that  $\Delta = \emptyset$  and  $\Gamma \vdash M : \mathbf{1}$ . By UNIT we can show  $\Gamma; \cdot \vdash E[\text{return } ()]$ , and by TERM we can show  $\Gamma; \cdot \vdash M$ . Recomposing, we arrive at  $\Gamma; \Delta \vdash E[\text{return } ()] \parallel M$  as required.

##### Case LIFT

Immediate by the inductive hypothesis.

*Case* LIFTV

Immediate by Lemma 1.

□

## B.2 $\lambda_{\text{act}}$ Preservation

**Lemma 38 (Replacement).** *If  $\Gamma \mid C \vdash E[M] : A$ ,  $\Gamma \mid C \vdash M : B$ , and  $\Gamma \mid C \vdash N : B$ , then  $\Gamma \mid C \vdash E[N] : B$ .*

*Proof.* By induction on the structure of  $E$ . □

### Theorem 12 (Preservation ( $\lambda_{\text{act}}$ configurations))

*If  $\Gamma; \Delta \vdash C_1$  and  $C_1 \longrightarrow C_2$ , then  $\Gamma; \Delta \vdash C_2$ .*

*Proof.* By induction on the derivation of  $C_1 \longrightarrow C_2$ , making implicit use of Lemma 38.

#### Case SPAWN

From the assumption that  $\Gamma; \Delta \vdash \langle a, E[\text{spawn } M], \vec{V} \rangle$ , we have that  $\Delta = a : C$ , that  $\Gamma \mid C \vdash \text{spawn } M : \text{ActorRef}(A)$  and  $\Gamma \mid A \vdash M : \mathbf{1}$ .

We can show  $\Gamma, b : \text{ActorRef}(A) \vdash b : \text{ActorRef}(A)$ ; and therefore that  $\Gamma, b : \text{ActorRef}(A) \vdash E[\text{return } b] : \mathbf{1}$ . By ACTOR, it follows that  $\Gamma, b : \text{ActorRef}(A); a : C \vdash \langle a, E[\text{return } b], \vec{V} \rangle$ .

By ACTOR, we can show  $\Gamma, b : \text{ActorRef}(A); b : A \vdash \langle b, M, \epsilon \rangle$ .

Finally, by PID and PAR, we have  $\Gamma; \Delta \vdash (\nu b)(\langle a, E[\text{return } b], \vec{V} \rangle \parallel \langle b, M, \epsilon \rangle)$  as required.

#### Case SEND

From the assumption that  $\Gamma; \Delta \vdash \langle a, E[\text{send } V' b], \vec{V} \rangle \parallel \langle b, M, \vec{W} \rangle$ , we have that  $\Gamma; a : A \vdash \langle a, E[\text{send } V' b], \vec{V} \rangle$  and  $\Gamma; b : C \vdash \langle b, M, \vec{W} \rangle$ . Consequently, we can write  $\Delta = a : A, b : C$ . From this, we know that  $\Gamma \mid A \vdash \text{send } V' b : \mathbf{1}$ , so we can write  $\Gamma = \Gamma', b : \text{ActorRef}(C)$ , and  $\Gamma \vdash V' : C$ . Additionally, we know that  $\Gamma; b : C \vdash \langle b, M, \vec{W} \rangle$  and thus that  $(\Gamma \vdash W_i : C)$  for each entry  $W_i \in \vec{W}$ .

As  $\Gamma \vdash V' : C$ , it follows that  $\Gamma \vdash \vec{W} \cdot V'$  and therefore that  $\Gamma; b : C \vdash \langle b, M, \vec{W} \cdot V' \rangle$ . We can also show that  $\Gamma \mid C \vdash \text{return } () : \mathbf{1}$ , and therefore it follows that  $\Gamma; a : A \vdash \langle a, E[\text{return } ()], \vec{V} \rangle$ .

Recomposing, we have that  $\Gamma; \Delta \vdash \langle a, E[\text{return } ()], \vec{V} \rangle \parallel \langle b, M, \vec{W} \cdot V' \rangle$  as required.

#### Case RECEIVE

By ACTOR, we have that  $\Gamma; \Delta \vdash \langle a, E[\text{receive}], W \cdot \vec{V} \rangle$ . From this, we know that  $\Gamma \mid A \vdash E[\text{receive}] : \mathbf{1}$  (and thus  $\Gamma \mid A \vdash \text{receive} : A$ ) and  $\Gamma \vdash W : A$ .

Consequently, we can show  $\Gamma \vdash E[\text{return } W] : \mathbf{1}$ . By ACTOR, we arrive at  $\Gamma; \Delta \vdash \langle a, E[\text{return } W], \vec{V} \rangle$  as required.

#### Case SELF

By ACTOR, we have that  $\Gamma; \Delta \vdash \langle a, E[\text{self}], \vec{V} \rangle$ , and thus that  $\Gamma \mid A \vdash E[\text{self}] : \mathbf{1}$  and  $\Gamma \mid A \vdash \text{self} : \text{ActorRef}(A)$ . We also know that  $\Gamma = \Gamma', a : \text{ActorRef}(A)$ , and that  $\Delta = a : A$ .

Trivially, we can show  $\Gamma', a : \text{ActorRef}(A) \vdash a : \text{ActorRef}(A)$ . Thus it follows that  $\Gamma', a : \text{ActorRef}(A) \vdash E[\text{return } a] : \mathbf{1}$  and thus it follows that  $\Gamma; \Delta \vdash \langle a, E[\text{return } a], \vec{V} \rangle$  as required.

*Case LIFT*

Immediate by the inductive hypothesis.

*Case LIFTV*

Immediate by Lemma 9. □

### B.3 Translation: $\lambda_{act}$ into $\lambda_{ch}$

#### Lemma 23

1. If  $\Gamma \vdash V : A$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket$  in  $\lambda_{ch}$ .
2. If  $\Gamma \mid B \vdash M : A$  in  $\lambda_{act}$ , then  $\llbracket \Gamma \rrbracket, \alpha : \mathbf{ChanRef}(\llbracket B \rrbracket) \vdash \llbracket M \rrbracket \alpha : \llbracket A \rrbracket$  in  $\lambda_{ch}$ .

*Proof.* By simultaneous induction on the derivations of  $\Gamma \vdash V : A$  and  $\Gamma \mid B \vdash M : A$ .

*Premise 1*

*Case  $\Gamma \vdash x : A$*

By the definition of  $\llbracket - \rrbracket$ , we have that  $\llbracket \alpha \rrbracket = \alpha$ . By the definition of  $\llbracket \Gamma \rrbracket$ , we have that  $\alpha : \llbracket A \rrbracket \in \llbracket \Gamma \rrbracket$ . Consequently, it follows that  $\llbracket \Gamma \rrbracket \vdash \alpha : \llbracket A \rrbracket$ .

*Case  $\Gamma \vdash \lambda x.M : A \rightarrow B$*

From the assumption that  $\Gamma \vdash \lambda x.M : A \rightarrow^C B$ , we have that  $\Gamma, x : A \mid C \vdash M : B$ . By the inductive hypothesis (premise 2), we have that  $\llbracket \Gamma \rrbracket, x : \llbracket A \rrbracket, ch : \mathbf{ChanRef}(\llbracket C \rrbracket) \vdash \llbracket M \rrbracket ch : \llbracket B \rrbracket$ .

By two applications of ABS, we have  $\llbracket \Gamma \rrbracket \vdash \lambda x.\lambda ch.\llbracket M \rrbracket ch : \llbracket A \rrbracket \rightarrow \mathbf{ChanRef}(\llbracket C \rrbracket) \rightarrow \llbracket B \rrbracket$  as required.

*Case  $\Gamma \vdash () : \mathbf{1}$*

Immediate.

*Case  $\Gamma \vdash (V, W) : (A \times B)$*

From the assumption that  $\Gamma \vdash (V, W) : (A, B)$  we have that  $\Gamma \vdash V : A$  and  $\Gamma \vdash W : B$ . By the inductive hypothesis (premise 1) and PAIR, we can show  $\llbracket \Gamma \rrbracket \vdash (\llbracket V \rrbracket, \llbracket W \rrbracket) : (\llbracket A \rrbracket \times \llbracket B \rrbracket)$  as required.

*Premise 2*

*Case  $\Gamma \mid C \vdash V W : B$*

From the assumption that  $\Gamma \mid C \vdash V W : B$ , we have that  $\Gamma \vdash V : A \rightarrow^C B$  and  $\Gamma \vdash W : B$ . By the inductive hypothesis (premise 1), we have that  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket \rightarrow \mathbf{ChanRef}(\llbracket C \rrbracket) \rightarrow \llbracket B \rrbracket$ , and  $\llbracket \Gamma \rrbracket \vdash \llbracket W \rrbracket : \llbracket B \rrbracket$ .

By extending the context  $\Gamma$  with a  $ch : \mathbf{ChanRef}(\llbracket C \rrbracket)$ , we can show that  $\llbracket \Gamma \rrbracket, ch : \mathbf{ChanRef}(\llbracket C \rrbracket) \vdash \llbracket V \rrbracket \llbracket W \rrbracket ch : \llbracket B \rrbracket$  as required.

Case  $\Gamma \mid C \vdash \text{let } x \Leftarrow M \text{ in } N : B$

From the assumption that  $\Gamma \mid C \vdash \text{let } x \Leftarrow M \text{ in } N : B$ , we have that  $\Gamma \mid C \vdash M : A$  and that  $\Gamma, x : A \mid C \vdash N : B$ .

By the inductive hypothesis (premise 2), we have that  $\llbracket \Gamma \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash \llbracket M \rrbracket ch : \llbracket A \rrbracket$  and  $\llbracket \Gamma \rrbracket, x : \llbracket A \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash \llbracket N \rrbracket ch : \llbracket B \rrbracket$ .

By EFFLET, it follows that  $\llbracket \Gamma \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash \text{let } x \Leftarrow \llbracket M \rrbracket ch \text{ in } \llbracket N \rrbracket ch : \llbracket B \rrbracket$  as required.

Case  $\Gamma \mid C \vdash \text{return } V : A$

From the assumption that  $\Gamma \mid C \vdash \text{return } V : A$ , we have that  $\Gamma \vdash V : A$ .

By the inductive hypothesis (premise 1), we have that  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket$ .

By weakening (as we do not use the mailbox channel), we can show that  $\llbracket \Gamma \rrbracket, y : \text{ChanRef}(\llbracket C \rrbracket) \vdash \text{return } \llbracket V \rrbracket : \llbracket A \rrbracket$  as required.

Case  $\Gamma \mid C \vdash \text{send } V W : \mathbf{1}$

From the assumption that  $\Gamma \mid C \vdash \text{send } V W : \mathbf{1}$ , we have that  $\Gamma \vdash V : A$  and  $\Gamma \vdash W : \text{ChanRef}(A)$ .

By the inductive hypothesis (premise 1) we have that  $\llbracket \Gamma \rrbracket \vdash \llbracket V \rrbracket : \llbracket A \rrbracket$  and  $\llbracket \Gamma \rrbracket \vdash \llbracket W \rrbracket : \text{ChanRef}(\llbracket A \rrbracket)$ .

By GIVE, we can show that  $\llbracket \Gamma \rrbracket \vdash \text{give } \llbracket V \rrbracket \llbracket W \rrbracket : \mathbf{1}$ , and by weakening we have that  $\llbracket \Gamma \rrbracket, y : \text{ChanRef}(\llbracket C \rrbracket) \vdash \text{give } \llbracket V \rrbracket \llbracket W \rrbracket : \mathbf{1}$  as required.

Case  $\Gamma \mid C \vdash \text{receive} : C$

Given a  $ch : \text{ChanRef}(\llbracket C \rrbracket)$ , we can show that  $\llbracket \Gamma \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash ch : \text{ChanRef}(\llbracket C \rrbracket)$  and therefore that  $\llbracket \Gamma \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash \text{take } ch : \llbracket C \rrbracket$  as required.

Case  $\Gamma \mid C \vdash \text{spawn } M : \text{ActorRef}(A)$

From the assumption that  $\Gamma \mid C \vdash \text{spawn } M : \text{ActorRef}(A)$ , we have that  $\Gamma \mid A \vdash M : A$ .

By the inductive hypothesis (premise 2), we have that  $\llbracket \Gamma \rrbracket, chMb : \text{ChanRef}(\llbracket A \rrbracket) \vdash \llbracket M \rrbracket chMb : \llbracket A \rrbracket$ . By FORK and RETURN, we can show that  $\llbracket \Gamma \rrbracket, chMb : \text{ChanRef}(\llbracket A \rrbracket) \vdash \text{fork } (\llbracket M \rrbracket chMb); \text{return } chMb : \text{ChanRef}(\llbracket A \rrbracket)$ .

By NEWCH and EFFLET, we can show that  $\llbracket \Gamma \rrbracket \vdash \text{let } chMb \Leftarrow \text{newCh in fork } (\llbracket M \rrbracket chMb); \text{return } chMb : \text{ChanRef}(\llbracket A \rrbracket)$ .

Finally, by weakening, we have that

$$\llbracket \Gamma \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash \text{let } chMb \Leftarrow \text{newCh in (fork } \llbracket M \rrbracket chMb); \text{return } chMb : \text{ChanRef}(\llbracket A \rrbracket)$$

as required.

Case  $\Gamma \mid C \vdash \text{self} : \text{ActorRef}(C)$

Given a  $ch : \text{ChanRef}(\llbracket C \rrbracket)$ , we can show that  $\llbracket \Gamma \rrbracket, ch : \text{ChanRef}(\llbracket C \rrbracket) \vdash \text{return } ch : \text{ChanRef}(\llbracket C \rrbracket)$  as required.

□

**Theorem 18** *If  $\Gamma; \Delta \vdash \mathcal{C}$ , then  $\llbracket \Gamma \rrbracket; \llbracket \Delta \rrbracket \vdash \llbracket \mathcal{C} \rrbracket$ .*

*Proof.* By induction on the derivation of  $\Gamma \vdash \mathcal{C}$ .

*Case Par*

From the assumption that  $\Gamma; \Delta \vdash \mathcal{C}_1 \parallel \mathcal{C}_2$ , we have that  $\Delta$  splits as  $\Delta_1, \Delta_2$  such that  $\Gamma; \Delta_1 \vdash \mathcal{C}_1$  and  $\Gamma; \Delta_2 \vdash \mathcal{C}_2$ . By the inductive hypothesis, we have that  $\llbracket \Gamma \rrbracket; \llbracket \Delta_1 \rrbracket \vdash \llbracket \mathcal{C}_1 \rrbracket$  and  $\llbracket \Gamma \rrbracket; \llbracket \Delta_2 \rrbracket \vdash \llbracket \mathcal{C}_2 \rrbracket$ . Recomposing by PAR, we have that  $\llbracket \Gamma \rrbracket; \llbracket \Delta_1 \rrbracket, \llbracket \Delta_2 \rrbracket \vdash \llbracket \mathcal{C}_1 \rrbracket \parallel \llbracket \mathcal{C}_2 \rrbracket$  as required.

*Case Pid*

From the assumption that  $\Gamma; \Delta \vdash (\nu a)\mathcal{C}$ , we have that  $\Gamma, a : \text{ActorRef}(A); \Delta, a : A \vdash \mathcal{C}$ . By the inductive hypothesis, we have that  $\llbracket \Gamma \rrbracket, a : \text{ChanRef}(\llbracket A \rrbracket); \llbracket \Delta \rrbracket, a : \llbracket A \rrbracket \vdash \llbracket \mathcal{C} \rrbracket$ . Recomposing by PID, we have that  $\llbracket \Gamma \rrbracket \vdash (\nu a)\llbracket \mathcal{C} \rrbracket$  as required.

*Case Actor*

From the assumption that  $\Gamma, a : \text{ActorRef}(A); a : A \vdash \langle a, M, \vec{V} \rangle$ , we have that  $\Gamma, a : \text{ActorRef}(A) \mid A \vdash M : \mathbf{1}$ . By Lemma 17, we have that  $\llbracket \Gamma \rrbracket, a : \text{ChanRef}(\llbracket A \rrbracket) \vdash \llbracket M \rrbracket a : \mathbf{1}$ . It follows straightforwardly that  $\llbracket \Gamma \rrbracket, a : \text{ChanRef}(\llbracket A \rrbracket); \cdot \vdash \llbracket M \rrbracket a$ .

We can also show that  $\llbracket \Gamma \rrbracket, a : \text{ChanRef}(\llbracket A \rrbracket); a : \llbracket A \rrbracket \vdash a(\llbracket \vec{V} \rrbracket)$  (where  $\llbracket \vec{V} \rrbracket = \llbracket V_1 \rrbracket \cdot \dots \cdot \llbracket V_n \rrbracket$ ), by repeated applications of Lemma 17.

By TERM and PAR, we have that  $\llbracket \Gamma \rrbracket, a : \text{ChanRef}(\llbracket A \rrbracket); a : \llbracket A \rrbracket \vdash a(\llbracket \vec{V} \rrbracket) \parallel \llbracket M \rrbracket a$  as required.  $\square$

**Theorem 21**

*If  $\Gamma \vdash \mathcal{C}_1$  and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then there exists some  $\mathcal{D}$  such that  $\llbracket \mathcal{C}_1 \rrbracket \longrightarrow^* \mathcal{D}$ , with  $\mathcal{D} \equiv \llbracket \mathcal{C}_2 \rrbracket$ .*

*Proof.* By induction on the derivation of  $\mathcal{C} \longrightarrow \mathcal{C}'$ .

**Case Spawn**

$$\begin{aligned}
 \text{Assumption} & \quad \llbracket \langle a, E[\text{spawn } M], \vec{V} \rangle \rrbracket & (1) \\
 \text{Definition of } \llbracket - \rrbracket & \quad a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket[\text{let } c \leftarrow \text{newCh in fork}(\llbracket M \rrbracket c); \text{return } c] a) & (2) \\
 \text{newCh reduction} & \quad a(\llbracket \vec{V} \rrbracket) \parallel (\nu b)((\llbracket E \rrbracket[\text{let } c \leftarrow \text{return } b \text{ in fork}(\llbracket M \rrbracket c); \text{return } c] a) \parallel b(\epsilon)) & (3) \\
 \text{Let reduction} & \quad a(\llbracket \vec{V} \rrbracket) \parallel (\nu b)((\llbracket E \rrbracket[\text{fork}(\llbracket M \rrbracket b); \text{return } b] a) \parallel b(\epsilon)) & (4) \\
 \text{Fork reduction} & \quad a(\llbracket \vec{V} \rrbracket) \parallel (\nu b)((\llbracket E \rrbracket[\text{return } (); \text{return } b] a) \parallel (\llbracket M \rrbracket b) \parallel b(\epsilon)) & (5) \\
 \text{Let reduction} & \quad a(\llbracket \vec{V} \rrbracket) \parallel (\nu b)((\llbracket E \rrbracket[\text{return } b] a) \parallel (\llbracket M \rrbracket b) \parallel b(\epsilon)) & (6) \\
 \equiv & \quad (\nu b)(a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket[\text{return } b] a) \parallel b(\epsilon) \parallel (\llbracket M \rrbracket b)) & (7) \\
 = & \quad \llbracket (\nu b)(\langle a, E[\text{return } b], \vec{V} \rangle \parallel \langle b, M, \epsilon \rangle) \rrbracket & (8)
 \end{aligned}$$

**Case Self**

$$\begin{aligned}
&\text{Assumption} && \llbracket \langle a, E[\text{self}], \vec{V} \rangle \rrbracket && (1) \\
&\text{Definition of } \llbracket - \rrbracket && a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket [\text{return } a] a) && (2) \\
&= && \llbracket \langle a, E[\text{return } a], \vec{V} \rangle \rrbracket &&
\end{aligned}$$

**Case Send**

$$\begin{aligned}
&\text{Assumption} && \llbracket \langle a, E[\text{send } V' b], \vec{V} \rangle \parallel \langle b, M, \vec{W} \rangle \rrbracket && (1) \\
&\text{Definition of } \llbracket - \rrbracket && a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket [\text{give } \llbracket V' \rrbracket b] a) \parallel b(\llbracket \vec{W} \rrbracket) \parallel (\llbracket M \rrbracket b) && (2) \\
&\text{Give reduction} && a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket [\text{return } ()] a) \parallel b(\llbracket \vec{W} \rrbracket \cdot \llbracket V' \rrbracket) \parallel (\llbracket M \rrbracket b) && (3) \\
&= && \llbracket \langle a, E[\text{return } ()], \vec{V} \rangle \parallel \langle b, M, \vec{W} \cdot V' \rangle \rrbracket && (4)
\end{aligned}$$

**Case Receive**

$$\begin{aligned}
&\text{Assumption} && \llbracket \langle a, E[\text{receive}], W \cdot \vec{V} \rangle \rrbracket && (1) \\
&\text{Definition of } \llbracket - \rrbracket && a(\llbracket W \rrbracket \cdot \llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket [\text{take } a] a) && (2) \\
&\text{Take reduction} && a(\llbracket \vec{V} \rrbracket) \parallel (\llbracket E \rrbracket [\text{return } \llbracket W \rrbracket] a) && (3) \\
&= && \llbracket \langle a, E[\text{return } W], \vec{V} \rangle \rrbracket && (4)
\end{aligned}$$

Lift is immediate from the inductive hypothesis, and LiftV is immediate from Lemma 19.  $\square$



#### B.4 Translation: $\lambda_{\text{ch}}$ into $\lambda_{\text{act}}$

##### Lemma 23

1. If  $\{B\} \Gamma \vdash V : A$ , then  $\langle \Gamma \rangle B \vdash \langle V \rangle : \langle A \rangle B$ .
2. If  $\{B\} \Gamma \vdash M : A$ , then  $\langle \Gamma \rangle B \mid \langle B \rangle B \vdash \langle M \rangle : \langle A \rangle B$ .

*Proof.* By simultaneous induction on the derivations of  $\{B\} \Gamma \vdash V : A$  and  $\{B\} \Gamma \vdash M : A$ .

*Premise 1*

*Case VAR*

From the assumption that  $\{B\} \Gamma \vdash \alpha : A$ , we know that  $\alpha : A \in \Gamma$ . By the definition of  $\langle \Gamma \rangle B$ , we have that  $\alpha : \langle A \rangle B \in \langle \Gamma \rangle B$ . Since  $\langle \alpha \rangle = \alpha$ , it follows that  $\langle \Gamma \rangle B \vdash \alpha : \langle A \rangle B$  as required.

*Case ABS*

From the assumption that  $\{C\} \Gamma \vdash \lambda x.M : A \rightarrow B$ , we have that  $\{C\} \Gamma, x : A \vdash M : B$ . By the inductive hypothesis (Premise 2), we have that  $\langle \Gamma \rangle C, x : \langle A \rangle C \mid \langle C \rangle C \vdash \langle M \rangle : \langle A \rangle C$ . By ABS, we can show that  $\langle \Gamma \rangle C \mid \langle C \rangle C \vdash \lambda x.\langle M \rangle : \langle A \rangle C \rightarrow^{\langle C \rangle C} \langle B \rangle C$  as required.

*Case REC*

Similar to ABS.

*Case UNIT*

Immediate.

*Case PAIR*

From the assumption that  $\{C\} \Gamma \vdash (V, W) : A \times B$ , we have that  $\{C\} \Gamma \vdash V : A$  and that  $\{C\} \Gamma \vdash W : B$ .

By the inductive hypothesis (premise 1), we have that  $\langle \Gamma \rangle C \vdash \langle V \rangle : \langle A \rangle C$  and that  $\langle \Gamma \rangle C \vdash \langle W \rangle : \langle B \rangle C$ .

It follows by PAIR that  $\langle \Gamma \rangle C \vdash (\langle V \rangle, \langle W \rangle) : (\langle A \rangle C \times \langle B \rangle C)$  as required.

*Cases INL, INR, and LISTCONS*

Similar to PAIR.

*Case EMPTYLIST*

Immediate.

*Premise 2*

*Case APP*

From the assumption that  $\{C\} \Gamma \vdash VW : B$ , we have that  $\{C\} \Gamma \vdash V : A \rightarrow B$  and  $\{C\} \Gamma \vdash W : A$ . By the inductive hypothesis (premise 1), we have that  $\langle \Gamma \rangle C \vdash \langle V \rangle : \langle A \rangle C \rightarrow^{\langle C \rangle C} \langle B \rangle C$  and that  $\langle \Gamma \rangle C \vdash \langle W \rangle : \langle A \rangle C$ .

By APP, it follows that  $\langle \Gamma \rangle C \mid \langle C \rangle C \vdash \langle V \rangle \langle W \rangle : \langle B \rangle C$  as required.

*Case RETURN*

From the assumption that  $\{C\} \Gamma \vdash \text{return } V : A$ , we have that  $\{C\} \Gamma \vdash V : A$ . By the inductive hypothesis we have that  $\langle \Gamma \rangle C \vdash \langle V \rangle : \langle A \rangle C$  and thus by RETURN we can show that  $\langle \Gamma \rangle C \mid \langle C \rangle C \vdash \text{return } \langle V \rangle : \langle A \rangle C$  as required.

*Case EFFLET*

From the assumption that  $\{C\} \Gamma \vdash \text{let } x \leftarrow M \text{ in } N : B$ , we have that  $\{C\} \Gamma \vdash M : A$  and  $\{C\} \Gamma, x : A \vdash N : B$ .

By the inductive hypothesis (premise 2), we have that  $\langle \Gamma \rangle C \mid \langle C \rangle C \vdash \langle M \rangle : \langle A \rangle C$  and  $\langle \Gamma \rangle C, x : \langle A \rangle C \mid \langle C \rangle C \vdash \langle N \rangle : \langle B \rangle C$ . Thus by EFFLET it follows that  $\langle \Gamma \rangle C \mid \langle C \rangle C \vdash \text{let } x \leftarrow \langle M \rangle \text{ in } \langle N \rangle : \langle B \rangle C$ .

*Case LETPAIR*

Similar to EFFLET.

*Case CASE*

From the assumption that  $\{C\} \Gamma \vdash \text{case } V \{ \text{inl } x \mapsto M; \text{inr } y \mapsto N \} : B$ , we have that  $\{C\} \Gamma \vdash V : A + A'$ , that  $\{C\} \Gamma, x : A \vdash M : B$ , and that  $\{C\} \Gamma, y : A' \vdash N : B$ .

By the inductive hypothesis (premise 1) we have that  $\langle \Gamma \rangle C \vdash \langle V \rangle : \langle A \rangle C + \langle A' \rangle C$ , and by premise 2 we have that  $\langle \Gamma \rangle C, x : \langle A \rangle C \mid \langle C \rangle C \vdash \langle M \rangle : \langle B \rangle C$  and  $\langle \Gamma \rangle C, y : \langle A' \rangle C \mid \langle C \rangle C \vdash \langle N \rangle : \langle B \rangle C$ .

By LISTCASE, it follows that  $\langle \Gamma \rangle C \mid \langle C \rangle C \vdash \text{case}(\langle V \rangle) \{ \text{inl } x \mapsto \langle M \rangle; \text{inr } y \mapsto \langle N \rangle \} : \langle B \rangle C$ .

*Case LISTCASE*

Similar to CASE.

*Case FORK*

From the assumption that  $\{A\} \Gamma \vdash \text{fork } M : \mathbf{1}$ , we have that  $\{A\} \Gamma \vdash M : \mathbf{1}$ . By the inductive hypothesis, we have that  $\langle \Gamma \rangle C \mid \langle A \rangle A \vdash \langle M \rangle : \mathbf{1}$ .

We can show that  $\langle \Gamma \rangle A \mid \langle A \rangle A \vdash \text{spawn } \langle M \rangle : \text{ActorRef}(\mathbf{1})$  and also show that  $\langle \Gamma \rangle A, x : \text{ActorRef}(\mathbf{1}) \vdash \text{return } () : \mathbf{1}$ .

Thus by EFFLET, it follows that  $\langle \Gamma \rangle A \mid \langle A \rangle A \vdash \text{let } x \leftarrow \text{spawn } \langle M \rangle \text{ in return } () : \mathbf{1}$  as required.

*Case GIVE*

From the assumption that  $\{A\} \Gamma \vdash \text{give } VW : \mathbf{1}$ , we have that  $\{A\} \Gamma \vdash V : A$  and  $\{A\} \Gamma \vdash W : \text{Chan}$ . By the inductive hypothesis, we have that  $\langle \Gamma \rangle A \vdash \langle V \rangle : \langle A \rangle A$  and  $\langle \Gamma \rangle A \vdash \langle W \rangle : \text{ActorRef}(\langle A \rangle A + \text{ActorRef}(\langle A \rangle A))$ . We can show that  $\langle \Gamma \rangle A \vdash \text{inl } \langle V \rangle : \langle A \rangle A + \text{ActorRef}(\langle A \rangle A)$ , and thus it follows that  $\langle \Gamma \rangle A \mid \langle A \rangle A \vdash \text{send inl } V \langle W \rangle : \mathbf{1}$  as required.

*Case TAKE*

From the assumption that  $\{A\} \Gamma \vdash \text{take } V : A$ , we have that  $\{A\} \Gamma \vdash V : \text{Chan}$ .

By the inductive hypothesis (premise 1), we have that  $(\Gamma) A \vdash (V) : \text{ActorRef}((A) A + \text{ActorRef}((A) A))$ .

We can show that:

- $(\Gamma) A \mid (A) A \vdash \text{self} : \text{ActorRef}((A) A)$
- $(\Gamma) A \mid (A) A \vdash \text{inr self} : (A) + \text{ActorRef}((A) A)$
- $(\Gamma) A, \text{selfPid} : \text{ActorRef}((A) A) \mid (A) A \vdash \text{send inr selfPid } (V) : \mathbf{1}$
- $(\Gamma) A, \text{selfPid} : \text{ActorRef}((A) A), z : \mathbf{1} \mid (A) A \vdash \text{receive} : (A) A$

Thus by two applications of EFFLET (noting that we desugar  $M; N$  into  $\text{let } z \leftarrow M \text{ in } N$ , where  $z$  is fresh), we arrive at:

$$(\Gamma) A \mid (A) A \vdash \text{let selfPid} \leftarrow \text{self in send (inr selfPid) } (V); \text{receive} : (A) A$$

as required.

*Case NEWCH*

We have that  $\{A\} \Gamma; \Delta \vdash \text{newCh} : \text{Chan}$ .

Our goal is to show that  $(\Gamma) A \mid (A) A \vdash \text{spawn body } ([ ], [ ]) : \text{ActorRef}((A) A + \text{ActorRef}((A) A))$ . To do so amounts to showing that  $(\Gamma) A \mid (A) A + \text{ActorRef}((A) A) \vdash \text{body } ([ ], [ ]) : \mathbf{1}$ .

We sketch the proof as follows. Firstly, by the typing of `receive`, `recvVal` must have type  $(A) A + \text{ActorRef}((A) A)$ . By inspection of both case branches and LETPAIR, we have that `state` must have type  $\text{List}((A) A) \times \text{List}(\text{ActorRef}((A) A))$ .

We expect `drain` to have type

$$\text{List}((A) A) \times \text{List}(\text{ActorRef}((A) A)) \rightarrow^{(A) A} \text{List}((A) A) \times \text{List}(\text{ActorRef}((A) A))$$

since we use the returned value as a recursive call to  $g$ , which must have the same type of `state`. Inspecting the case split in `drain`, we have that the empty list case for values returns the input state, which of course has the same type. The same can be said for the empty list case of the `readers` case split.

In the case where both the values and readers lists are non-empty, we have that  $v$  has type  $(A) A$  and  $pid$  has type  $\text{ActorRef}((A) A)$ . Additionally, we have that  $vs$  has type  $\text{List}((A) A)$  and  $pids$  has type  $\text{List}(\text{ActorRef}((A) A))$ . We can show via `send` that `send v pid` has type  $\mathbf{1}$ . After desugaring  $M; N$  into an EFFLET, we can show that a recursive call to  $f$  is well-typed. □

**Theorem 24** *If  $\{A\} \Gamma; \Delta \vdash C$  with  $\Gamma \asymp \Delta$ , then  $(\Gamma) A; (\Delta) A \vdash (C)$ .*

*Proof.* By induction on the derivation of  $\{A\} \Gamma; \Delta \vdash C$ .

*Case PAR*

From the assumption that  $\{A\} \Gamma; \Delta \vdash \mathcal{C}_1 \parallel \mathcal{C}_2$ , we have that  $\Delta$  splits as  $\Delta_1, \Delta_2$  such that  $\{A\} \Gamma; \Delta_1 \vdash \mathcal{C}_1$  and  $\{A\} \Gamma; \Delta_2 \vdash \mathcal{C}_2$ .

By the inductive hypothesis, we have that  $\langle \Gamma \rangle C; \langle \Delta_1 \rangle C \vdash \langle \mathcal{C}_1 \rangle$  and  $\langle \Gamma \rangle C; \langle \Delta_2 \rangle C \vdash \langle \mathcal{C}_2 \rangle$ . By the definition of  $\langle - \rangle C$  on linear configuration environments, it follows that  $\langle \Delta_1 \rangle C, \langle \Delta_2 \rangle C = \langle \Delta \rangle C$ . Consequently, by PAR, we can show that  $\langle \Gamma \rangle C; \langle \Delta \rangle C \vdash \langle C \rangle_1 \parallel \langle C \rangle_2$  as required.

*Case CHAN*

From the assumption that  $\{A\} \Gamma; \Delta \vdash (\nu a)\mathcal{C}$ , we have that  $\{A\} \Gamma, a : \text{ChanRef}(A); \Delta, a : A \vdash \mathcal{C}$ . By the inductive hypothesis, we have that  $\langle \Gamma \rangle A, a : \text{ActorRef}(\langle A \rangle A + \text{ActorRef}(\langle A \rangle A)); \langle \Delta \rangle A, a : \langle A \rangle A + \text{ActorRef}(\langle A \rangle A) \vdash \langle C \rangle$ .

By PID, it follows that  $\langle \Gamma \rangle C; \langle \Delta \rangle A \vdash (\nu a)\langle C \rangle$  as required.

*Case TERM*

From the assumption that  $\{A\} \Gamma; \cdot \vdash M$ , we have that  $\{A\} \Gamma \vdash M : \mathbf{1}$ .

By Lemma 23, we have that  $\langle \Gamma \rangle A \mid \langle A \rangle A \vdash \langle M \rangle : \mathbf{1}$ . By weakening, we can show that  $\langle \Gamma \rangle A, a : \text{ActorRef}(\langle A \rangle A) \mid \langle A \rangle A \vdash \langle M \rangle : \mathbf{1}$ .

It follows that, by ACTOR, we can construct a configuration of the form  $\langle \Gamma \rangle A, a : \text{ActorRef}(\langle A \rangle A); a : \langle A \rangle A \vdash \langle a, \langle M \rangle, \epsilon \rangle$ , and by PID, we arrive at

$$\langle \Gamma \rangle A; \cdot \vdash (\nu a)(\langle a, \langle M \rangle, \epsilon \rangle)$$

as required.

*Case BUF*

We assume that  $\{A\} \Gamma; a : A \vdash a(\vec{V})$ , and since  $\Gamma \simeq \Delta$ , we have that we can write  $\Gamma = \Gamma', a : \text{ChanRef}(A)$ .

From the assumption that  $\{A\} \Gamma', a : \text{ChanRef}(A); a : A \vdash a(\vec{V})$ , we have that  $\{A\} \Gamma', a : \text{ChanRef}(A) \vdash V_i : A$  for each  $V_i \in \vec{V}$ .

By repeated application of Lemma 23, we have that  $\{A\} \langle \Gamma \rangle A, a : \text{ActorRef}(\langle A \rangle A + \text{ActorRef}(\langle A \rangle A)) \vdash \langle V_i \rangle : \langle A \rangle A$  for each  $V_i \in \vec{V}$ , and by LISTCONS and EMPTYLIST we can construct a list  $\langle V_0 \rangle :: \dots :: \langle V_n \rangle :: [ ]$  with type  $\text{List}(\langle A \rangle A)$ .

Relying on our previous analysis of the typing of `body`, we have that  $\langle \text{body} \rangle$  has type:

$$(\text{List}(\langle A \rangle A) \times \text{List}(\text{ActorRef}(\langle A \rangle A))) \rightarrow \langle A \rangle A + \text{ActorRef}(\langle A \rangle A) \quad \mathbf{1}$$

Thus it follows that

$$\langle \Gamma' \rangle B, a : \text{ActorRef}(\langle A \rangle A + \text{ActorRef}(\langle A \rangle A)) \mid \langle A \rangle A + \text{ActorRef}(\langle A \rangle A) \vdash \text{body}(\langle \vec{V} \rangle, [ ]) : \mathbf{1}$$

By ACTOR, we can see that

$$\langle \Gamma' \rangle A, a : \text{ActorRef}(\langle A \rangle A + \text{ActorRef}(\langle A \rangle A)); a : \langle A \rangle A + \text{ActorRef}(\langle A \rangle A) \vdash \\ \langle a, \text{body}(\langle \vec{V} \rangle), [ ] \rangle, \epsilon \rangle$$

as required. □

**Theorem 27**

If  $\{A\} \Gamma; \Delta \vdash \mathcal{C}_1$ , and  $\mathcal{C}_1 \longrightarrow \mathcal{C}_2$ , then there exists some  $\mathcal{D}$  such that  $\langle \mathcal{C}_1 \rangle \longrightarrow^* \mathcal{D}$  with  $\mathcal{D} \equiv \langle \mathcal{C}_2 \rangle$ .

*Proof.*

By induction on the derivation of  $\Gamma; \Delta \vdash \mathcal{C}_1$ .

Case GIVE

$$\begin{aligned} & \text{Assumption } E[\text{give } W \ a] \parallel a(\vec{V}) \\ \text{Definition of } \langle - \rangle & (\nu b) \langle \langle b, \langle E \rangle [\text{send}(\text{inl} \langle W \rangle) \ a], \epsilon \rangle \parallel \langle a, \text{body}(\langle \vec{V} \rangle), [ ] \rangle, \epsilon \rangle \\ & \equiv (\nu b) \langle \langle b, \langle E \rangle [\text{send}(\text{inl} \langle W \rangle) \ a], \epsilon \rangle \parallel \langle a, \text{body}(\langle \vec{V} \rangle), [ ] \rangle, \epsilon \rangle \\ \longrightarrow & \text{(Send)} (\nu b) \langle \langle b, \langle E \rangle [\text{return} \ \langle \rangle], \epsilon \rangle \parallel \langle a, \text{body}(\langle \vec{V} \rangle), [ ] \rangle, \text{inl} \langle W \rangle \rangle \end{aligned}$$

Now, let  $G[-] = (\nu b) \langle \langle b, \langle E \rangle [\text{return} \ \langle \rangle], \epsilon \rangle \parallel [-] \rangle$ .

We now have

$$G[\langle a, \text{body}(\langle \vec{V} \rangle), [ ] \rangle, (\text{inl} \langle W \rangle)]$$

which we can expand to

$$\begin{aligned} G[\langle a, & (\text{rec } g(\text{state}) . \\ & \text{let } \text{recvVal} \leftarrow \text{receive in} \\ & \text{let } (\text{vals}, \text{readers}) = \text{state in} \\ & \text{case } \text{recvVal} \{ \\ & \quad \text{inl } v \mapsto \text{let } \text{newVals} \leftarrow \text{vals} ++ [v] \text{ in} \\ & \quad \quad \text{let } \text{state}' \leftarrow \text{drain}(\text{newVals}, \text{readers}) \text{ in} \\ & \quad \quad \text{body}(\text{state}') \\ & \quad \text{inr } \text{pid} \mapsto \text{let } \text{newReaders} \leftarrow \text{readers} ++ [\text{pid}] \text{ in} \\ & \quad \quad \text{let } \text{state}' \leftarrow \text{drain}(\text{vals}, \text{newReaders}) \text{ in} \\ & \quad \quad \text{body}(\text{state}') \rangle \rangle (\langle \vec{V} \rangle, [ ] \rangle) \end{aligned}$$

Applying the arguments to the recursive function  $f$ ; performing the **receive**, and the **let** and **case** reductions, we have:

$$G[\langle a, \text{let } newVals \Leftarrow \langle \vec{V} \rangle \text{ ++ } \langle \langle W \rangle \rangle \text{ in } \_, \epsilon \rangle \\ \text{let } state' \Leftarrow \text{drain}(newVals, [ ]) \text{ in} \\ \text{body}(state')]$$

Next, we reduce the append operation, and note that since we pass a state without pending readers into `drain`, that the argument is returned unchanged:

$$G[\langle a, \text{let } state' \Leftarrow \text{return}(\langle \vec{V} \rangle :: \langle W \rangle :: [ ]) \text{ in } \_, \epsilon \rangle \\ \text{body } state']$$

Next, we apply the let-reduction and expand the evaluation context:

$$(\nu b)(\langle b, \langle E \rangle[\text{return}()], \epsilon \rangle \parallel \langle a, \text{body}(\langle \vec{V} \rangle :: \langle W \rangle :: [ ]), \epsilon \rangle)$$

which is structurally congruent to

$$(\nu b)(\langle b, \langle E \rangle[\text{return}()], \epsilon \rangle \parallel \langle a, \text{body}(\langle \vec{V} \rangle :: \langle W \rangle :: [ ]), \epsilon \rangle)$$

which is equal to

$$\langle E[\text{return}()] \parallel a(\langle \vec{V} \cdot W \rangle) \rangle$$

as required.

*Case TAKE*

$$\begin{aligned} & \text{Assumption } E[\text{take } a] \parallel a(W \cdot \vec{V}) \\ \text{Definition of } \langle - \rangle & (\nu b)(\langle b, \langle E \rangle[\text{let } selfPid \Leftarrow \text{self in } \_, \epsilon] \parallel \langle b, \text{body}(\langle W \rangle :: \langle \vec{V} \rangle, [ ]), \epsilon \rangle \\ & \quad \text{send}(\text{inr } selfPid) a; \\ & \quad \text{receive}] \\ & \equiv (\nu b)(\langle b, \langle E \rangle[\text{let } selfPid \Leftarrow \text{self in } \_, \epsilon] \parallel \langle b, \text{body}(\langle W \rangle :: \langle \vec{V} \rangle, [ ]), \epsilon \rangle \\ & \quad \text{send}(\text{inr } selfPid) a; \\ & \quad \text{receive}] \\ & \longrightarrow (\text{Self}) (\nu b)(\langle b, \langle E \rangle[\text{let } selfPid \Leftarrow \text{return } b \text{ in } \_, \epsilon] \parallel \langle b, \text{body}(\langle W \rangle :: \langle \vec{V} \rangle, [ ]), \epsilon \rangle \\ & \quad \text{send}(\text{inr } selfPid) a; \\ & \quad \text{receive}] \\ & \longrightarrow_M (\text{Let}) (\nu b)(\langle b, \langle E \rangle[\text{send}(\text{inr } b) a; \_, \epsilon] \parallel \langle b, \text{body}(\langle W \rangle :: \langle \vec{V} \rangle, [ ]), \epsilon \rangle \\ & \quad \text{receive}] \\ & \longrightarrow (\text{Send}) (\nu b)(\langle b, \langle E \rangle[\text{return}(); \text{receive}], \epsilon \rangle \parallel \langle b, \text{body}(\langle W \rangle :: \langle \vec{V} \rangle, [ ]), (\text{inr } b) \rangle) \\ & \longrightarrow_M (\text{Let}) (\nu b)(\langle b, \langle E \rangle[\text{receive}], \epsilon \rangle \parallel \langle b, \text{body}(\langle W \rangle :: \langle \vec{V} \rangle, [ ]), (\text{inr } b) \rangle) \end{aligned}$$

Now, let  $G[-] = (\nu b)(\langle b, \langle E \rangle[\text{receive}], \epsilon \rangle \parallel [-])$ .

Expanding, we begin with:

$$G[\langle a, (\text{rec } g(\text{state}) . \text{let } \text{recvVal} \leftarrow \text{receive in} \\ \text{let } (\text{vals}, \text{readers}) = \text{state in} \\ \text{case } \text{recvVal} \{ \\ \text{inl } v \mapsto \text{let } \text{newVals} \leftarrow \text{vals} ++ [v] \text{ in} \\ \text{let } \text{state}' \leftarrow \text{drain}(\text{newVals}, \text{readers}) \text{ in} \\ g(\text{state}') \\ \text{inr } \text{pid} \mapsto \text{let } \text{newReaders} \leftarrow \text{readers} ++ [\text{pid}] \text{ in} \\ \text{let } \text{state}' \leftarrow \text{drain}(\text{vals}, \text{newReaders}) \text{ in} \\ g(\text{state}') \} \rangle) \langle (W) :: (\vec{V}), [ ] \rangle \rangle, (\text{inr } b) \rangle]$$

Reducing the recursive function, receiving from the mailbox, splitting the pair, and then taking the second branch on the case statement, we have:

$$G[\langle a, \text{let } \text{newReaders} \leftarrow [ ] ++ [b] \text{ in} \\ \text{let } \text{state}' \leftarrow \text{drain}(\text{vals}, \text{newReaders}) \text{ in} \\ \text{body } \text{state}' \rangle, \epsilon \rangle]$$

Reducing the list append operation, expanding `drain`, and re-expanding  $G$ , we have:

$$(\nu b)(\langle b, E[\text{receive}], \epsilon \rangle \parallel \langle a, \text{let } \text{state}' \leftarrow (\text{rec } f(x). \\ \text{let } (\text{vals}, \text{readers}) = x \text{ in} \\ \text{case } \text{vals} \{ \\ [ ] \mapsto \text{return}(\text{vals}, \text{readers}) \\ v :: \text{vs} \mapsto \\ \text{case } \text{readers} \{ \\ [ ] \mapsto \text{return}(\text{vals}, \text{readers}) \\ \text{pid} :: \text{pids} \mapsto \text{send } v \text{ pid}; \\ f(\text{vs}, \text{pids}) \} \} \rangle) \langle (W) :: (\vec{V}), [b] \rangle \text{ in} \\ \text{body } \text{state}' \rangle, \epsilon \rangle]$$

Next, we reduce the recursive function and the case statements:

$$(\nu b)(\langle b, E[\text{receive}], \epsilon \rangle \parallel \langle a, \text{let } \text{state}' \leftarrow \text{send } \langle (W) \rangle b; \\ \text{drain } \langle (\vec{V}), [ ] \rangle \\ \text{body } \text{state}' \rangle, \epsilon \rangle]$$

We next perform the send operation, and note that the recursive call to `drain` will return the argument unchanged, since  $\langle (\vec{V}), [ ] \rangle$  has no pending requests.

Thus we have:

$$(\nu b)(\langle b, E[\text{receive}], \langle (W) \rangle \rangle \parallel \langle a, \text{body } \langle (\vec{V}), [ ] \rangle \rangle, \epsilon \rangle]$$

Finally, we perform the **receive** and apply a structural congruence to arrive at

$$(\nu b)(\langle b, E[\langle W \rangle], \epsilon \rangle \parallel \langle a, \mathbf{body}(\langle \langle \vec{V} \rangle, [ ] \rangle), \epsilon \rangle)$$

which is equal to

$$\langle E[\mathbf{return} W] \parallel a(\vec{V}) \rangle$$

as required.

*Case* NEWCH

Assumption  $E[\mathbf{newCh}]$

$$\begin{aligned} \text{Definition of } \langle - \rangle & (\nu a)(\langle a, \langle E \rangle[\mathbf{spawn}(\mathbf{body}([ ], [ ])), \epsilon \rangle) \\ & \longrightarrow (\nu a)(\nu b)(\langle a, \langle E \rangle[\mathbf{return} b], \epsilon \rangle \parallel \langle b, \mathbf{body}([ ], [ ]), \epsilon \rangle) \\ & \equiv (\nu b)(\nu a)(\langle a, \langle E \rangle[\mathbf{return} b], \epsilon \rangle \parallel \langle b, \mathbf{body}([ ], [ ]), \epsilon \rangle) \\ & = \langle (\nu b)(E[\mathbf{return} b] \parallel b(\epsilon)) \rangle \end{aligned}$$

as required.

*Case* FORK

Assumption  $E[\mathbf{fork} M]$

$$\begin{aligned} \text{Definition of } \langle - \rangle & (\nu a)(\langle a, \langle E \rangle[\mathbf{let} x \Leftarrow \mathbf{spawn} \langle M \rangle \mathbf{in return} ()], \epsilon \rangle) \\ & \longrightarrow (\nu a)(\nu b)(\langle a, \langle E \rangle[\mathbf{let} x \Leftarrow \mathbf{return} b \mathbf{in return} ()], \epsilon \rangle \parallel \langle a, \langle M \rangle, \epsilon \rangle) \\ & \longrightarrow_M (\nu a)(\nu b)(\langle a, \langle E \rangle[\mathbf{return} ()], \epsilon \rangle \parallel \langle b, \langle M \rangle, \epsilon \rangle) \\ & \equiv (\nu a)(\langle a, \langle E \rangle[\mathbf{return} ()], \epsilon \rangle \parallel (\nu b)(\langle b, \langle M \rangle, \epsilon \rangle)) \\ & = \langle E[\mathbf{return} ()] \parallel M \rangle \end{aligned}$$

as required.

*Case* LIFT

Immediate by the inductive hypothesis.

*Case* LIFTV

Immediate by Lemma 25.

□