Multi-Strategy Trading Utilizing Market Regimes

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- These are "model-based" methods. So, one makes assumptions (e.g., known expected returns) that may turn out to be troublesome.
- This issue spurred research into "model-free" approaches.

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- Universal portfolio (Cover 1991) - Sequential portfolio allocation to match the best constantly rebalanced portfolio in hindsight (for an arbitrary market process).
- Many extensions and follow-on work: multiplicative updates (Helmbold et al. 1998), efficient online computation (Kalai et al. 2002), Anticor (Borodin et al. 2004), kernel-weighted allocation (Györfi et al. 2006).

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- Major contribution of universal algorithms is an online procedure to solve this problem, within a target portfolio class.
- We seek online procedures that also allow us to utilize context in the spirit of (non-parametric) statistics.

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  - Make use of historical data to non-parametrically model regimes.
- Devise online algorithm for dynamically rebalancing portfolio, shaped by contextual information.

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- Regimes: Characterize context by relative profitability of primitive strategies.
  - Good trading strategies exploit recurring market dynamics that can be more prevalent in some time periods than in others.
  - Trends depend on hard to model latent variables - we seek alternate state description in an action-oriented representation.

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But, it is helpful to differentiate the static structure implied by the qualitative strategy from the dynamic evolution of regimes.

Portfolio Allocation Algorithm: A Template

Three major steps:

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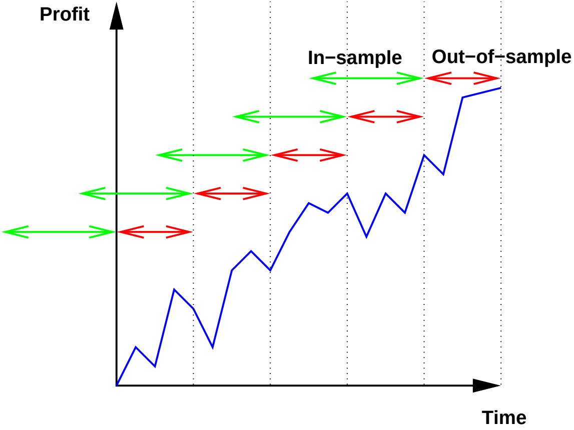
- (Optional) Use historical data to infer *set* of regimes, i.e., relative order between strategies.
- In-Sample: Identify the order (by performance) over regimes within a moving window, i.e., estimate current context.
- Out-Sample: Allocate working capital assuming persistence of the identified in-sample context.

Simplest Instantiation: Trade with Best In-Sample Strategy



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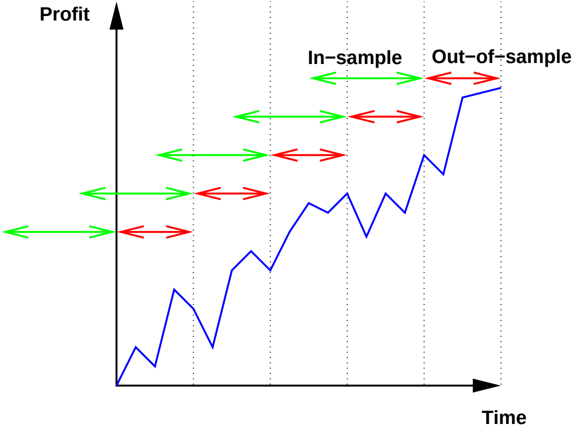

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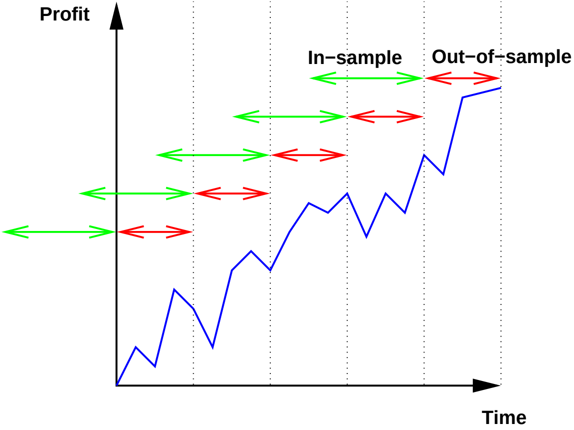
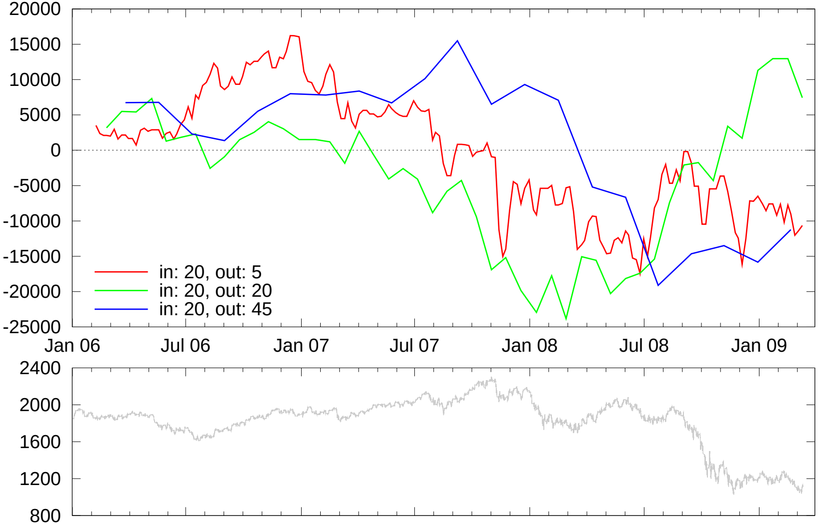


- In: Identify best quantitative strategy
- Out: Allocate capital to the best in-sample strategy

Performance: Trading with Best In-Sample Strategy



Observations: Trading with Best In-Sample Strategy

Can be profitable. However,

- Sensitivity to parameter choice, e.g., window size.
- Need fine-grained trading to follow changing trends.
- Wasteful chatter between different strategies.
  - Diversification could help solve some of these problems.

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This is more diversified, but tends not to suffice.
– we will see a few empirical results later in this presentation.

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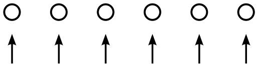

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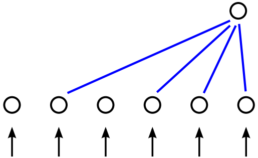

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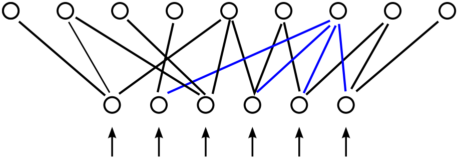

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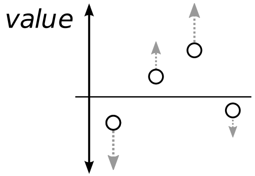

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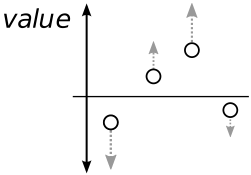

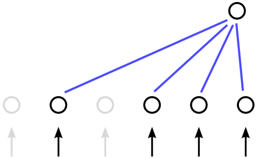

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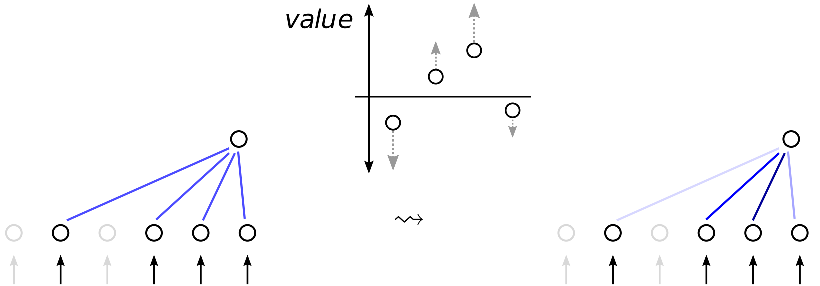

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The effective number of regimes may be significantly smaller than the number of underlying strategies.

- Dimensionality reduction would aid the state identification step
  - Previous instantiations may be considered special cases
- Possible to build predictive models in a space that is different from standard latent variable time series models.

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  - Find significantly correlated events among possible combinations of events e.g. using permutation tests

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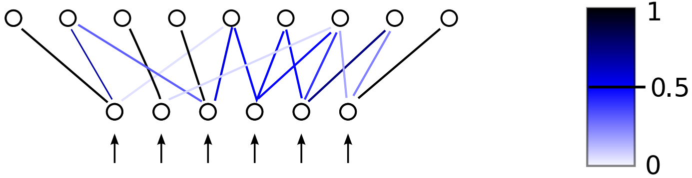
- In-sample period (Estimation of current regime):
  1. Compute values of all regimes by taking a weighted sum of the strategy fitnesses in the in-sample period



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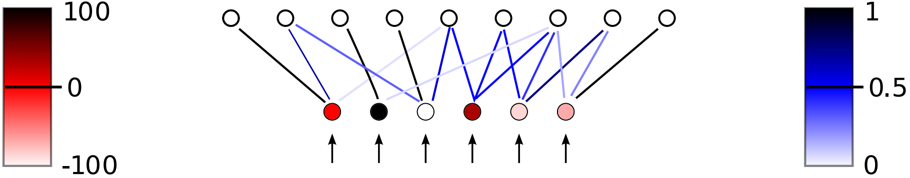

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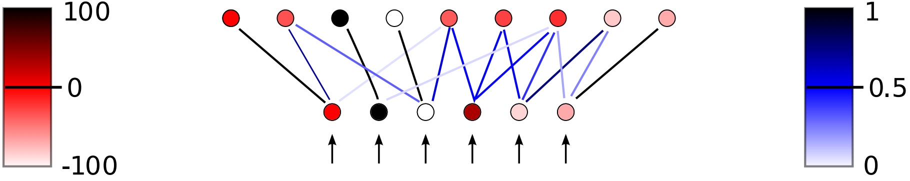

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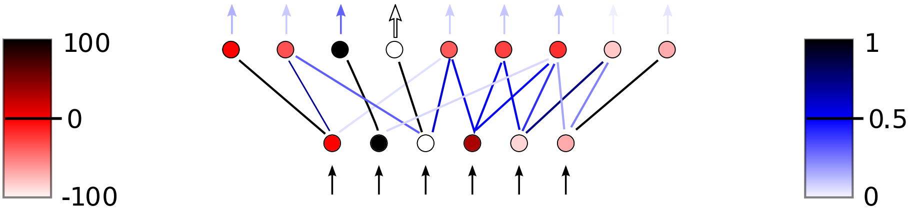

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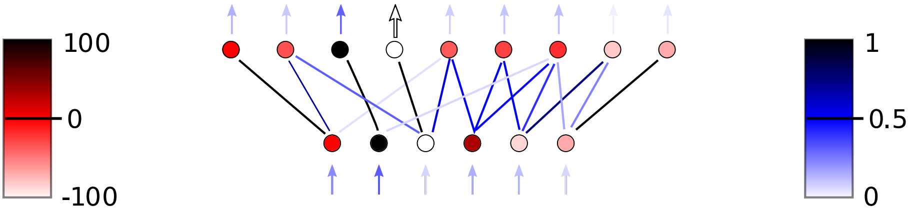
- Out-of-sample period (Trading): Multiplicative weight update for allocation between regimes



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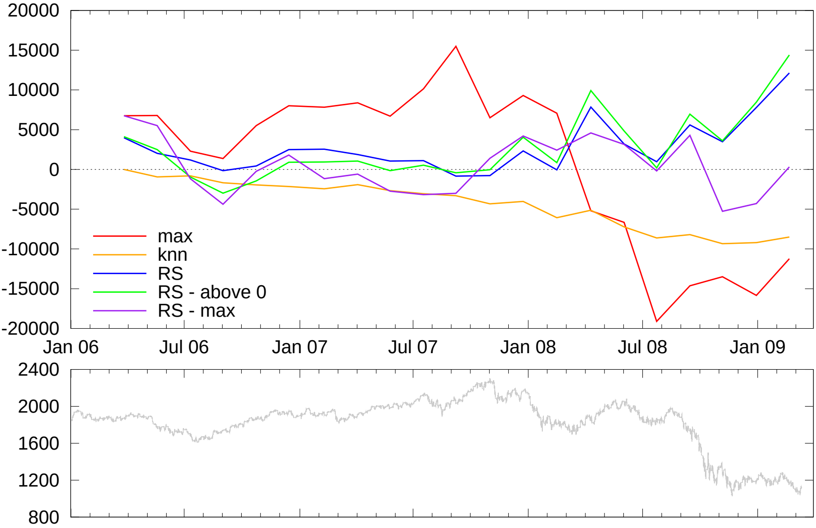

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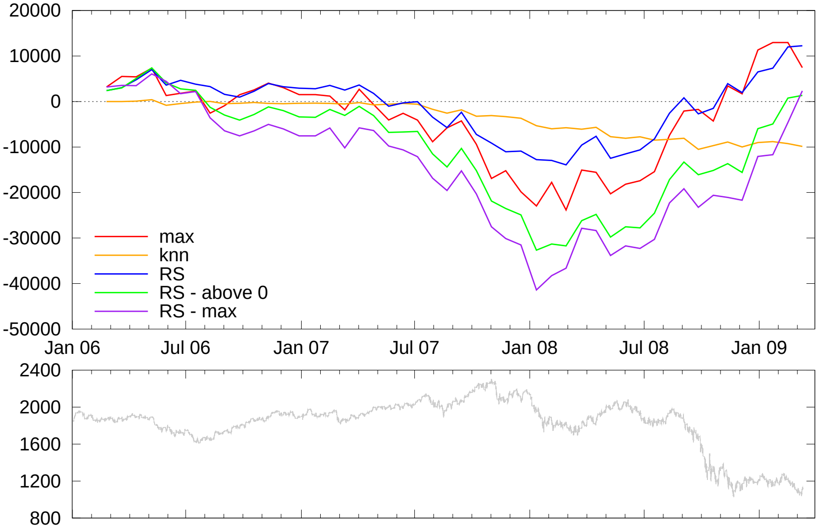

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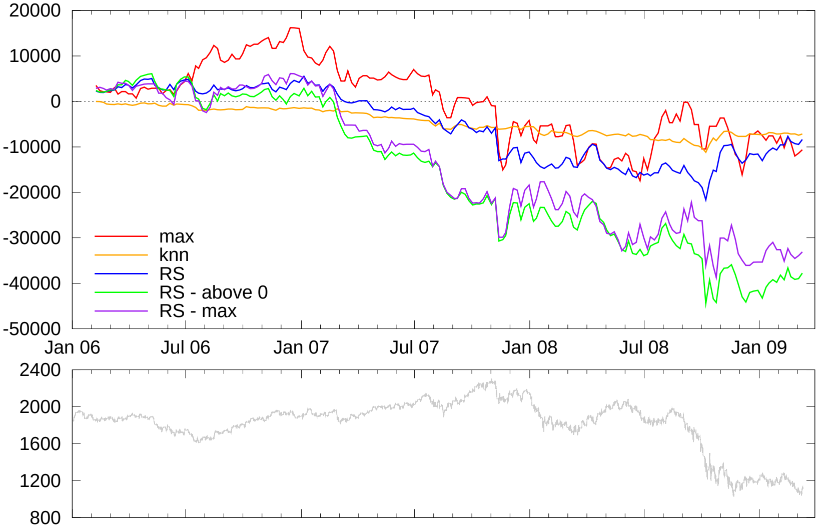

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  - Data allows us to identify possible correlation patterns within strategy space
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  - Structure in this space (e.g., low-dimensional regime subspaces) can be exploited to devise more efficient strategies.

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