Multi-Strategy Trading Utilizing Market Regimes

Hynek Mlnařík ¹
Subramanian Ramamoorthy ²
Rahul Savani ¹

¹Warwick Institute for Financial Computing
Department of Computer Science
University of Warwick

²Institute of Perception, Action and Behaviour
School of Informatics
University of Edinburgh
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- These are “model-based” methods. So, one makes assumptions (e.g., known expected returns) that may turn out to be troublesome.
- This issue spurred research into “model-free” approaches.
“Model-free” Portfolio Allocation

- Point of departure: Classic work on optimal bet sizing (Kelly 1956, Breiman 1961) - how much to bet given odds?


- Universal portfolio (Cover 1991) - Sequential portfolio allocation to match the best constantly rebalanced portfolio in hindsight (for an arbitrary market process).

- Many extensions and follow-on work: multiplicative updates (Helmbold et al. 1998), efficient online computation (Kalai et al. 2002), Anticor (Borodin et al. 2004), kernel-weighted allocation (Győrfi et al. 2006).
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Constantly rebalanced portfolios (Thorp 1971, Markovitz 1976, Bell+Cover 1988, Algoet+Cover 1988) - keep relative allocation of capital constant (still assuming known market return distributions).
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Utilizing Market Context

Market processes are not *entirely* arbitrary

Statistical view of Universal Portfolios (Belentepe 2005): Weights (constrained to a partition of unity) are conditional expectation of a multivariate normal distribution, $w \sim \mathcal{N}(\bar{\Sigma}^{-1}t\bar{r}_t, 1_t\bar{\Sigma}t)$.

Unconstrained version is the standard log-optimal investment. Major contribution of universal algorithms is an online procedure to solve this problem, within a target portfolio class. We seek online procedures that also allow us to utilize context in the spirit of (non-parametric) statistics.
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- Devise online algorithm for dynamically rebalancing portfolio, shaped by contextual information.
**Our Approach – Two Major Concepts**

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  - Good trading strategies exploit *recurring market dynamics* that can be *more prevalent in some time periods than in others*.
  - Trends depend on hard to model latent variables - we seek alternate state description in an action-oriented representation.
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Three major steps:

- **(Optional)** Use historical data to infer set of regimes, i.e., relative order between strategies.
Portfolio Allocation Algorithm: A Template

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- **In-Sample:** Identify the order (by performance) over regimes within a moving window, i.e., estimate current context.
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- **In-Sample: Identify the order (by performance) over regimes within a moving window, i.e., estimate current context.**

- **Out-Sample: Allocate working capital assuming persistence of the identified in-sample context.**
Simplest Instantiation: Trade with Best In-Sample Strategy

In: Identify best quantitative strategy
Out: Allocate capital to the best in-sample strategy
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- **In**: Identify best quantitative strategy
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Performance: Trading with Best In-Sample Strategy

[Graph showing performance over time with various strategies and their performance metrics.]
Observations: Trading with Best In-Sample Strategy

Can be profitable. However,

- Sensitivity to parameter choice, e.g., window size.
- Need fine-grained trading to follow changing trends.
- Wasteful chatter between different strategies.
  - Diversification could help solve some of these problems.
Another Simple Instantiation: $k$–NN

Execute the weighted average action derived from $k$-nearest market states in a historical database.
Another Simple Instantiation: k-NN

Execute the weighted average action derived from k-nearest market states in a historical database.

This is more diversified, but tends not to suffice. – we will see a few empirical results later in this presentation.
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During training, weights are adjusted between strategies according to fitness.

Fitness of a regime is defined as a weighted sum of its strategy fitnesses, similar to a mixture model.

The effective number of regimes may be significantly smaller than the number of underlying strategies.

Dimensionality reduction would aid the state identification step.

Previous instantiations may be considered special cases.

Possible to build predictive models in a space that is different from standard latent variable time series models.
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Algorithm: Regime Detection and Strategy Optimization

Training - Acquire regimes from data
- Explicitly specified by expert

Find significantly correlated events among possible combinations of events e.g. using permutation tests
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Trading - Allocate capital based on regime-level performance

- **In-sample period (Estimation of current regime):**
  1. Compute values of all regimes by taking a weighted sum of the strategy fitnesses in the in-sample period
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```
0   0.5    100   -100   0
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Performance of RED-STOP Algorithm

![Graph showing the performance of RED-STOP Algorithm from Jan 06 to Jan 09. The graph plots various metrics such as max, knn, RS, RS - above 0, and RS - max, with data points for each month.]
Performance of RED-STOP Algorithm
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![Graph showing the performance of RED-STOP Algorithm over time, with lines for max, knn, RS, RS - above 0, and RS - max. The x-axis represents months from January 2006 to January 2009, and the y-axis represents values ranging from -50,000 to 2,400.](image)
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- Relationship to alternate regime-switching models:
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  - Possible to devise sophisticated time-series models based on EM and MCMC techniques but they can be fragile in on-line ‘covariance-shifted’ scenarios.

- What is the role of historical data? Or, what happens in novel scenarios?
  - Data allows us to identify possible correlation patterns within strategy space – structure induced by latent market dynamics – no parametric assumptions regarding the latent dynamics. Structure in this space (e.g., low-dimensional regime subspaces) can be exploited to devise more efficient strategies.
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Future Work:
- Systematic empirical evaluation (across multiple markets)
- Explore use of low-dimensional structure in regime space.
- Risk-sensitive optimization and predictive-modelling.
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