Information Dynamics

Samson Abramsky

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Information Dynamics: the very idea

Robin's lecture at MFPS 2000.

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From *Information, Processes and Games*, in *Handbook of Philosophy of Information*, ed. Johan van Benthem and Pieter Adriaans, Elsevier 2008:

> *What, then, is this nascent field? We would like to use the term Information Dynamics, which was proposed some time ago by Robin Milner, to suggest how the area of Theoretical Computer Science usually known as "Semantics" might emancipate itself from its traditional focus on interpreting the syntax of pre-existing programming languages, and become a more autonomous study of the fundamental structures of Informatics.*

Computing as a science

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From Robin's essay on *Semantic Ideas in Computing*, in *Computing Tomorrow*, ed. Ian Wand and Robin Milner, Cambridge 1996:

> *Are there distinct principles and concepts which underlie computing, so that we are justified in calling it an independent science?*
>
> *. . .*
>
> *In this essay I argue that a rich conceptual development is in progress, to which we cannot predict limits, and whose outcome will be a distinct science.*
>
> *. . .*
>
> *In the previous section we found that the domain model can be understood in terms of amounts of information, and also that sequential computation corresponds to a special discipline imposed on the flow of information. In the present section, we have found that a key to understanding concurrent or interactive computation lies in the structure of this information flow.*
>
> *. . .*
>
> *Thus both applications and theories converge upon the phenomena of information flow; in my view this indicates a new scientific identity.*

Mathematical structures for information flow

From the preface to Robin's book *The space and motion of communicating agents*:

> *Large informatic systems are complex, and any rigorous model must control this complexity by means of adequate structure. After many years seeking such models, I am convinced that categories provide this structure most convincingly.*

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Diagrammatic representations ('string diagrams') will play a key role. The *same* pictures and the *same* diagrammatic transformations show up in all these, apparently very different contexts.

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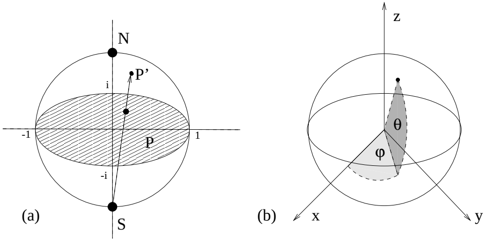
Geometric picture: the Bloch sphere



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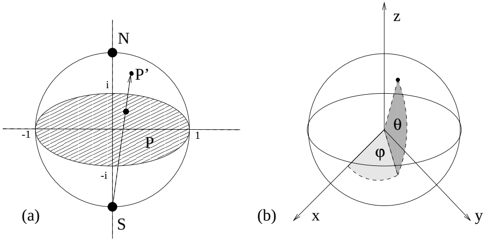

Things get interesting with *n*-qubit registers

$$\sum_i \alpha_i |i\rangle, \qquad i \in \{0,1\}^n.$$

Quantum Entanglement

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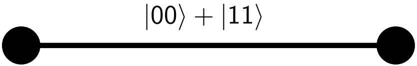
Bell state:



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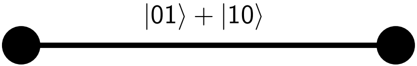

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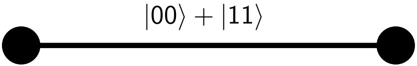

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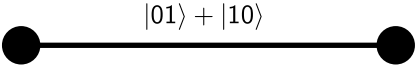

Compound systems are represented by *tensor product*: $\mathcal{H}_1 \otimes \mathcal{H}_2$. Typical element:

$$\sum_i \lambda_i \cdot \phi_i \otimes \psi_i$$

Superposition encodes *correlation*.

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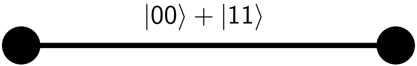

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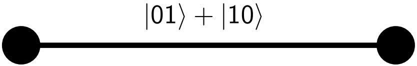

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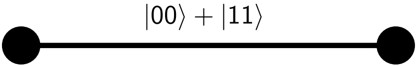

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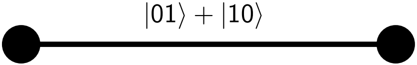

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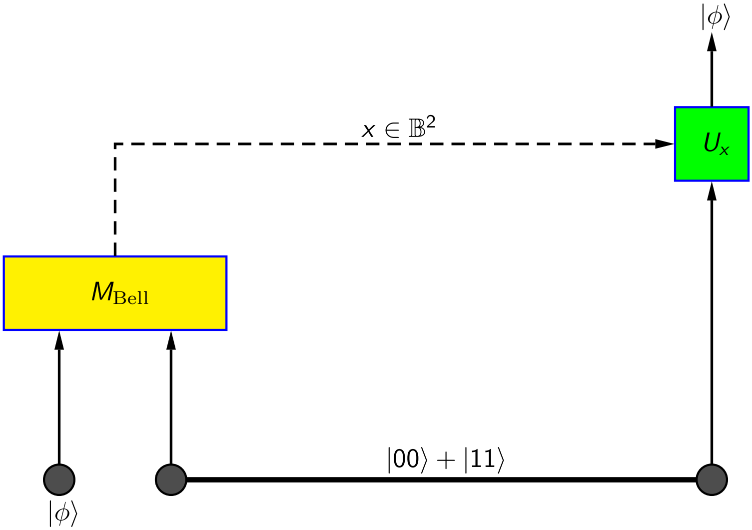
Bell's theorem: QM is *essentially non-local*.

From ‘paradox’ to ‘feature’: Teleportation



Entangled states as linear maps

$\mathcal{H}_1 \otimes \mathcal{H}_2$ is spanned by

$$\begin{matrix} |11\rangle & \cdots & |1m\rangle \\ \vdots & \ddots & \vdots \\ |n1\rangle & \cdots & |nm\rangle \end{matrix}$$

hence

$$\sum_{i,j} \alpha_{ij} |ij\rangle \quad \longleftrightarrow \quad \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{nm} \end{pmatrix} \quad \longleftrightarrow \quad |i\rangle \mapsto \sum_j \alpha_{ij} |j\rangle$$

Pairs $|\psi_1, \psi_2\rangle$ are a special case — $|ij\rangle$ in a well-chosen basis.

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$$\mathbf{Hom}(A, B) \cong A^* \otimes B.$$

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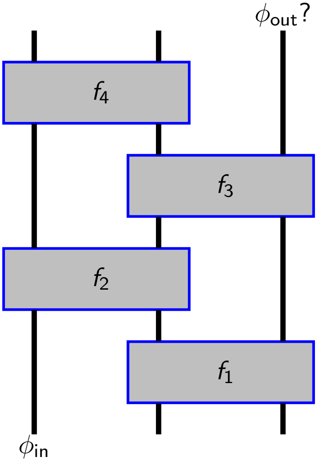
Does this remind you of λ-calculus a little bit? ...

What is the output?



$$(\mathbf{P}_{f_4} \otimes 1) \circ (1 \otimes \mathbf{P}_{f_3}) \circ (\mathbf{P}_{f_2} \otimes 1) \circ (1 \otimes \mathbf{P}_{f_1}) : \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \longrightarrow \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$$

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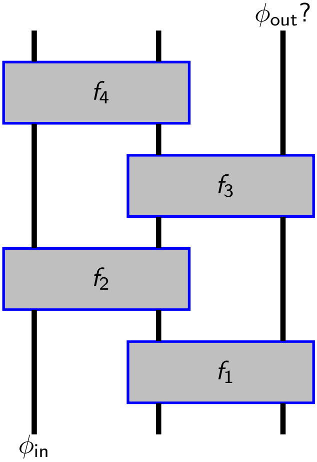

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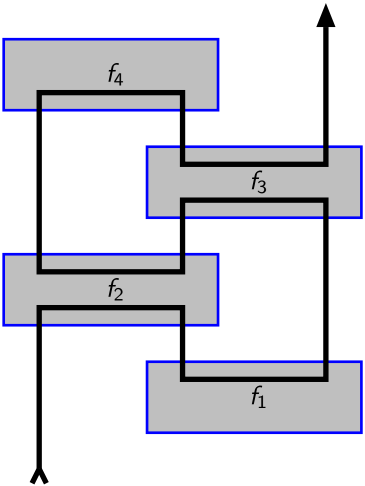
$$\phi_{out} = f_3 \circ f_4 \circ f_2^{\dagger} \circ f_3^{\dagger} \circ f_1 \circ f_2(\phi_{in})$$

Follow the line!



$$f_3 \circ f_4 \circ f_2^{\dagger} \circ f_3^{\dagger} \circ f_1 \circ f_2$$

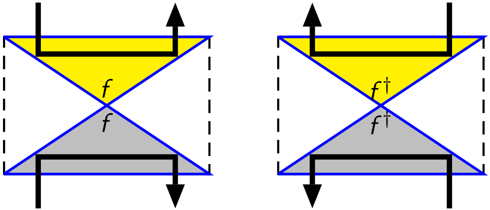
Bipartite Projectors

Information flow in entangled states can be captured mathematically by the isomorphism

$$\mathbf{Hom}(A, B) \cong A^* \otimes B.$$

This leads to a *decomposition* of bipartite projectors into "names" (preparations) and "conames" (measurements).

In graphical notation:



Graphical Calculus for Information Flow

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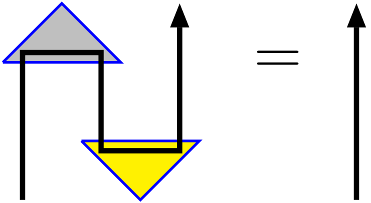
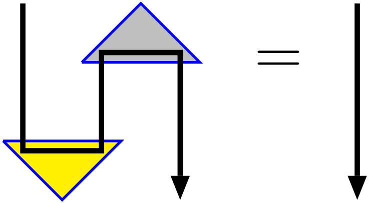
Compact Closure: The basic algebraic laws for units and counits.



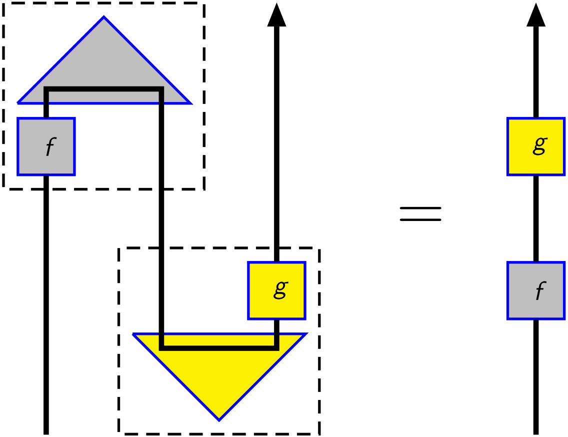
$$(\epsilon_A \otimes 1_A) \circ (1_A \otimes \eta_A) = 1_A$$



$$(1_{A^*} \otimes \epsilon_A) \circ (\eta_A \otimes 1_{A^*}) = 1_{A^*}$$

Compositionality

The key algebraic fact from which teleportation (and many other protocols) can be derived.



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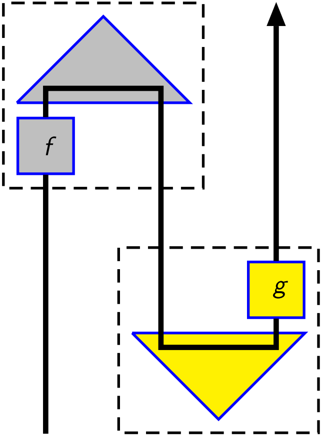

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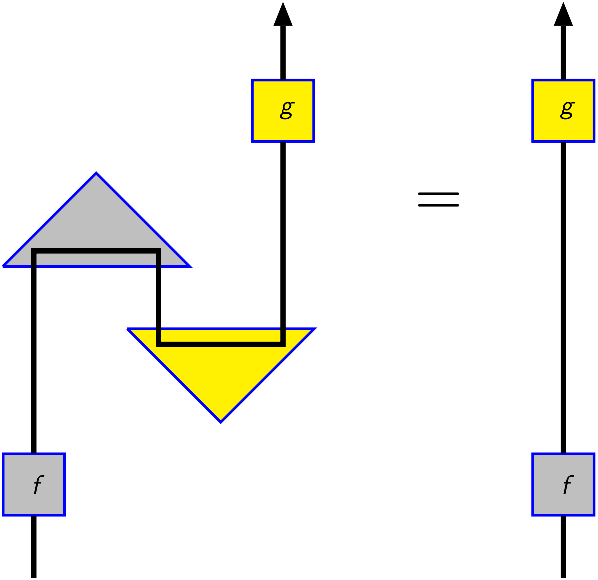
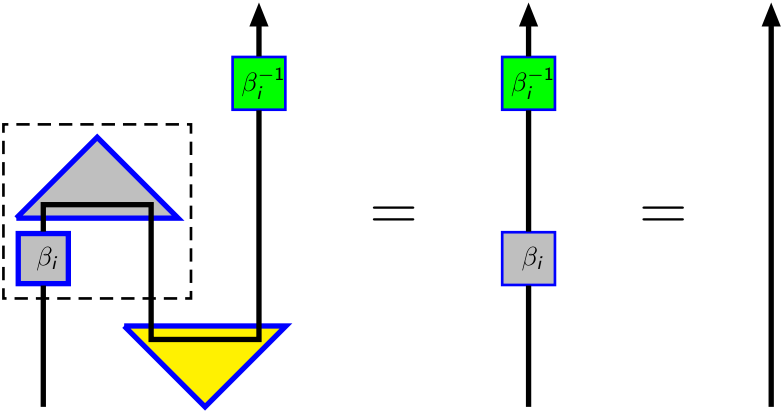


Teleportation diagrammatically



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- Software tool support: Quantomatic. Tactics, graph rewriting, visual interface.
- Applications. Formalization of quantum protocols, QKD, measurement-based quantum computation, etc. Analysis of determinism in MBQC, compositional structure of multipartite entanglement. Foundational topics: e.g. analysis of non-locality.

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We will trace a path through some of these ...

The Temperley-Lieb Algebra

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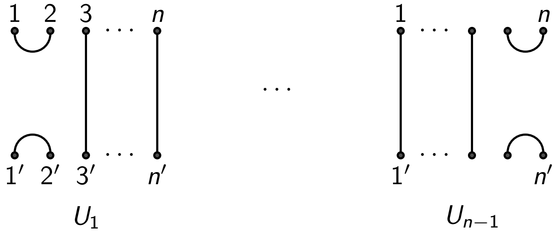
Generators:

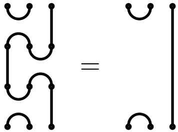


The Temperley-Lieb Algebra

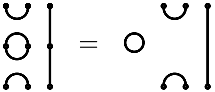
Generators:

U_1 ... U_{n-1}

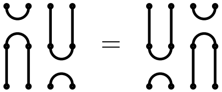
Relations:



$U_1 U_2 U_1 = U_1$



$U_1^2 = \delta U_1$



$U_1 U_3 = U_3 U_1$

Structure of Temperley-Lieb category

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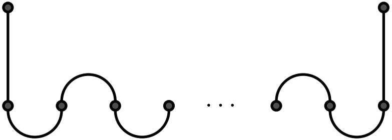

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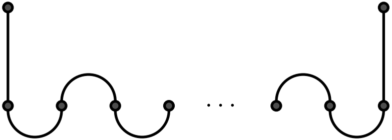

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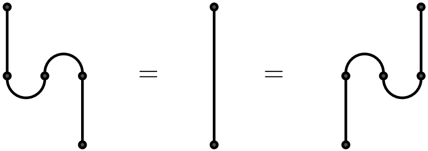

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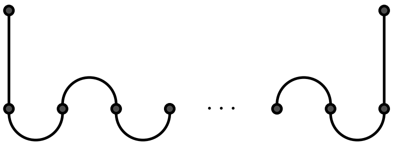

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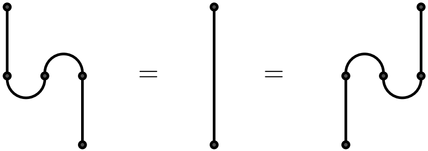
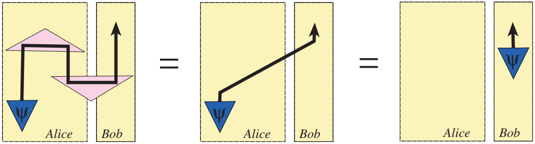


The same structure which accounts for teleportation:



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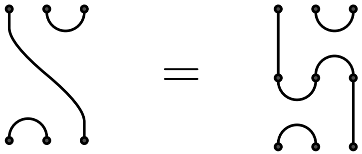

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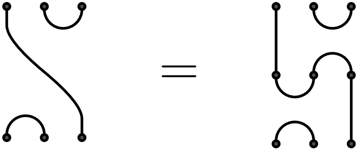
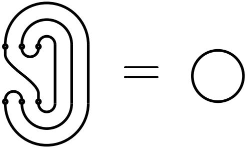
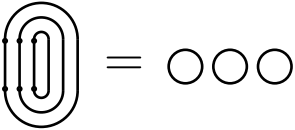


Diagrammatic trace:



The Ear is a Circle



Trace of Identity is the Dimension

The Connection to Knots

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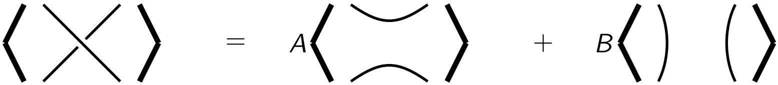

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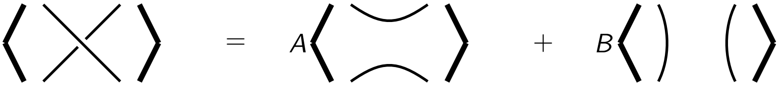

Each over-crossing in a knot or link is evaluated to a weighted sum of the two possible planar smoothings in the Temperley-Lieb algebra. With suitable choices for the coefficients A and B (as Laurent polynomials), this is invariant under the second and third Reidemeister moves. With an ingenious choice of normalizing factor, it becomes invariant under the first Reidemeister move — and yields the Jones polynomial!

Computation: back to the λ-calculus

We shall consider the *bracketing combinator*

$$\mathbf{B} \equiv \lambda x.\lambda y.\lambda z.\, x(yz) : (B \to C) \to (A \to B) \to (A \to C).$$

This is characterized by the equation $\mathbf{B}abc = a(bc)$.

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We take $A = B = C = \mathbf{1}$ in $\mathbf{TL}$. The interpretation of the open term

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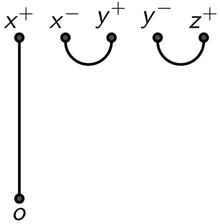
is as follows:



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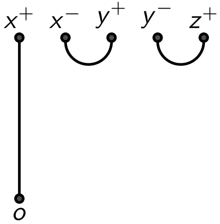

Here x^+ is the output of x, and x^- the input, and similarly for y. The output of the whole expression is o.

Diagrammatic Simplification as β-Reduction

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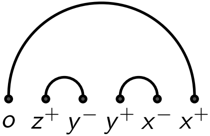
When we abstract the variables, we obtain the following caps-only diagram:



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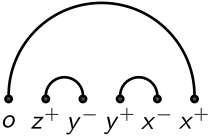
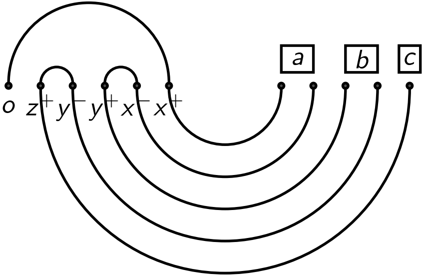


Now we consider an application $\mathbf{B}abc$ (where application is represented by cups):



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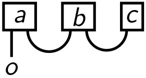

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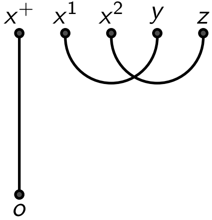

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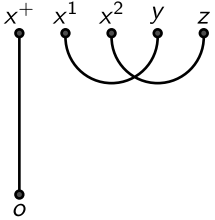

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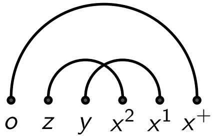

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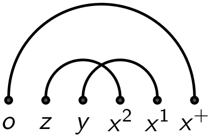

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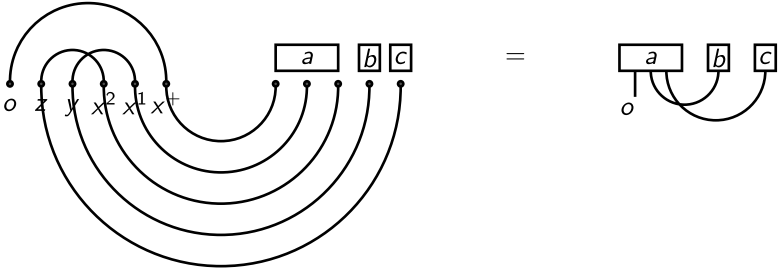

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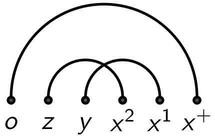

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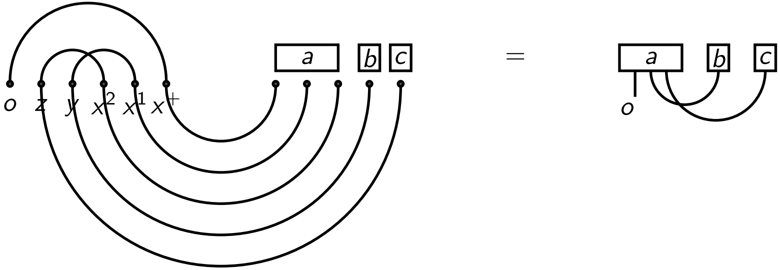

With **BCI** combinators one can interpret *Linear* λ*-calculus*. With just **BI** one has *planar* λ*-calculus*.

Linguistics

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Does	he	like	her?		question
$s\, i^l \pi^l$	π	$i\, o^l$	o	$\rightarrow$	s

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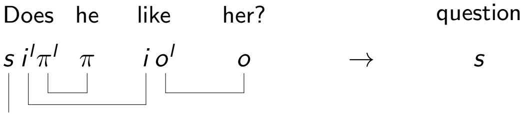

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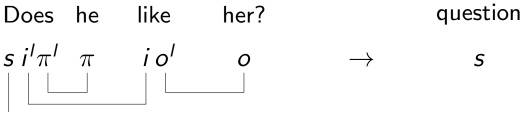

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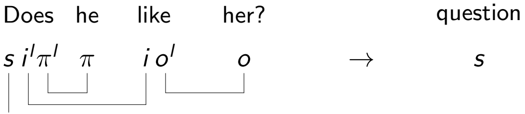

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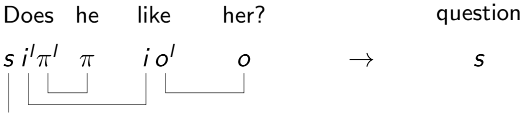

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Implementations and benchmarks look promising: see recent work by Sadrzadeh and Graefenstette.

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- As we have seen, the same structures reach into a wide range of other disciplines.
- There are other promising ingredients for a general theory of information flow. In particular, sheaves as a general 'logic of contextuality'. See my paper with Adam Brandenburger in *New Journal of Physics* (2011).

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