Semi-Markov PEPA

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What is PEPA?

- a stochastic process algebra
- Markovian or exponential distributions
- fast, sleek, no reward cards, spreads straight from the fridge

PEPA: Process Algebra

- Syntax:

$$P \quad ::= \quad (a,\lambda).P \;\Big|\; P + P \;\Big|\; P \underset{S}{\bowtie} P \;\Big|\; P/L \;\Big|\; A$$

- $(a,\lambda).P$: prefix operation $\qquad P + P$: competitive choice

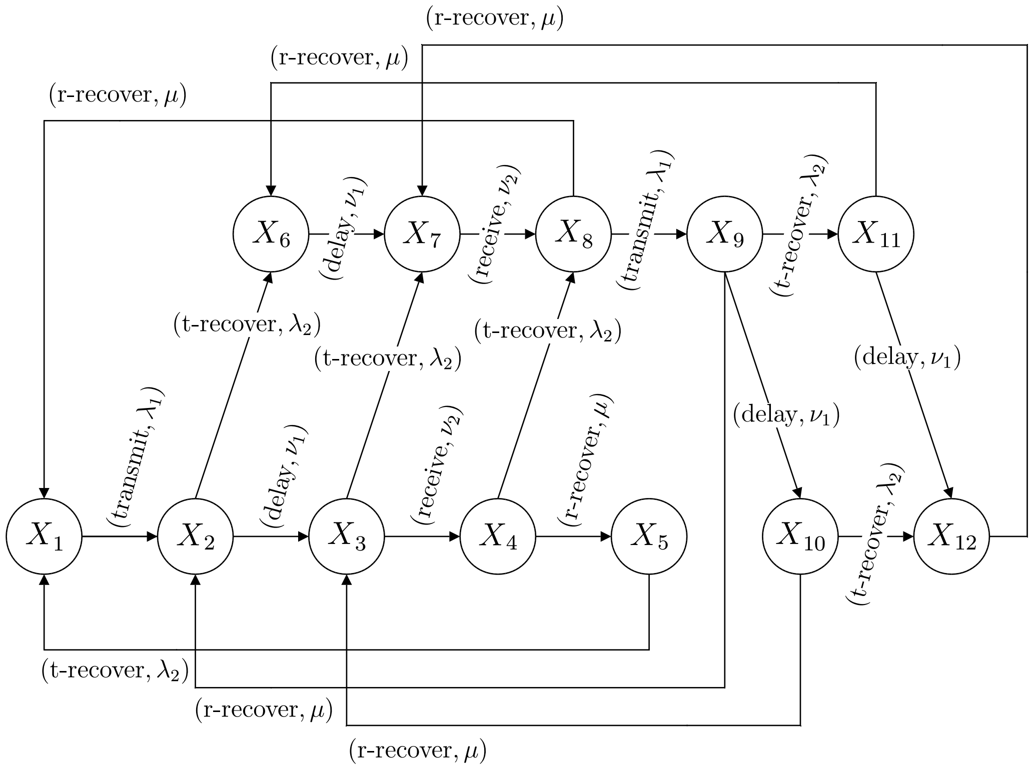
- $P \underset{S}{\bowtie} P$: component cooperation $\qquad P/L$: action hiding

PEPA: Example

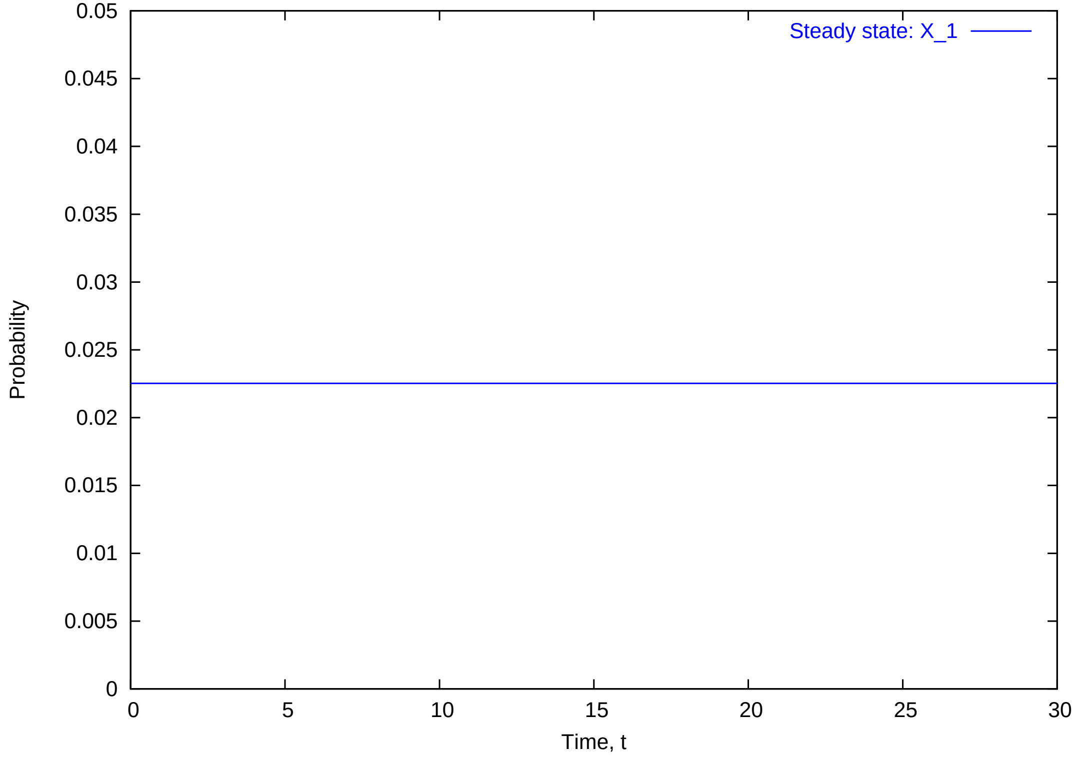
- A simple transmitter-receiver network:

$$
\begin{aligned}
Transmitter &\stackrel{\mathsf{def}}{=} (\mathsf{transmit}, \lambda_1).(\mathsf{t\text{-}recover}, \lambda_2).Transmitter \\
Receiver &\stackrel{\mathsf{def}}{=} (\mathsf{receive}, \top).(\mathsf{r\text{-}recover}, \mu).Receiver \\
Network &\stackrel{\mathsf{def}}{=} (\mathsf{transmit}, \top).(\mathsf{delay}, \nu_1).(\mathsf{receive}, \nu_2).Network \\
System &\stackrel{\mathsf{def}}{=} (Transmitter \underset{\emptyset}{\bowtie} Receiver) \underset{\{\mathsf{transmit},\mathsf{receive}\}}{\bowtie} Network
\end{aligned}
$$

Global State Space



Transient and Steady State



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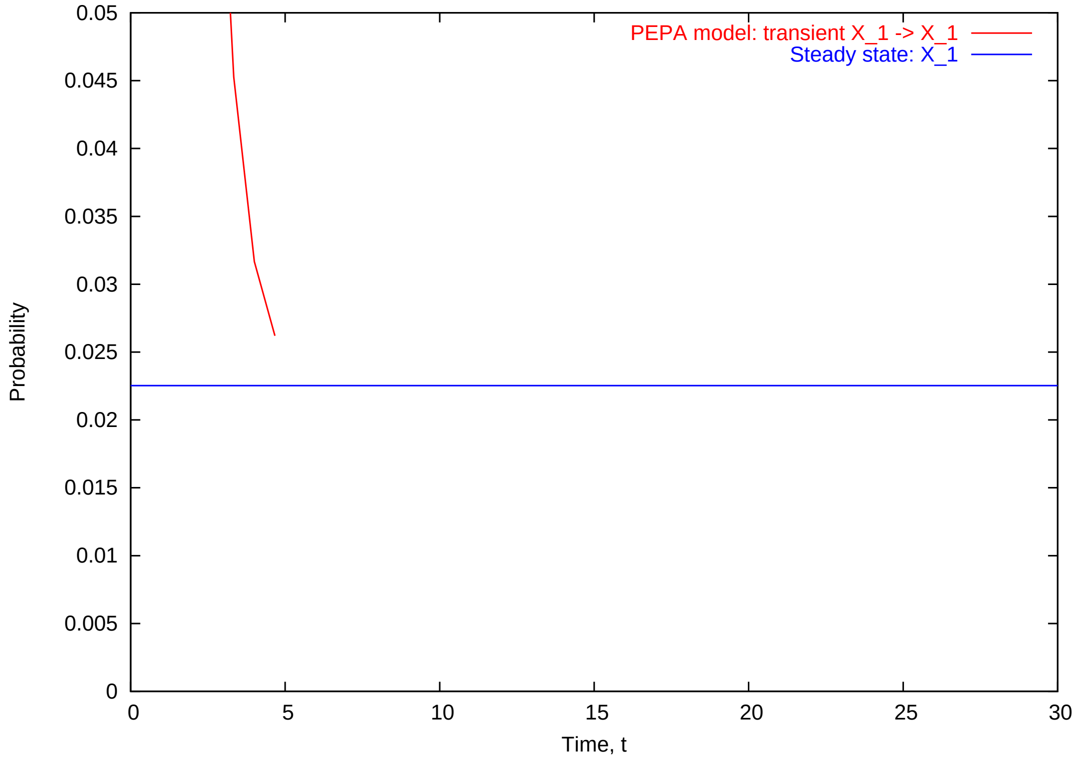

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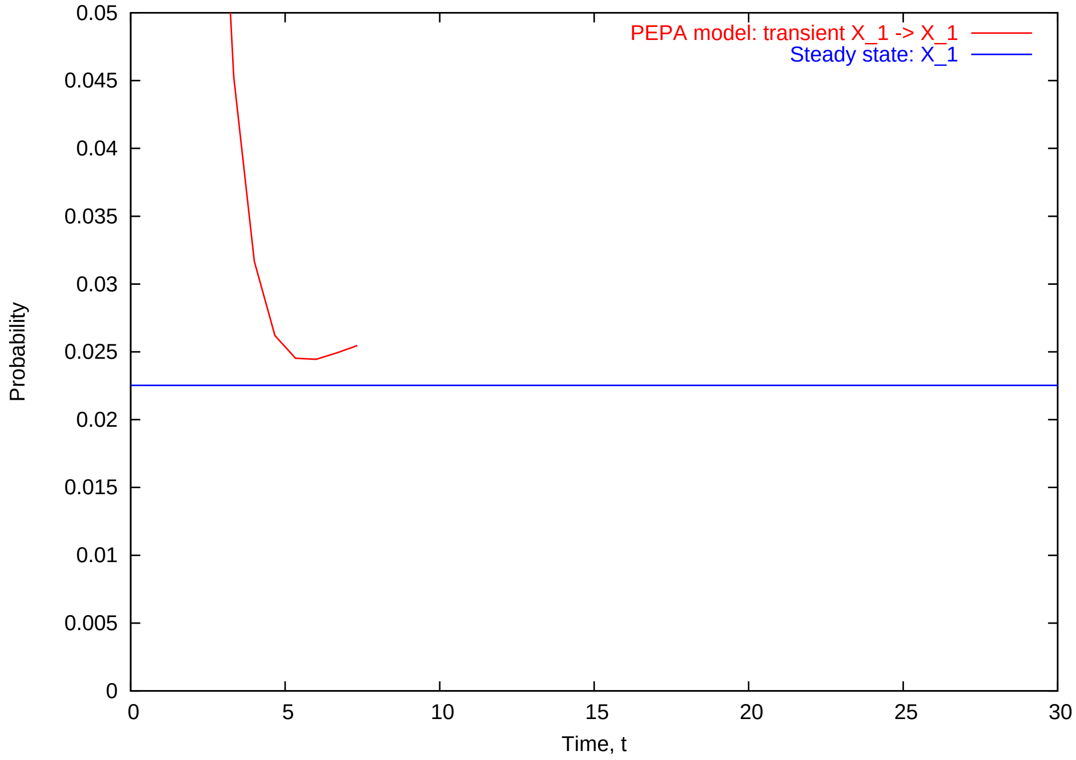

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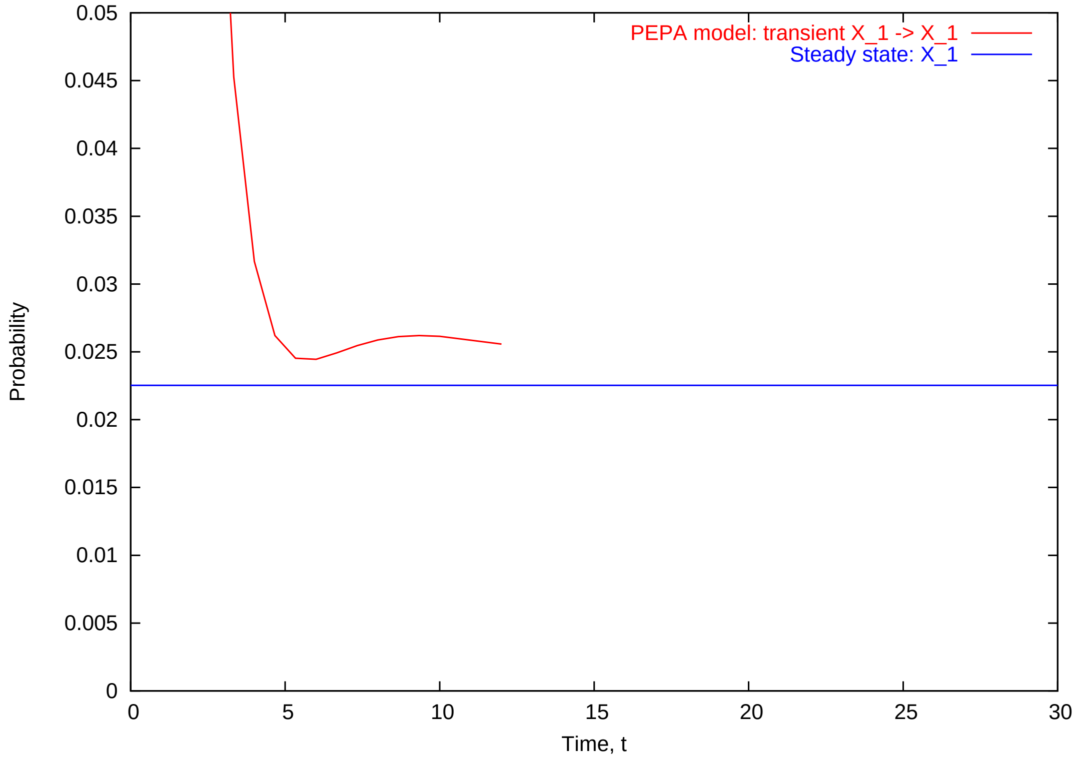

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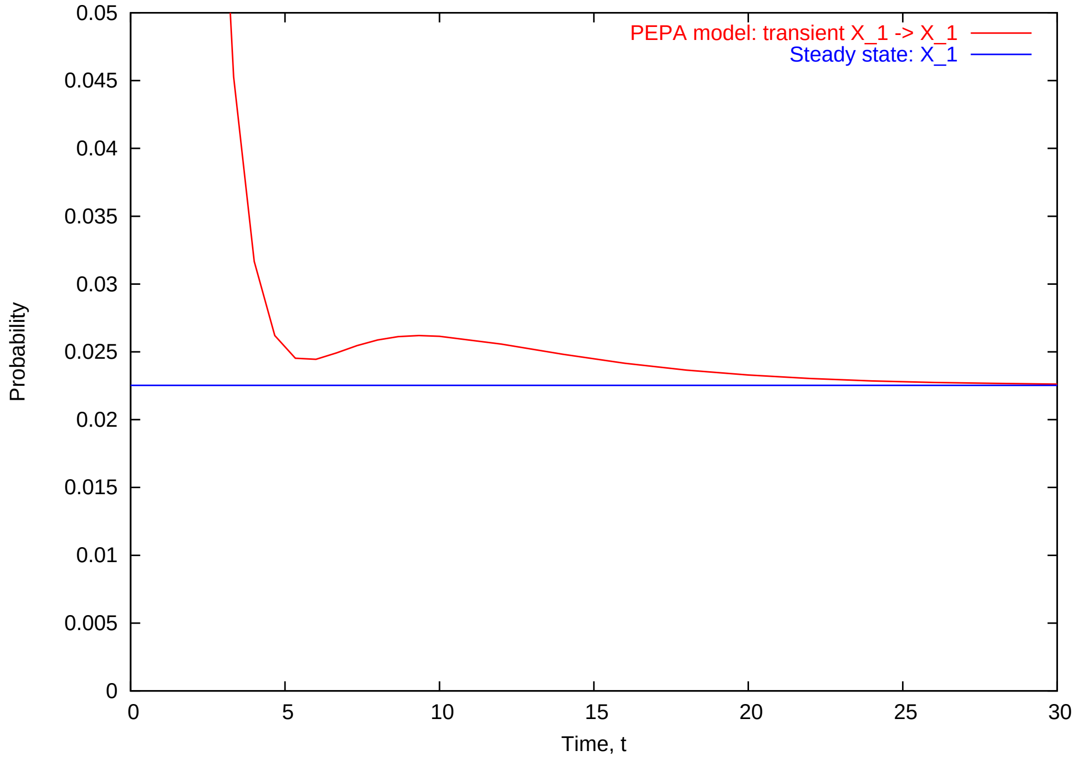
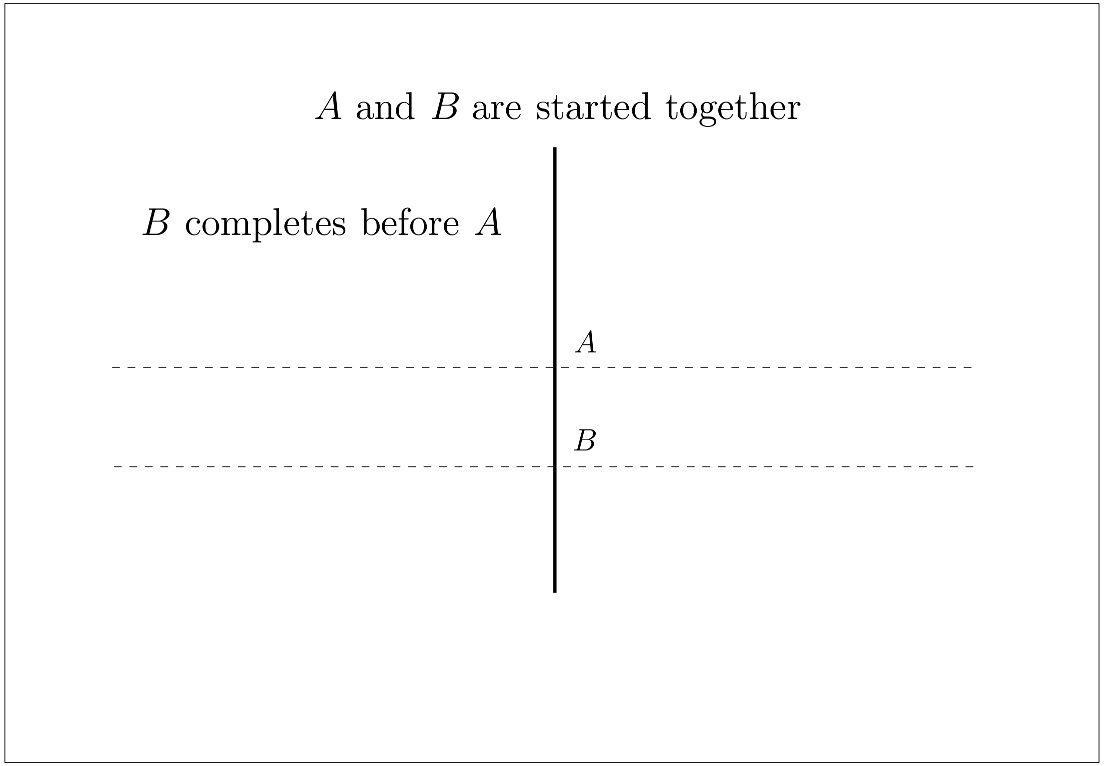


Concurrent Interleaving



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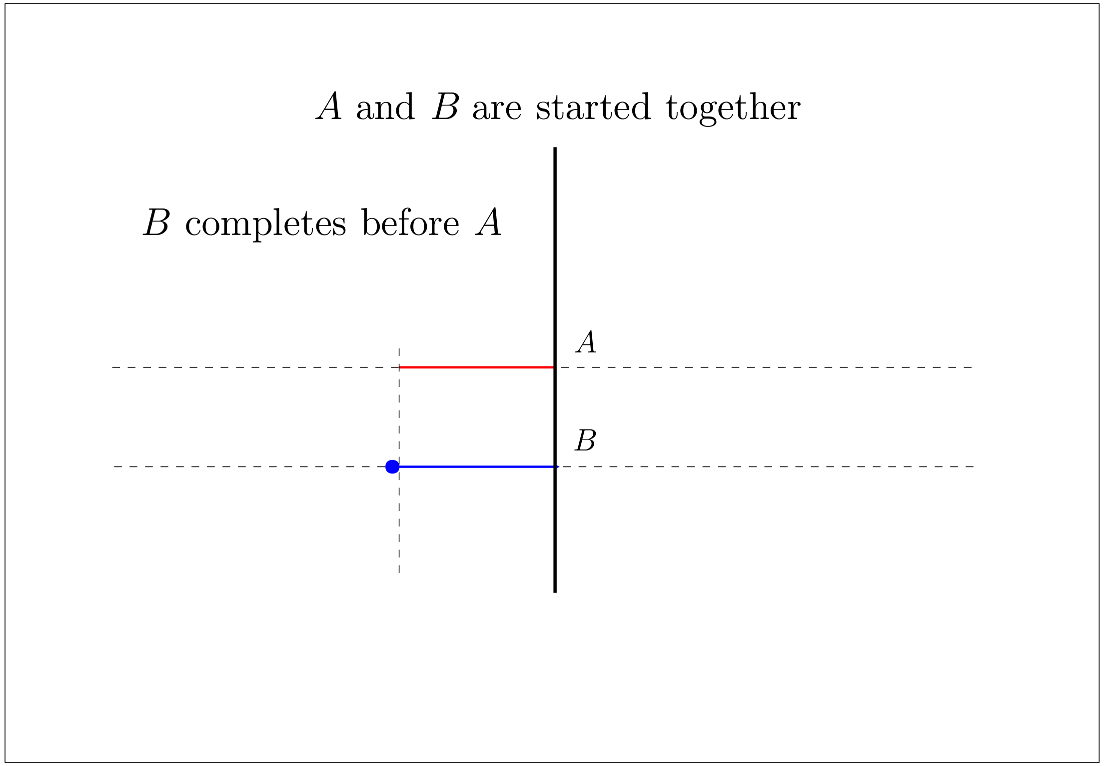

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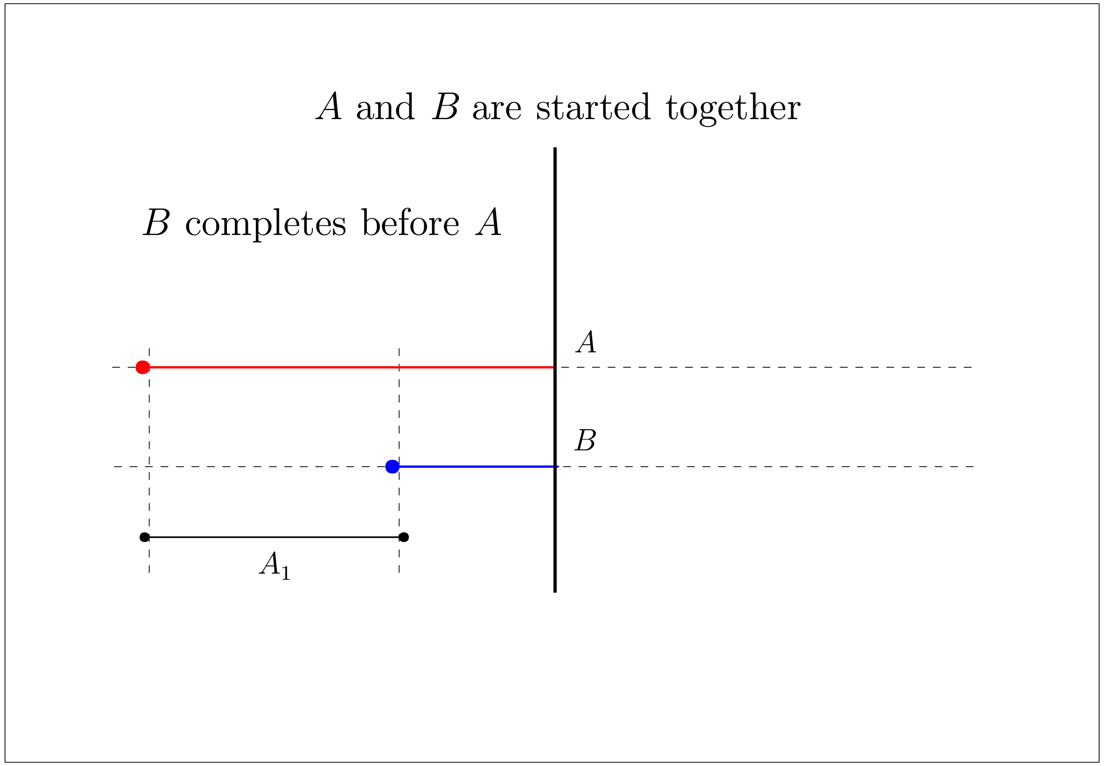

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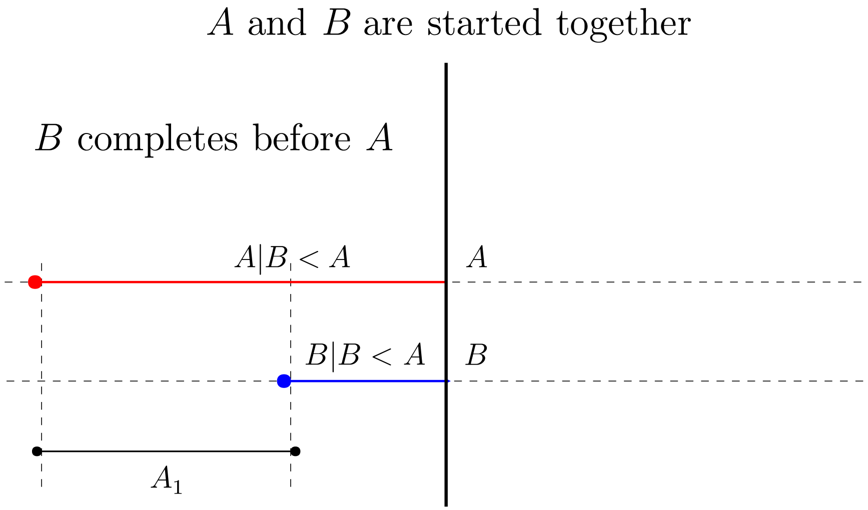
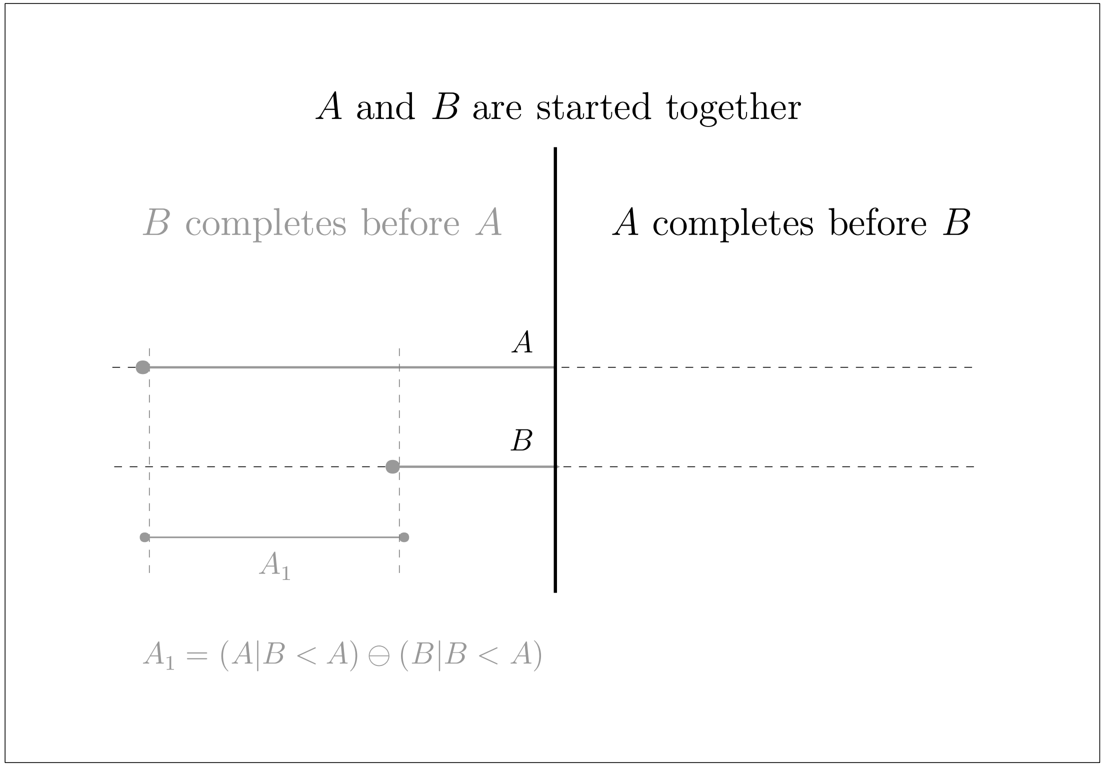


$A_1 = (A|B < A) \ominus (B|B < A)$

Concurrent Interleaving



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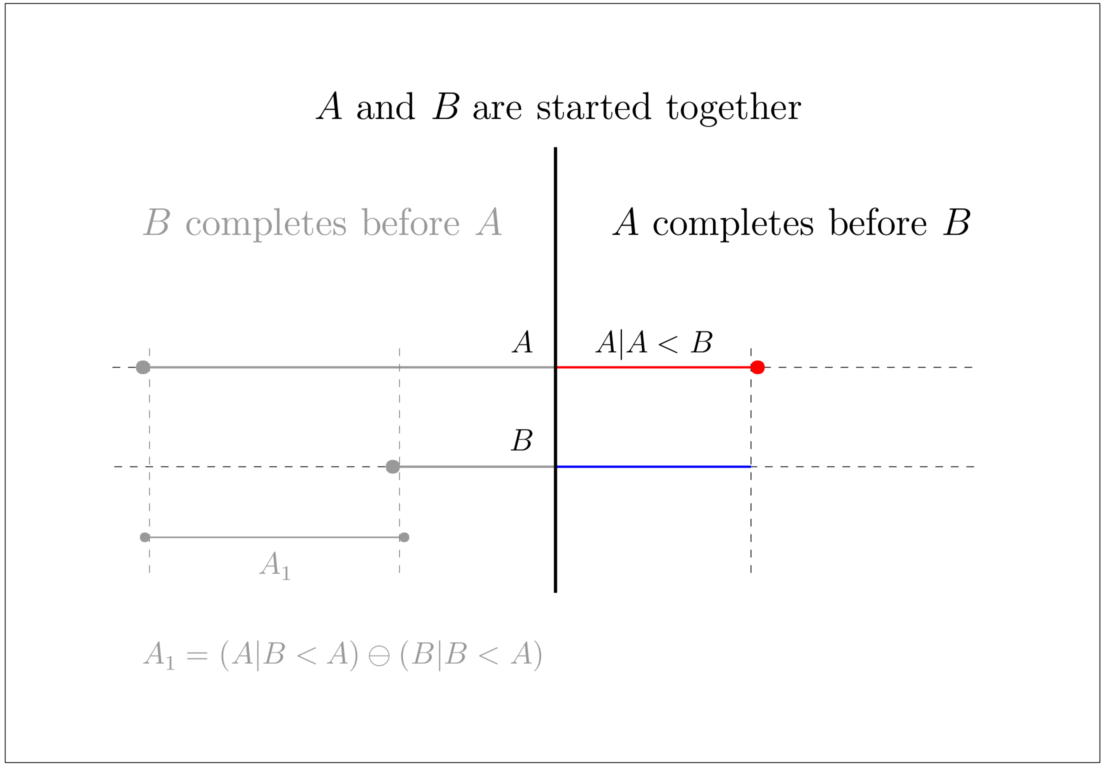

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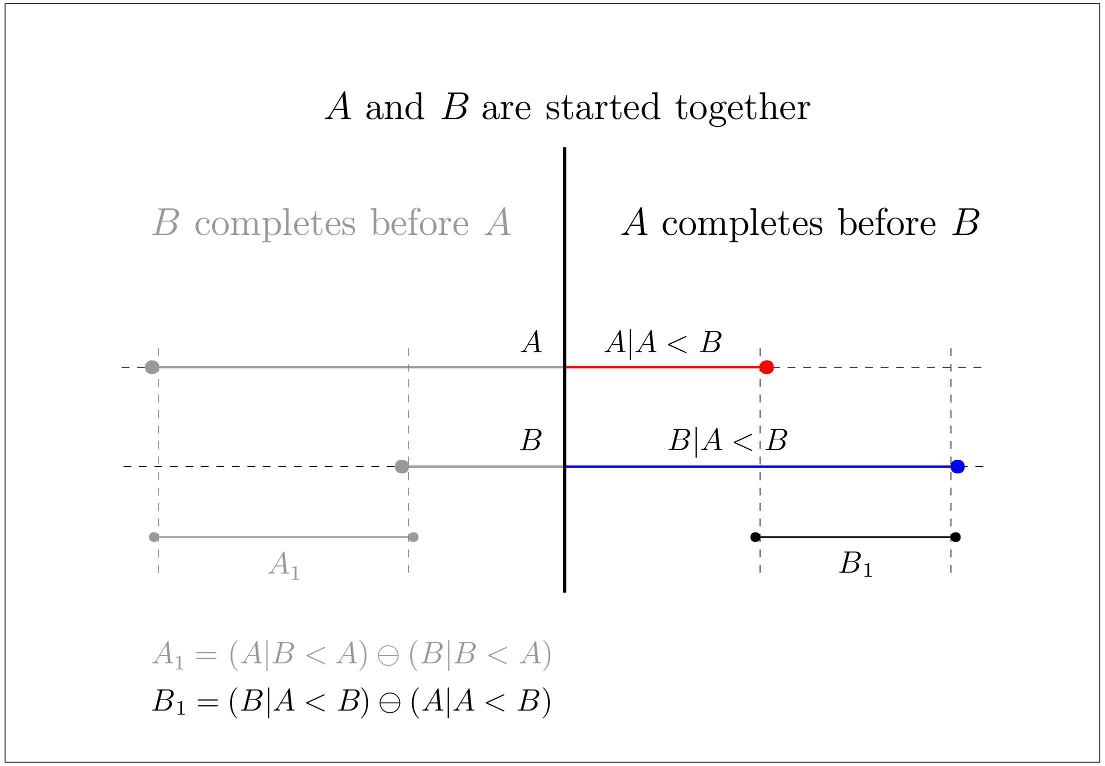
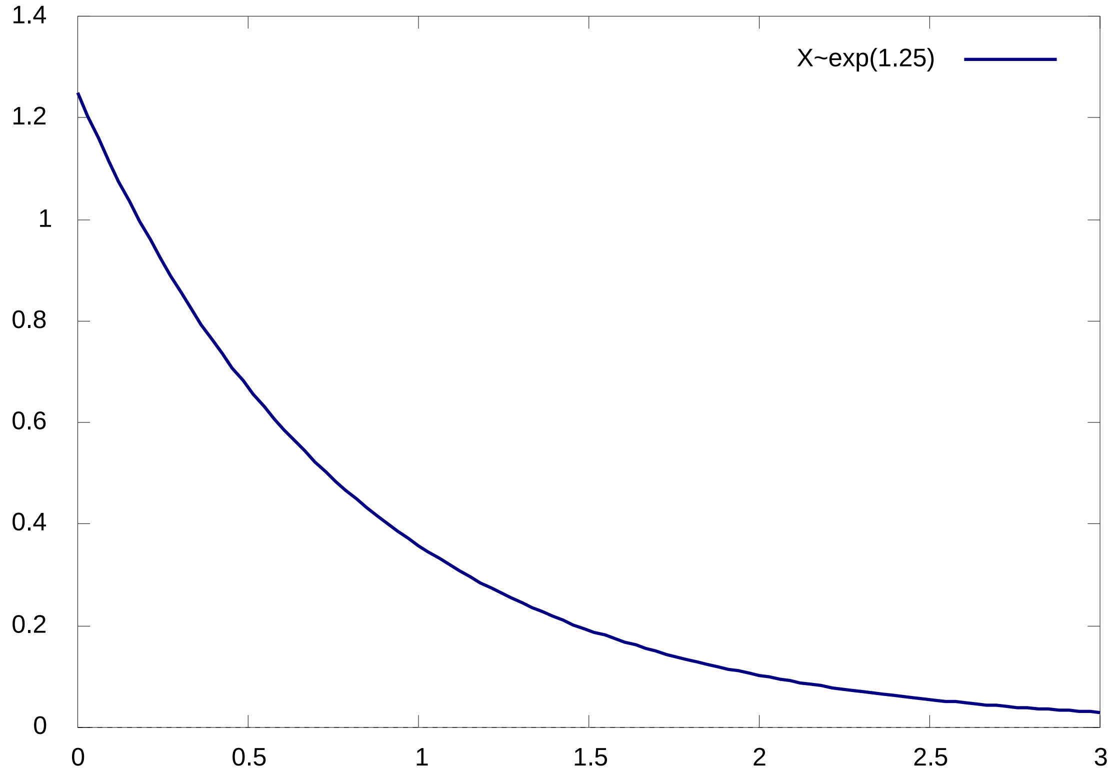


Exponential memorylessness



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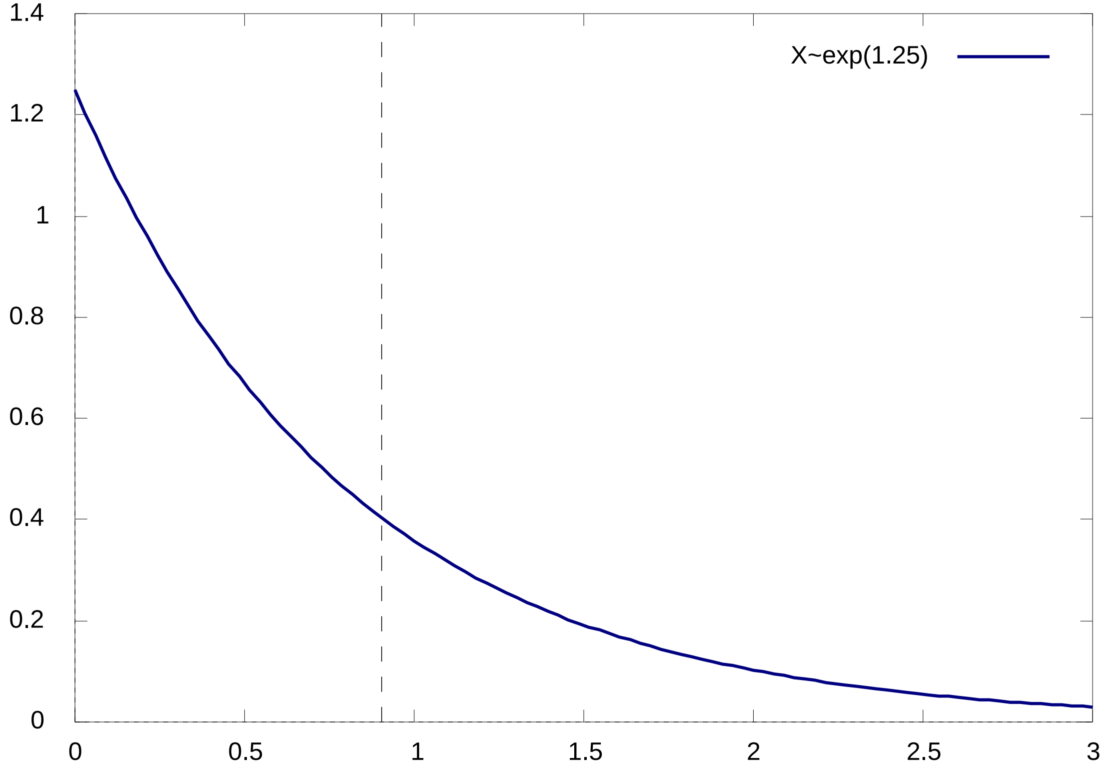

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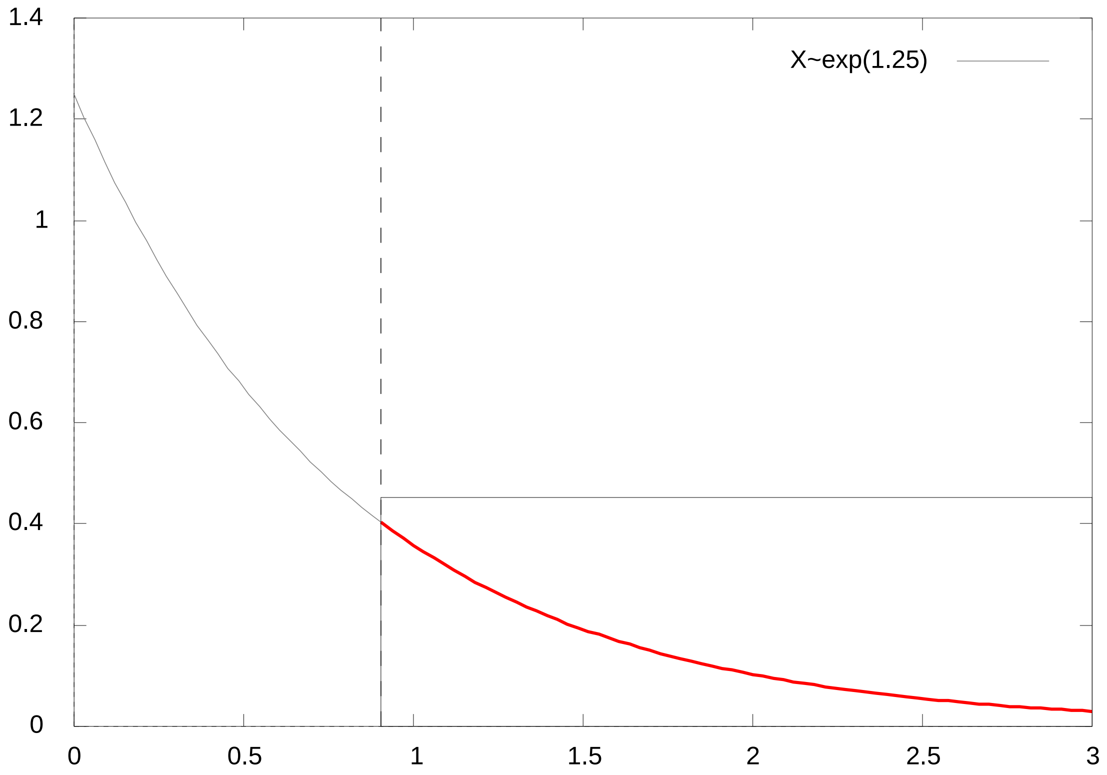

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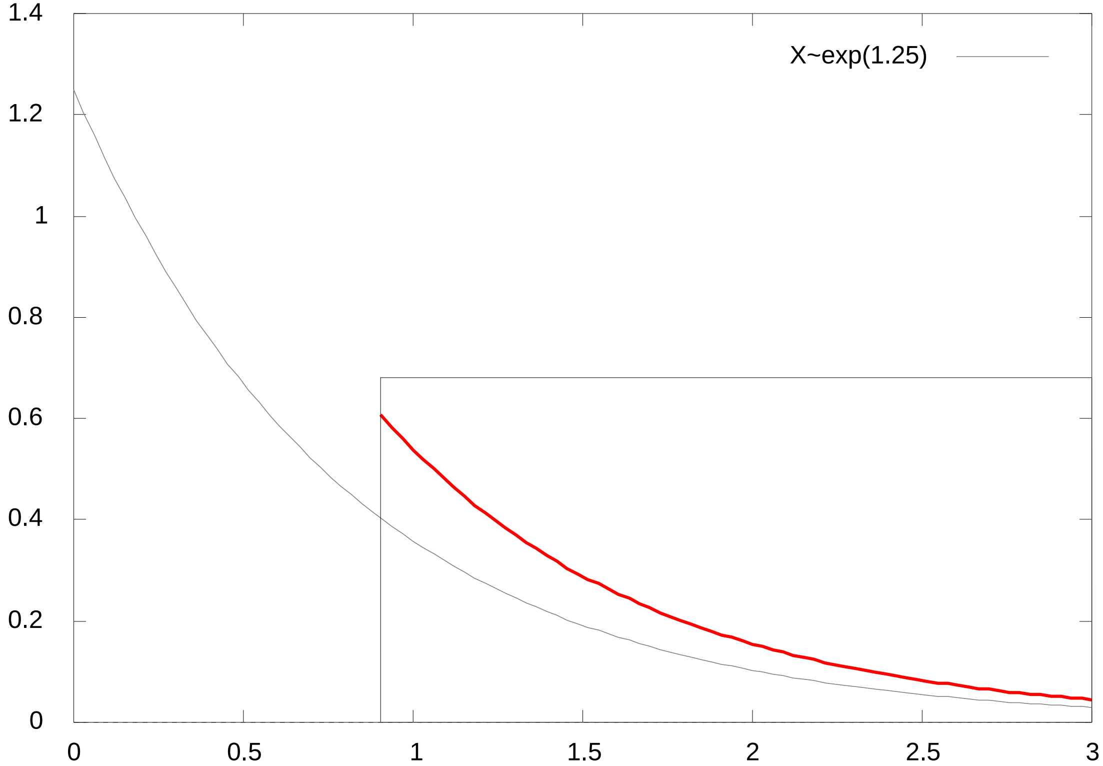

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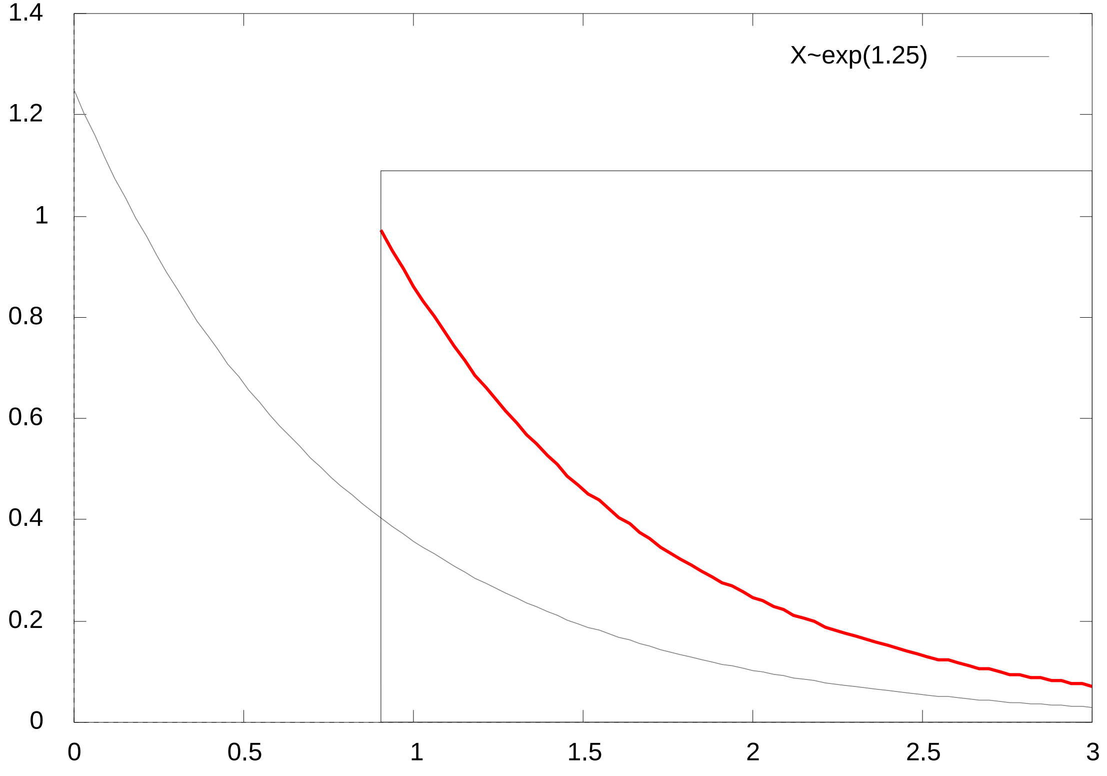

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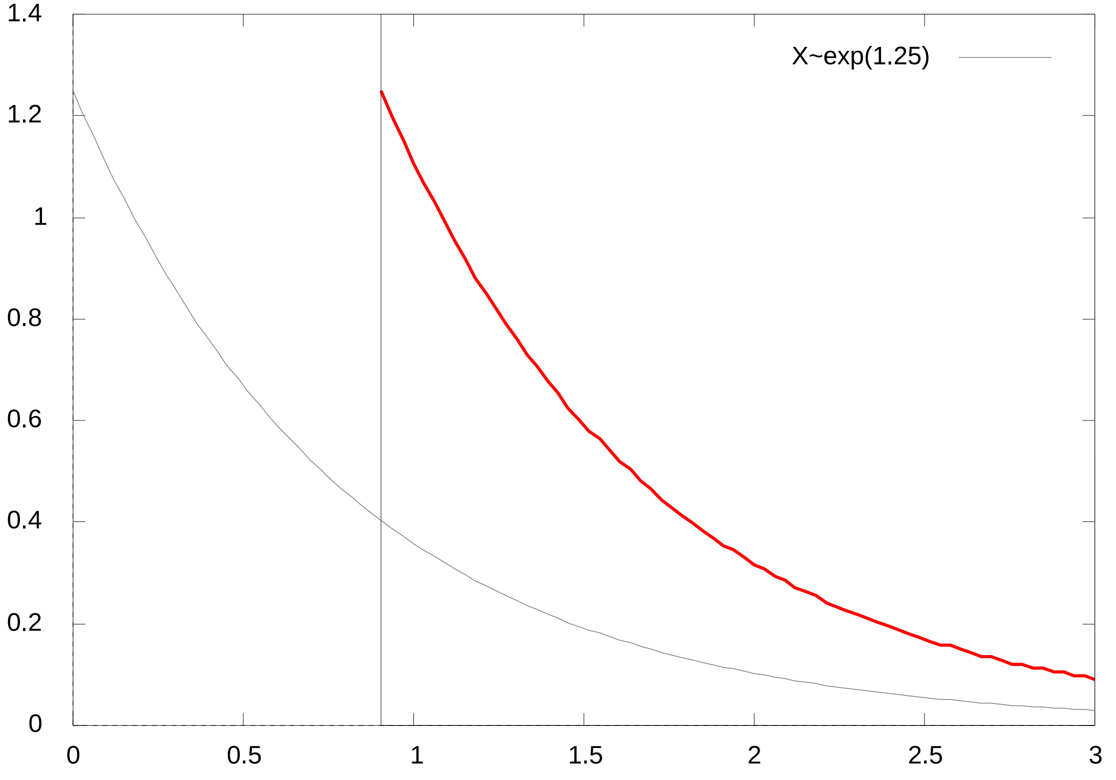

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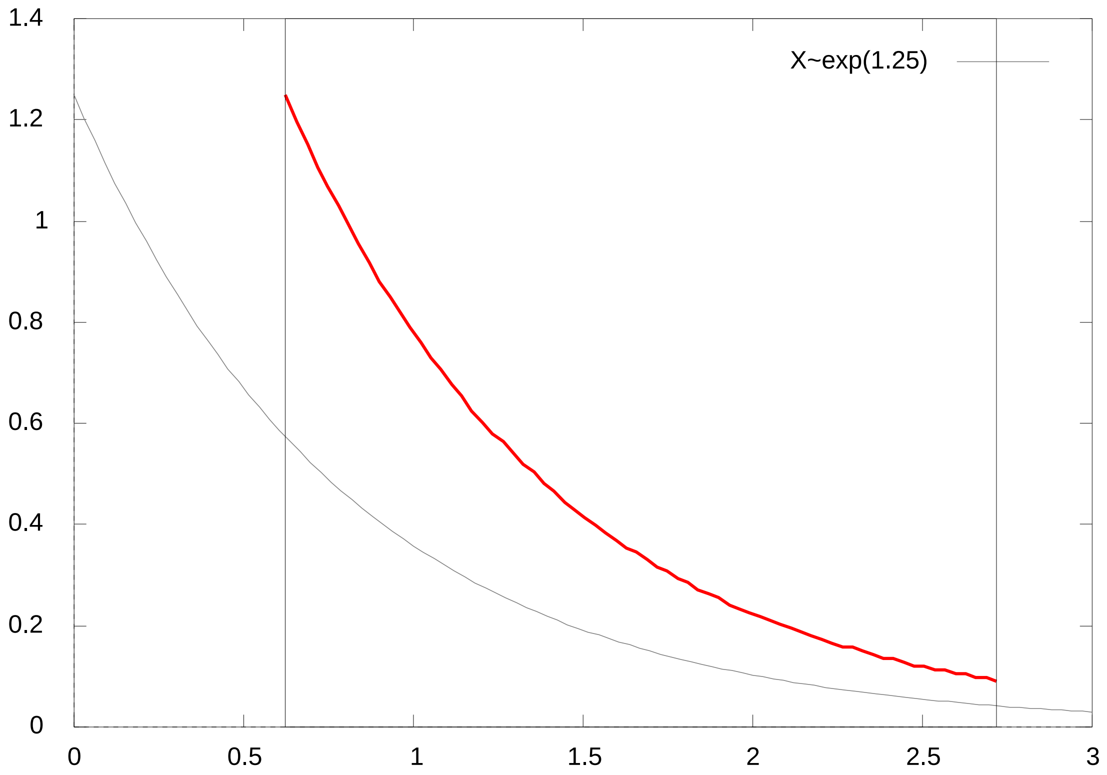

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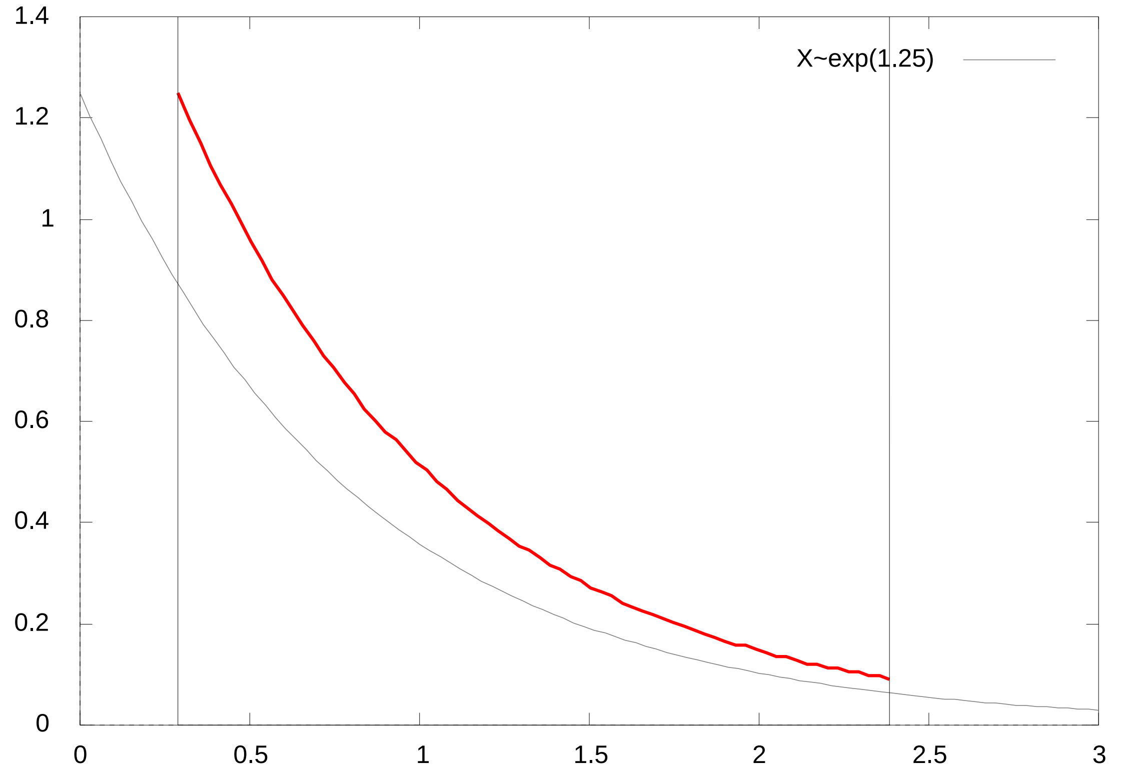

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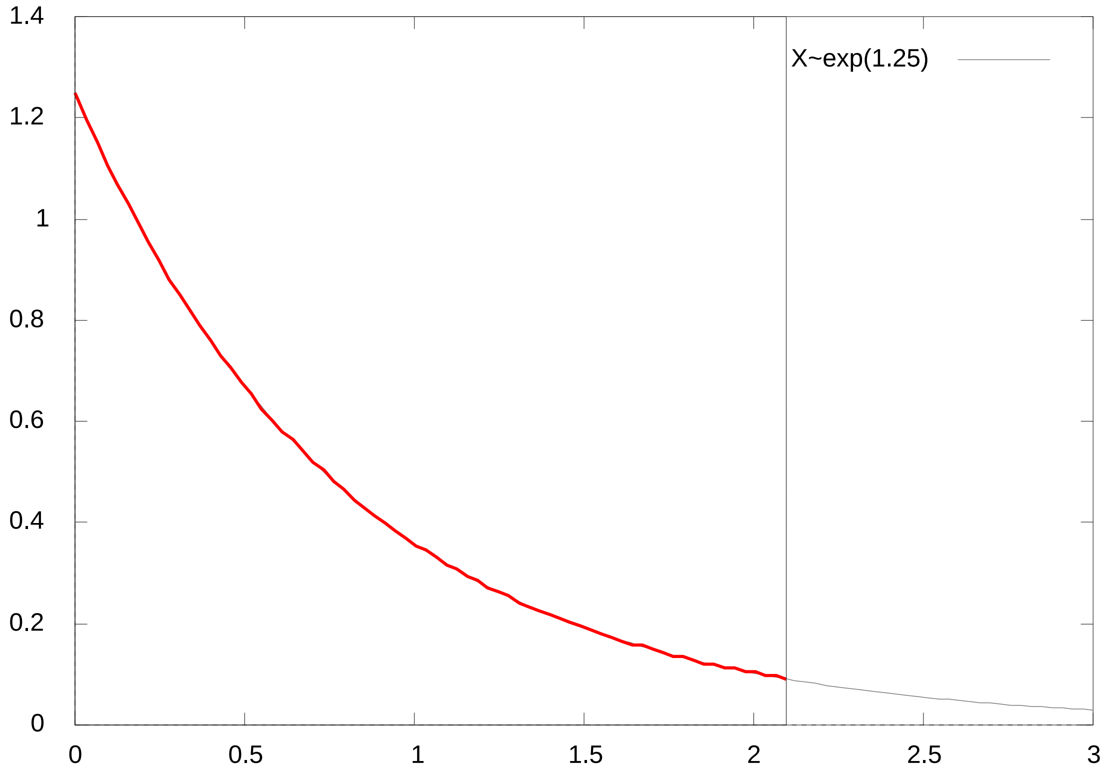


Semi-Markov Processes

- Semi-Markov processes use arbitrary distributions
- But no support for competitive choice or concurrent execution
- So what use are semi-Markov processes as an underlying formalism for PEPA?

PEPA and SMPs?

Two motivations:

1. Systems with Markovian concurrency that have areas of mutual exclusion
2. Fully generally distributed concurrent design, but run on a single threaded architecture

Semi-Markov PEPA

- Syntax:

$$P \ ::= \ (a, D).P \,\big|\, P + P \,\big|\, P \underset{S}{\bowtie} P \,\big|\, P/L \,\big|\, A$$

$$D \ ::= \ \lambda \,\big|\, \omega : L(s)$$

- new prefix operator: $(a, D).P$
  - λ: normal exponential rate parameter
  - $\omega : L(s)$: a selection weight, ω, and a general distribution description, $L(s)$

Semi-Markov Example I

- Mutual exclusion modelling:

$$\begin{aligned} A &\stackrel{\text{def}}{=} (\text{think}, \lambda_1).(\text{recover}, \lambda_2).A \\ &\quad + (\text{error}, \lambda_3).(\text{mutex}, 1 : L_1(s)).A \end{aligned}$$

$$S_n \stackrel{\text{def}}{=} \underbrace{A \underset{\emptyset}{\bowtie} A \underset{\emptyset}{\bowtie} \cdots \underset{\emptyset}{\bowtie} A}_{n}$$

- Areas of Markovian concurrency interspersed with semi-Markov sequential behaviour

Semi-Markov Example II

- Web-server/database model:

$$
\begin{aligned}
Server &\stackrel{\text{def}}{=} (\mathsf{get}, 5 : exp(1.5, s))).Server_1 \\
Server_1 &\stackrel{\text{def}}{=} (\mathsf{static\_page}, 1 : det(3, s)).Server \\
&\quad + (\mathsf{dbase\_fetch}, 2 : gamma(2.2, 3.2, s)).Server_2 \\
Server_2 &\stackrel{\text{def}}{=} (\mathsf{dbase\_rtn}, \top).(\mathsf{dynamic\_page}, 1 : uniform(2, 5, s)).Server \\
Dbase &\stackrel{\text{def}}{=} (\mathsf{dbase\_fetch}, \top).(\mathsf{dbase\_rtn}, 4 : exp(2.3, s)).Dbase \\
Sys &\stackrel{\text{def}}{=} Server \underset{\{\mathsf{dbase\_fetch},\mathsf{dbase\_rtn}\}}{\bowtie} Dbase
\end{aligned}
$$

- Concurrent design. Single-threaded architecture with weighted process selection

Some Conclusions

- Proposal for semi-Markov PEPA

- Incorporates PEPA functionality as a subset

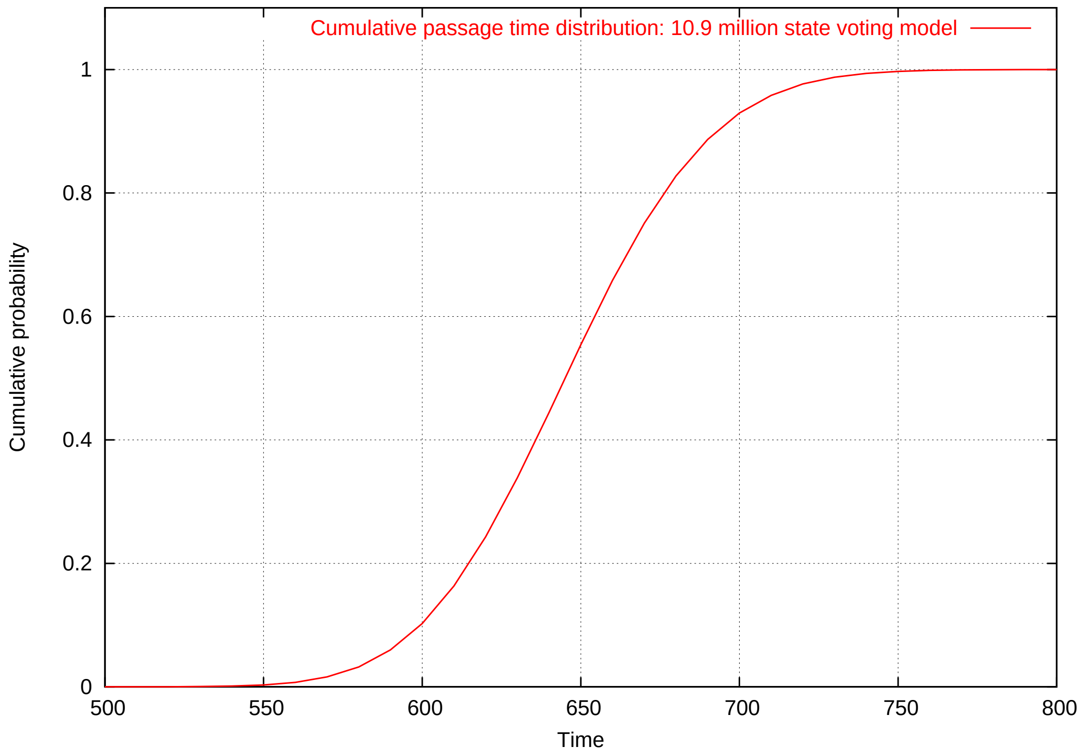
- Has 2 genuine application areas

Tool Support

- ipc: PEPA to DNAmaca

- SM-SPN DNAmaca
  - Transient distributions
  - Passage-time distributions

Semi-Markov Passage-time



Semi-Markov Passage-time