



One hundred years of the PEPA tools

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12th June 2003



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PEPA is ten years old!



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One year programming in ML \equiv ten years programming in Java



Background

- Performance Evaluation Process Algebra (PEPA) is used as a formal description language for Markov chain modelling. PEPA is a compact language with a small number of primitive operations.

Prefix: $(\alpha, r).P$ performs α at rate r to become P .

Choice: $P + Q$ sets up a race between P and Q . The first to perform an action wins: the other is discarded.

Cooperation: $P \bowtie_L Q$ runs P and Q in parallel, synchronising on activities in L .

Hiding: P/L hides the activities in L , preventing cooperands from synchronising on them.



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A first PEPA tool: The PEPA Workbench

- Our first PEPA tool was the PEPA Workbench, implemented in Standard ML.
- The PEPA syntax can be represented simply as an ML datatype.

```
datatype Component =
  PREFIX of (Activity * Rate) * Component      (* . *)
| CHOICE of Component * Component             (* + *)
| COOP of Component * Component * Activity list (* ⊗ *)
| HIDING of Component * Activity list         (* / *)
| VAR of Identifier                           (* X *)
| DEF of Identifier * Component * Component    (* def = *)
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The PEPA Workbench: derivatives

```
fun derivative E (PREFIX (a as (alpha, rate), P)) = [(a, P)]
| derivative E (CHOICE (P, Q)) =
    (derivative E P) @ (derivative E Q)
| derivative E (COOP (P, Q, L)) =
    let
        val (dP, dQ) = (derivative E P, derivative E Q)
        val (fP, fQ) = (filterout dP L, filterout dQ L)
    in
        (map (fn (a, P') => (a, COOP (P', Q, L))) fP)
        @ (map (fn (a, Q') => (a, COOP (P, Q', L))) fQ)
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| derivative E (HIDING (P, L)) = ...
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Beyond ML

- With the ML edition of the PEPA Workbench it was possible to solve small models using exterior solvers such as Maple and Matlab.
- However, users of the workbench wanted to make more detailed models (with larger state spaces).
- The ML edition of the PEPA Workbench could not solve Robert Holton's robotic workcell model efficiently enough so we interfaced it with an external solver written in C.
- Other users wanted to run the workbench on Solaris, Windows and Linux machines so we ported the Workbench and the solver to Java.



The PEPA Workbench [Java edition, 21-1-2003]

Status	Complete
54 P14 <{reg14}> S14' <{reg1	
55 P15 <{reg14}> S14' <{reg1	
56 P16 <{reg14}> S14' <{reg1	
57 P14 <{reg14}> S14' <{reg1	
58 P15 <{reg14}> S14' <{reg1	
59 P15 <{reg14}> S14 <{reg1	
60 P14 <{reg14}> S14 <{reg16, rep14}> DB15 <{rep16}> S16' <{reg15, rep15}> S15	
61 P14 <{reg14}> S14 <{reg16, rep14}> DB16 <{rep16}> S16 <{reg15, rep15}> S15'	
62 P15 <{reg14}> S14 <{reg16, rep14}> DB16 <{rep16}> S16' <{reg15, rep15}> S15'	
63 P14 <{reg14}> S14 <{reg16, rep14}> DB16 <{rep16}> S16' <{reg15, rep15}> S15	
64 P15 <{reg14}> S14' <{reg16, rep14}> DB15 <{rep16}> S16' <{reg15, rep15}> S15'	
65 P14 <{reg14}> S14' <{reg16, rep14}> DB15 <{rep16}> S16' <{reg15, rep15}> S15	
66 P14 <{reg14}> S14' <{reg16, rep14}> DB16 <{rep16}> S16 <{reg15, rep15}> S15'	
67 P15 <{reg14}> S14' <{reg16, rep14}> DB16 <{rep16}> S16' <{reg15, rep15}> S15'	
68 P14 <{reg14}> S14' <{reg16, rep14}> DB16 <{rep16}> S16' <{reg15, rep15}> S15	
69 P14 <{reg14}> S14 <{reg16, rep14}> DB15 <{rep16}> S16' <{reg15, rep15}> S15'	
70 P14 <{reg14}> S14 <{reg16, rep14}> DB16 <{rep16}> S16' <{reg15, rep15}> S15'	
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72 P14 <{reg14}> S14' <{reg16, rep14}> DB16 <{rep16}> S16' <{reg15, rep15}> S15'	

Solve for steady state solution
Set steady state solver parameters
Solve for transient solution
Set transient solver parameters
Solve via successive over-relaxation
Set SOR solver parameters

Stop

States found	72	Transitions found	240
Number of iterations		Error value	



Beyond the PEPA Workbench

- Graham Clark had extended the ML edition of the workbench and developed the Java edition of the workbench from his Peparoni simulator for PEPA.
- He then implemented an editor for PEPA in the Möbius multi-paradigm modelling framework, extending PEPA to $PEPA_k$ with guards and parameters.

```
Consume [a,b] = [a > 0] => (outa, ar).Consume [a-1,b]
+ [(b > 0) && (a == 0)] => (outb, br).Consume [a,b-1];
```

```
Breakdown = (outa, T).Breakdown
+ (fail,fr).(recover,RecoverRate).Breakdown ;
```

```
System = Consume [0,0] <outa> Breakdown;
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```
MultiProcessorExample
```

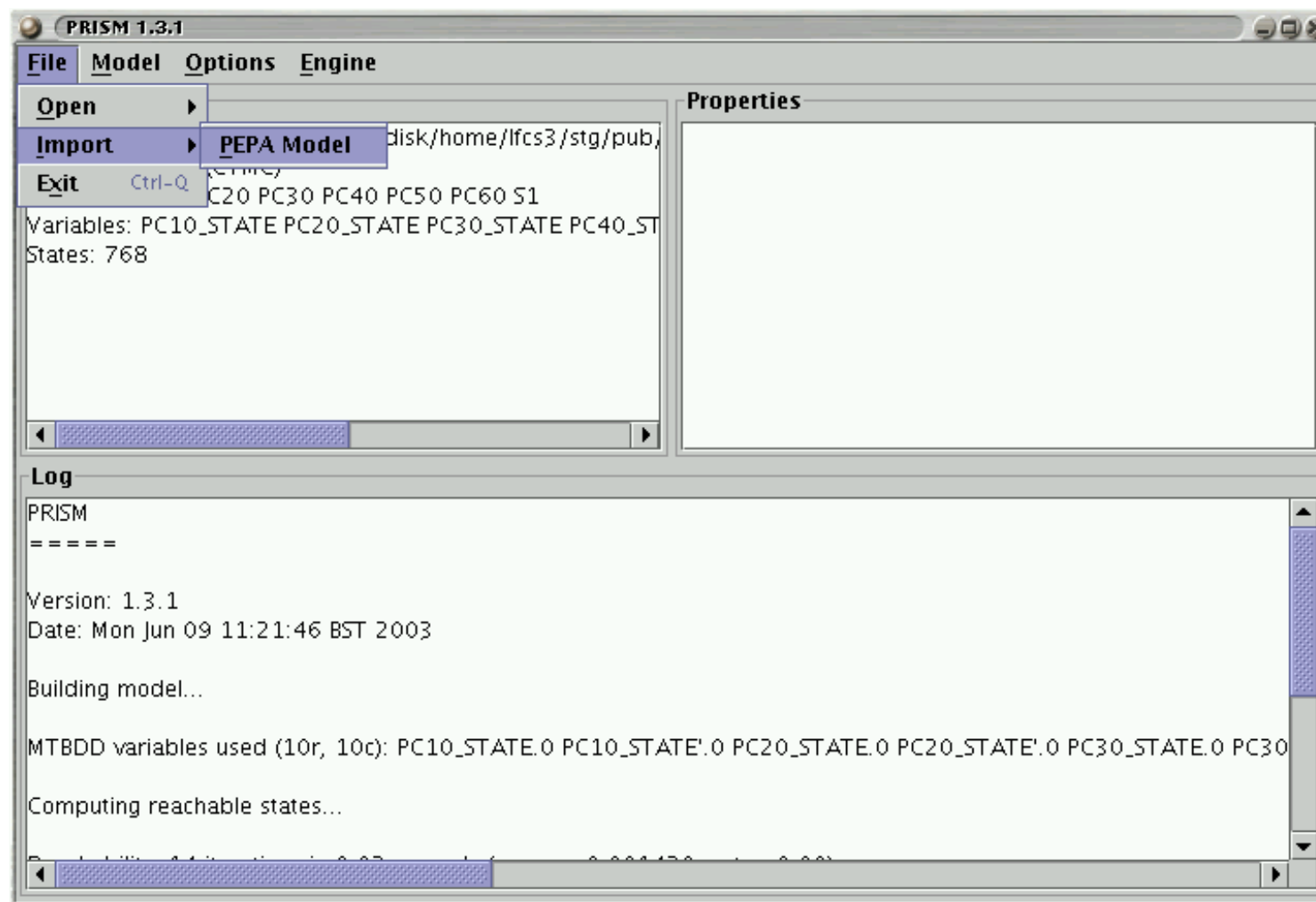
```
Mem1 := (getM1,-).(relM1,-).Mem1;  
Mem2 := (getM2,-).(relM2,-).Mem2;  
  
Bus := (getM1,g1).(relM1,r).Bus;  
      + (getM2,g2).(relM2,r).Bus;  
  
Proc := (getM1,-).(use,u1).(relM1,-).(update,p1).(think,t).Proc;  
       + (getM2,-).(use,u2).(relM2,-).(update,p2).(think,t).Proc;  
  
S := {getM1, getM2, relM1, relM2};  
System := (Proc | Proc | Proc) <S> Bus <S> (Mem1 | Mem2) |
```

UltraSAN/Möbius PEPA Editor 1.0 alpha
MultiProcessorExample Version Number: 1



PEPA and PRISM

- PRISM is a probabilistic model checker which supports modelling in DTMCs, CTMCs and MDPs with PTCL and CSL model checking.
- The matrix storing the state space of the system is expressed as an MTBDD built using the CUDD package.
- Support for the PEPA language in PRISM was provided in two steps:
 1. extending the PRISM input language with a new **system** construct providing the PEPA composition operators for synchronisation over activity sets and hiding; and
 2. compiling the PEPA language into the extended PRISM language.





PEPA modelling with PRISM

- PEPA modelling with PRISM has proved to be very effective in practice. The largest PEPA model so far solved has been solved with PRISM.
- However, there are a number of places where the user needs to understand the tool chain thoroughly:
 - The PEPA-to-PRISM compiler rejects (valid) PEPA models which use active/active synchronisation or anonymous components;
 - The compiler can fail during compilation with Java stack overflow;
 - PRISM can reject models which the PEPA-to-PRISM compiler outputs;
 - The CUDD package can fail with out-of-memory errors and need to be reconfigured.



PEPA and IPC/Dnamaca

The Imperial PEPA compiler (IPC) compiles PEPA models into Petri nets which are solved with the Dnamaca solver. Dnamaca provides a number of numerical solvers and outperforms PRISM on small PEPA models.

```
P1 = (start, r1).P2; =  
    \transition{P1_start} {  
        %% PEPA action type { start }  
        \condition{ P1 > 0 }  
        \action {  
            next -> P1 = P1 - 1;  
            next -> P2 = P2 + 1;  
        }  
        \priority{1}  
        \rate{ PEPA_r1 }  
    }
```



Synchronisation in Dnamaca

Suppose that two copies of P synchronise on the **run** activity.

$P2 = (\text{run}, r2).P3; =$

```

\transition{P2_run__P2_1_run} {
  %% PEPA action type { run }
  \condition{ P2 > 0 && P2_1 > 0 }
  \action {
    next -> P2_1 = P2_1 - 1;
    next -> P3_1 = P3_1 + 1;
    next -> P2 = P2 - 1;
    next -> P3 = P3 + 1;
  }
  \priority{1}
  \rate{ PEPA_r2 }
}

```



PEPA modelling with IPC and Dnamaca

- More of the PEPA language is supported by IPC/Dnamaca than by PRISM. Active/active synchronisation and anonymous components are supported.
- However, there are still a number of places where the user needs to understand the tool chain thoroughly:
 - The IPC compiler can fail during compilation with Haskell memory exhaustion;
 - Dnamaca can reject models which IPC outputs; and
 - Dnamaca's numerical procedures can fail to converge.



Dnamaca features and PEPA extensions

- Because Dnamaca supports non-Markovian modelling, beyond the models which are expressible in PEPA, it would be possible to support PEPA extensions with Dnamaca:
 - PEPA_k guards and parameters; *[Clark, Sanders, '01]*
 - Weighted (WSCCS-style) PEPA; *[Bradley, '02]*
 - PEPA nets with priorities; *[Gilmore, Hillston, Ribaud, Kloul, '03]*
 - Semi-Markov PEPA; *[Bradley, '03]*
 -



PEPA nets

- PEPA nets are Petri nets with PEPA tokens. An example token is

$$\begin{aligned} \text{Agent} &\stackrel{\text{def}}{=} (\mathbf{go}, \lambda). \text{Agent}' \\ \text{Agent}' &\stackrel{\text{def}}{=} (\text{interrogate}, r_i). \text{Agent}'' \\ \text{Agent}'' &\stackrel{\text{def}}{=} (\mathbf{return}, \mu). \text{Agent}''' \\ \text{Agent}''' &\stackrel{\text{def}}{=} (\text{dump}, r_d). \text{Agent} \end{aligned}$$

go and **return** are *firings* of the PEPA net. **interrogate** and **dump** are local transitions.

- A PEPA net can be processed with the PEPA Workbench for PEPA nets or compiled to PEPA using the PEPA net compiler.



The PEPA nets compiler

- The PEPA nets compiler compiles out token movement.

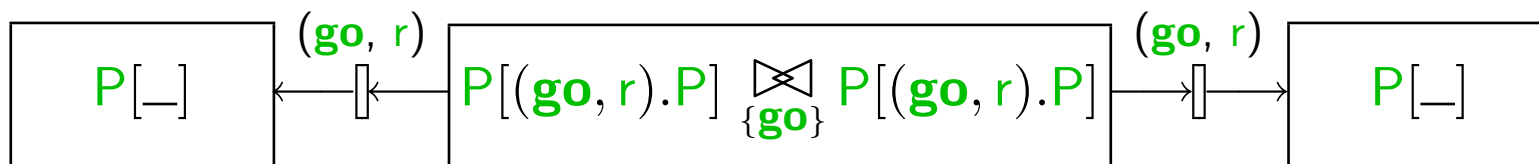
$$\begin{aligned}
 \text{Agent_at_P2} &\stackrel{\text{def}}{=} (\text{go_to_P1}, \lambda). \text{Agent}'_at_P1 \\
 &\quad + (\text{go_to_P3}, \lambda). \text{Agent}'_at_P3 \\
 \text{Agent}'_at_P1 &\stackrel{\text{def}}{=} (\text{interrogate_at_P1}, r_i). \text{Agent}''_at_P1 \\
 \text{Agent}'_at_P3 &\stackrel{\text{def}}{=} (\text{interrogate_at_P3}, r_i). \text{Agent}''_at_P3 \\
 \text{Agent}''_at_P1 &\stackrel{\text{def}}{=} (\text{return_to_P2}, \mu). \text{Agent}'''_at_P2 \\
 \text{Agent}''_at_P3 &\stackrel{\text{def}}{=} (\text{return_to_P2}, \mu). \text{Agent}'''_at_P2 \\
 \text{Agent}'''_at_P2 &\stackrel{\text{def}}{=} (\text{dump_at_P2}, r_d). \text{Agent_at_P2}
 \end{aligned}$$

- (Strictly speaking, tokens must specify their cells within places. Different cells at the same place fall under different synchronisation sets.)



Loss of expressivity

- In PEPA nets it is possible for tokens to synchronise on their exit actions from a place:

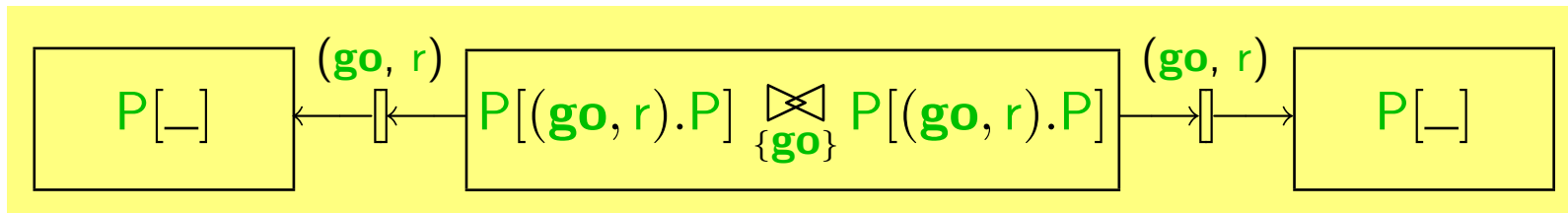


- The PEPA nets compiler cannot compile this idiom because the two tokens must go to different cells—cells can only contain a single token—and their exit activities must specify the destination cell. Therefore these activity renamings are distinct and so synchronisation is not possible.
- We consider this to be a small loss of expressivity.



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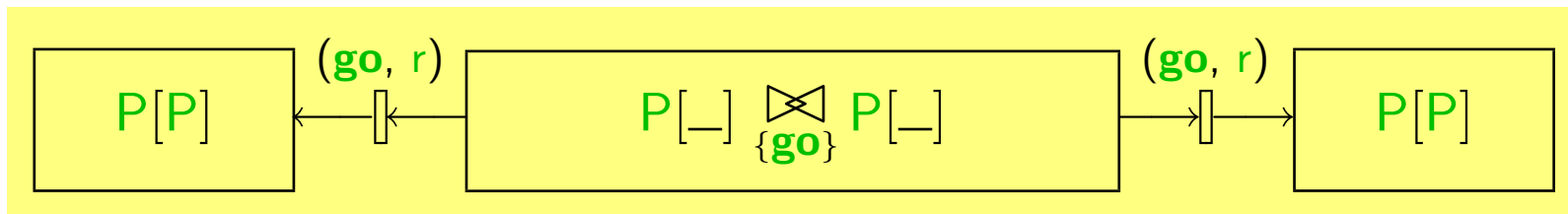


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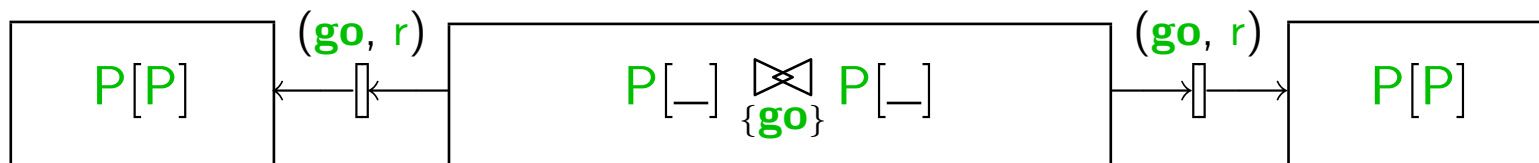


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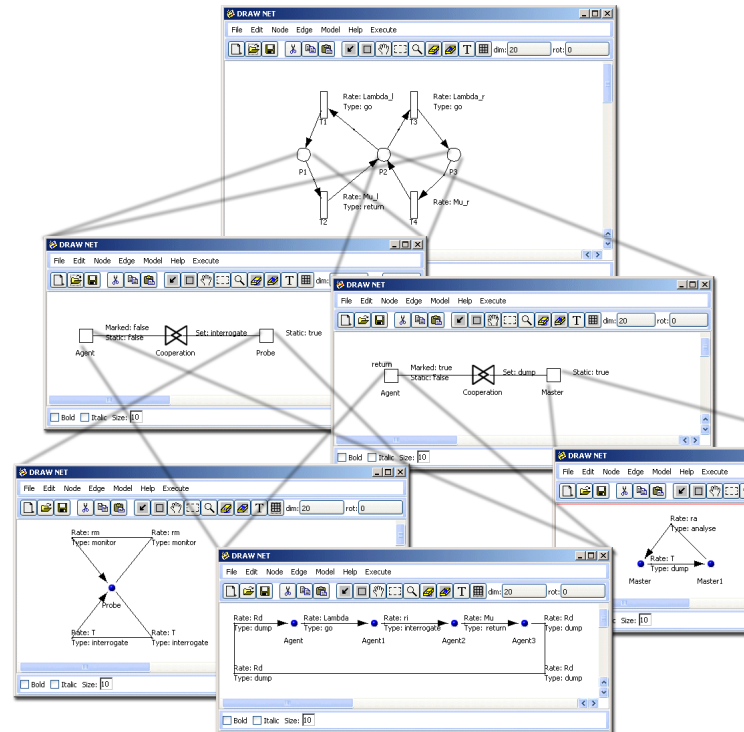
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PEPA nets in DrawNET





Conclusions

- Compiling PEPA and PEPA net models to other formalisms seems to be a very profitable activity.
- However, there are typically many small details in the translation which need to be taken care of.
- It is tempting to lift features of the host tool back to the PEPA level but sometimes desirable properties of the PEPA language are lost.
- It is important to strike a balance between exploiting opportunities and losing theoretical properties.