One hundred years of the PEPA tools

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12th June 2003
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PEPA is ten years old!
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One year programming in ML ≡ ten years programming in Java
Background

• Performance Evaluation Process Algebra (PEPA) is used as a formal description language for Markov chain modelling. PEPA is a compact language with a small number of primitive operations.

Prefix:  \((\alpha, r).P\) performs \(\alpha\) at rate \(r\) to become \(P\).

Choice:  \(P + Q\) sets up a race between \(P\) and \(Q\). The first to perform an action wins: the other is discarded.

Cooperation:  \(P \bowtie_L Q\) runs \(P\) and \(Q\) in parallel, synchronising on activities in \(L\).

Hiding:  \(P/L\) hides the activities in \(L\), preventing cooperands from synchronising on them.
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A first PEPA tool: The PEPA Workbench

- Our first PEPA tool was the PEPA Workbench, implemented in Standard ML.
- The PEPA syntax can be represented simply as an ML datatype.

```
datatype Component =
  | PREFIX of (Activity * Rate) * Component (* . *)
  | CHOICE of Component * Component (* + *)
  | COOP of Component * Component * Activity list (*PIPE*)
  | HIDING of Component * Activity list (* / *)
  | VAR of Identifier (* X *)
  | DEF of Identifier * Component * Component (* def *)
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```plaintext
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    | COOP of Component * Component * Activity list
    | HIDING of Component * Activity list
    | VAR of Identifier
    | DEF of Identifier * Component * Component
```

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PEPA tools

PASTA workshop, Edinburgh
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```
The PEPA Workbench: derivatives

fun derivative E (PREFIX (a as (alpha, rate), P)) = [(a, P)]
| derivative E (CHOICE (P, Q)) =
    (derivative E P) @ (derivative E Q)
| derivative E (COOP (P, Q, L)) =
    let
        val (dP, dQ) = (derivative E P, derivative E Q)
        val (fP, fQ) = (filterout dP L, filterout dQ L)
    in
        (map (fn (a, P’) => (a, COOP (P’, Q, L))) fP)
        @ (map (fn (a, Q’) => (a, COOP (P, Q’, L))) fQ)
        @ cooperations dP dQ L
    end
| derivative E (HIDING (P, L)) = ...
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  end
| derivative E (HIDING (P, L)) = ...
Beyond ML

- With the ML edition of the PEPA Workbench it was possible to solve small models using exterior solvers such as Maple and Matlab.

- However, users of the workbench wanted to make more detailed models (with larger state spaces).

- The ML edition of the PEPA Workbench could not solve Robert Holton’s robotic workcell model efficiently enough so we interfaced it with an external solver written in C.

- Other users wanted to run the workbench on Solaris, Windows and Linux machines so we ported the Workbench and the solver to Java.
### PEPA Workbench

**File** | **Options** | **Run** | **Show** | **Solver** | **Experiment** | **Simulate**
---|---|---|---|---|---|---
**Status** | Complete | **Solve for steady state solution** | **Set steady state solver parameters** | **Solve for transient solution** | **Set transient solver parameters** | **Solve via successive over-relaxation** | **Set SOR solver parameters**

**States found**: 72
**Transitions found**: 240
**Number of iterations**: 72
**Error value**: 240

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Beyond the PEPA Workbench

- Graham Clark had extended the ML edition of the workbench and developed the Java edition of the workbench from his Peparoni simulator for PEPA.

- He then implemented an editor for PEPA in the Möbius multi-paradigm modelling framework, extending PEPA to PEPA\(_k\) with guards and parameters.

\[
\text{Consume}[a, b] = \begin{cases} 
[a > 0] => (\text{outa}, \text{ar}).\text{Consume}[a-1, b] \\
+ [(b > 0) \&\& (a == 0)] => (\text{outb}, \text{br}).\text{Consume}[a, b-1]; 
\end{cases}
\]

\[
\text{Breakdown} = (\text{outa}, \text{T}).\text{Breakdown} \\
+ (\text{fail}, \text{fr}).(\text{recover}, \text{RecoverRate}).\text{Breakdown};
\]

\[
\text{System} = \text{Consume}[0, 0] <\text{outa}> \text{Breakdown};
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\]

\[
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\]
DEM0: MultiProcessorExample

MEM1 := (getM1,-).(relM1,-).MEM1;
MEM2 := (getM2,-).(relM2,-).MEM2;
BUS := (getM1,g1).(relM1,r).BUS;
  + (getM2,g2).(relM2,r).BUS;
PROC := (getM1,-).(use,u1).(relM1,-).(update,p1).(think,t).PROC;
  + (getM2,-).(use,u2).(relM2,-).(update,p2).(think,t).PROC;
S := (getM1, getM2, relM1, relM2);
SYSTEM := (PROC | PROC | PROC) <> BUS <> (MEM1 | MEM2)

UltraSAN/Mobius PEPA Editor 1.0 alpha

MultiProcessorExample Version Number: 1
PEPA and PRISM

- PRISM is a probabilistic model checker which supports modelling in DTMCs, CTMCs and MDPs with PTCL and CSL model checking.

- The matrix storing the state space of the system is expressed as an MTBDD built using the CUDD package.

- Support for the PEPA language in PRISM was provided in two steps:
  1. extending the PRISM input language with a new system construct providing the PEPA composition operators for synchronisation over activity sets and hiding; and
  2. compiling the PEPA language into the extended PRISM language.
PEPA modelling with PRISM

- PEPA modelling with PRISM has proved to be very effective in practice. The largest PEPA model so far solved has been solved with PRISM.

- However, there are a number of places where the user needs to understand the tool chain thoroughly:
  - The PEPA-to-PRISM compiler rejects (valid) PEPA models which use active/active synchronisation or anonymous components;
  - The compiler can fail during compilation with Java stack overflow;
  - PRISM can reject models which the PEPA-to-PRISM compiler outputs;
  - The CUDD package can fail with out-of-memory errors and need to be reconfigured.
PEPA and IPC/Dnamaca

The Imperial PEPA compiler (IPC) compiles PEPA models into Petri nets which are solved with the Dnamaca solver. Dnamaca provides a number of numerical solvers and outperforms PRISM on small PEPA models.

\begin{verbatim}
\transition{P1_start} {
  \%
  PEPA action type { start }
  \condition{ P1 > 0 }
  \action {
    next -> P1 = P1 - 1;
    next -> P2 = P2 + 1;
  }
  \priority{1}
  \rate{ PEPA_r1 }
}
\end{verbatim}

P1 = (start, r1).P2; =
Synchronisation in Dnamaca

Suppose that two copies of P synchronise on the run activity.

\begin{verbatim}
\transition{P2_run__P2_1_run} {
  \textbf{\% PEPA action type \{ run \}}
  \condition{ P2 > 0 && P2_1 > 0 }
  \action{
    \textbf{next} \rightarrow P2_1 = P2_1 - 1;
    \textbf{next} \rightarrow P3_1 = P3_1 + 1;
    \textbf{next} \rightarrow P2 = P2 - 1;
    \textbf{next} \rightarrow P3 = P3 + 1;
  }
  \priority{1}
  \rate{ PEPA_r2 }
}
\end{verbatim}

P2 = (run, r2).P3; =
PEPA modelling with IPC and Dnamaca

- More of the PEPA language is supported by IPC/Dnamaca than by PRISM. Active/active synchronisation and anonymous components are supported.

- However, there are still a number of places where the user needs to understand the tool chain thoroughly:
  - The IPC compiler can fail during compilation with Haskell memory exhaustion;
  - Dnamaca can reject models which IPC outputs; and
  - Dnamaca’s numerical procedures can fail to converge.
Dnamaca features and PEPA extensions

- Because Dnamaca supports non-Markovian modelling, beyond the models which are expressible in PEPA, it would be possible to support PEPA extensions with Dnamaca:
  - PEPA_{k} guards and parameters;  \([Clark, Sanders, '01]\)
  - Weighted (WSCCS-style) PEPA; \([Bradley, '02]\)
  - PEPA nets with priorities; \([Gilmore, Hillston, Ribaudo, Kloul, '03]\)
  - Semi-Markov PEPA; \([Bradley, '03]\)
  - . . .
PEPA nets

- PEPA nets are Petri nets with PEPA tokens. An example token is

\[
\begin{align*}
\text{Agent} & \overset{\text{def}}{=} (\text{go}, \lambda).\text{Agent}' \\
\text{Agent}' & \overset{\text{def}}{=} (\text{interrogate}, r_i).\text{Agent}'' \\
\text{Agent}'' & \overset{\text{def}}{=} (\text{return}, \mu).\text{Agent}''' \\
\text{Agent}''' & \overset{\text{def}}{=} (\text{dump}, r_d).\text{Agent}
\end{align*}
\]

\text{go} and \text{return} are \textbf{firings} of the PEPA net. \text{interrogate} and \text{dump} are local transitions.

- A PEPA net can be processed with the PEPA Workbench for PEPA nets or compiled to PEPA using the PEPA net compiler.
The PEPA nets compiler

- The PEPA nets compiler compiles out token movement.

\[
\begin{align*}
\text{Agent\_at\_P2} & \triangleq (\text{go\_to\_P1, } \lambda).\text{Agent\_at\_P1} \\
& \quad + (\text{go\_to\_P3, } \lambda).\text{Agent\_at\_P3} \\
\text{Agent\_at\_P1} & \triangleq (\text{interrogate\_at\_P1, } r_i).\text{Agent\_at\_P1} \\
\text{Agent\_at\_P3} & \triangleq (\text{interrogate\_at\_P3, } r_i).\text{Agent\_at\_P3} \\
\text{Agent\_at\_P1} & \triangleq (\text{return\_to\_P2, } \mu).\text{Agent\_at\_P2} \\
\text{Agent\_at\_P3} & \triangleq (\text{return\_to\_P2, } \mu).\text{Agent\_at\_P2} \\
\text{Agent\_at\_P2} & \triangleq (\text{dump\_at\_P2, } r_d).\text{Agent\_at\_P2}
\end{align*}
\]

- (Strictly speaking, tokens must specify their cells within places. Different cells at the same place fall under different synchronisation sets.)
Loss of expressivity

- In PEPA nets it is possible for tokens to synchronise on their exit actions from a place:

- The PEPA nets compiler cannot compile this idiom because the two tokens must go to different cells—cells can only contain a single token—and their exit activities must specify the destination cell. Therefore these activity renamings are distinct and so synchronisation is not possible.

- We consider this to be a small loss of expressivity.
Loss of expressivity

- In PEPA nets it is possible for tokens to synchronise on their exit actions from a place:

\[
P[\_] \xrightarrow{(\text{go, } r)} P[(\text{go, } r).P] \xrightarrow{\{\text{go}\}} P[(\text{go, } r).P] \xrightarrow{(\text{go, } r)} P[\_]
\]

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Loss of expressivity

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```
[ P \rightarrow \{ go \} \rightarrow P \leftarrow ]
```

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PEPA nets in DrawNET
Conclusions

• Compiling PEPA and PEPA net models to other formalisms seems to be a very profitable activity.

• However, there are typically many small details in the translation which need to be taken care of.

• It is tempting to lift features of the host tool back to the PEPA level but sometimes desirable properties of the PEPA language are lost.

• It is important to strike a balance between exploiting opportunities and losing theoretical properties.