

PEPA Queues

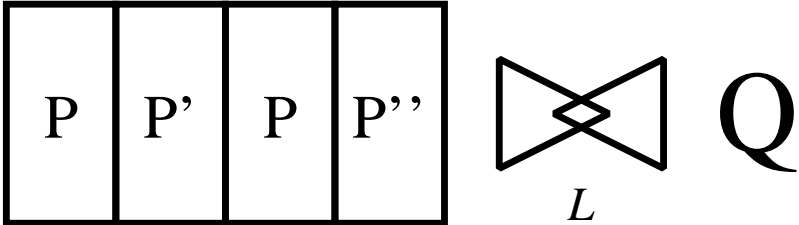
Ashok Argent-Katwala
Imperial College London

PASTA 2005

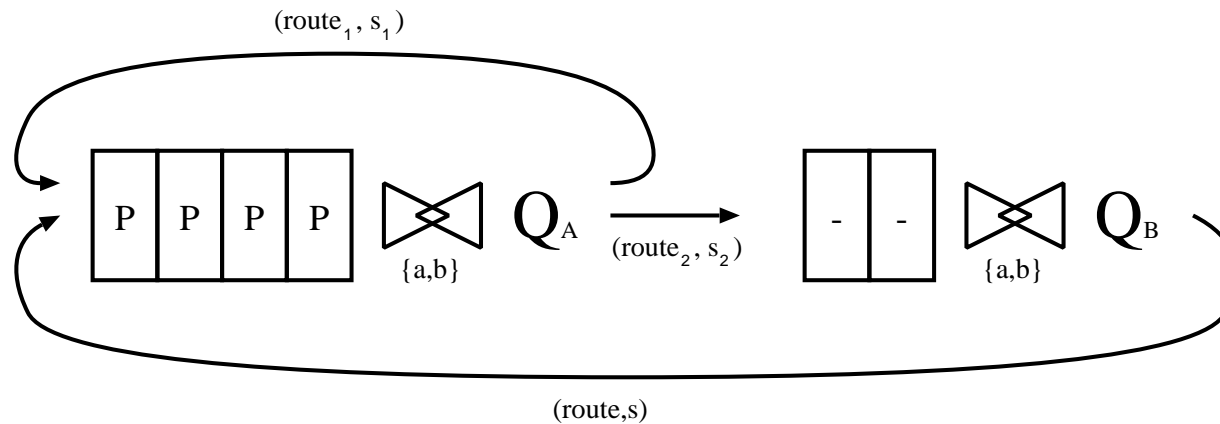
7–8 September

PEPA and PEPA nets

A PEPA Queue



Network of PEPA Queues



$$Q_A \stackrel{def}{=} (a, \top).Q_A + (b, \top).Q'_A$$

$$Q'_A \stackrel{def}{=} (b, \top).Q''_A$$

$$Q''_A \stackrel{def}{=} (route_1, \top).Q_A + (route_2, \top).Q'_A$$

$$Q_B \stackrel{def}{=} (a, \top).Q_B + (b, \top).Q_B + (hold, r).Q'_B$$

$$Q'_B \stackrel{def}{=} (reset, \top).Q_B + (route, \top).Q'_B$$

$$P \stackrel{def}{=} (a, r_1).P' + (b, r_2).P$$

$$P' \stackrel{def}{=} (b, r_2).P + (c, r_3).P$$

Translating to PEPA

- One component per queue to track the queue length.
- One copy of each customer (or customer derivative) per queue slot.
- Expand the actions to carry spatial information about where service leads to.
- Rewritten copies of the stationary queue components for the new actions

For example, the P at the head of queue A:

$$\begin{aligned} P_{A1} &= (a_{A,r_1}).P'_{A1} + (b_{A,r_2}).P_{A1} \\ &\quad + (route_2_sB0,top).P_{B1} + (route_2_sB1,top).P_{B2} \\ P'_{A1} &= (b_{A,r_2}).P_{A1} + (c,r_3).P_{A1} \\ &\quad + (route_2_sB0,top).P'_{B1} + (route_2_sB1,top).P'_{B2} \end{aligned}$$

$route_{2_sBi}$ is the variant of the $route_2$ action that is only enabled when there are i customers in queue B.

Putting them to use

Links

PEPAdb tools

<http://logicwand.com/pepadb/tools/>

Model from paper

```
-- PEPA layer
-- -- Stationary components
Q_A = (a,top).Q_A + (b,top).Q_A'
Q_A' = (b,top).Q_A''
Q_A'' = (route_1,top).Q_A + (route_2,top).Q_A'
Q_B = (a,top).Q_B + (b,top).Q_B + (hold,r).Q_B'
Q_B' = (reset,top).Q_B + (route,top).Q_B'

-- -- Customers
P = (a,r_1).P' + (b,r_2).P
P' = (b,r_2).P + (c,r_3).P

-- Queue layer
Q:A = (route_1,s_1).A + (route_2,s_2).B
Q:B = (route,s).A

-- Network layer
QNet:Sys = Q:A[P,P,P,P]<a,b>Q_A ; Q:B[_,_]<a,b>Q_B
```


Full translation

```
QA_pop0 = (route_sA0,top).QA_pop1
QA_pop1 = (route_sA1,top).QA_pop2
          + (route_1_sA1,top).QA_pop1
          + (route_2_sB0,top).QA_pop0
          + (route_2_sB1,top).QA_pop0
QA_pop2 = (route_sA2,top).QA_pop3
          + (route_1_sA2,top).QA_pop2
          + (route_2_sB0,top).QA_pop1
          + (route_2_sB1,top).QA_pop1
QA_pop3 = (route_sA3,top).QA_pop4
          + (route_1_sA3,top).QA_pop3
          + (route_2_sB0,top).QA_pop2
          + (route_2_sB1,top).QA_pop2
QA_pop4 = (route_2_sB0,top).QA_pop3
          + (route_2_sB1,top).QA_pop3
```

```
QB_pop0 = (route_2_sB0,top).QB_pop1
QB_pop1 = (route_2_sB0,top).QB_pop2
          + (route_sA0,top).QB_pop0
          + (route_sA1,top).QB_pop0
          + (route_sA2,top).QB_pop0
          + (route_sA3,top).QB_pop0
QB_pop2 = (route_sA0,top).QB_pop1
          + (route_sA1,top).QB_pop1
          + (route_sA2,top).QB_pop1
          + (route_sA3,top).QB_pop1
```

$$Q_A = (a_{A,top}).Q_A + (b_{A,top}).Q_{A'}$$

$$Q_A = (a_{A,top}).Q_A + (b_{A,top}).Q_{A'}$$

$$Q_{A'} = (b_{A,top}).Q_{A''}$$

$$\begin{aligned} Q_{A''} = & (route_1_sA1,top).Q_A \\ & + (route_1_sA2,top).Q_A \\ & + (route_1_sA3,top).Q_A \\ & + (route_1_sA4,top).Q_A \\ & + (route_2_sB0,top).Q_{A'} \\ & + (route_2_sB1,top).Q_{A'} \end{aligned}$$

$$Q_B = (a_{B,top}).Q_B + (b_{B,top}).Q_B + (hold,r).Q_{B'}$$

$$\begin{aligned} Q_{B'} = & (reset,top).Q_B \\ & + (route_sA0,top).Q_{B'} + (route_sA1,top).Q_{B'} \\ & + (route_sA2,top).Q_{B'} + (route_sA3,top).Q_{B'} \end{aligned}$$

```

P_A4 = (route_1_sA0,top).P_A3
      + (route_1_sA1,top).P_A3
      + (route_1_sA2,top).P_A3
      + (route_1_sA3,top).P_A3
      + (route_2_sB0,top).P_A3
      + (route_2_sB1,top).P_A3
P'_A4 = (c,r_3).P_A4
        + (route_1_sA0,top).P'_A3
        + (route_1_sA1,top).P'_A3
        + (route_1_sA2,top).P'_A3
        + (route_1_sA3,top).P'_A3
        + (route_2_sB0,top).P'_A3
        + (route_2_sB1,top).P'_A3

P_A3 = (route_1_sA0,top).P_A2
      + (route_1_sA1,top).P_A2
      + (route_1_sA2,top).P_A2
      + (route_1_sA3,top).P_A2
      + (route_2_sB0,top).P_A2
      + (route_2_sB1,top).P_A2
P'_A3 = (c,r_3).P_A3
        + (route_1_sA0,top).P'_A2
        + (route_1_sA1,top).P'_A2
        + (route_1_sA2,top).P'_A2
        + (route_1_sA3,top).P'_A2
        + (route_2_sB0,top).P'_A2
        + (route_2_sB1,top).P'_A2

```

```
P_A2 = (route_1_sA0,top).P_A1
      + (route_1_sA1,top).P_A1
      + (route_1_sA2,top).P_A1
      + (route_1_sA3,top).P_A1
      + (route_2_sB0,top).P_A1
      + (route_2_sB1,top).P_A1
P'_A2 = (c,r_3).P_A2
        + (route_1_sA0,top).P'_A1
        + (route_1_sA1,top).P'_A1
        + (route_1_sA2,top).P'_A1
        + (route_1_sA3,top).P'_A1
        + (route_2_sB0,top).P'_A1
        + (route_2_sB1,top).P'_A1
```

$$\begin{aligned}
P_B2 &= (\text{route_sA3,top}) \cdot P_B1 \\
&\quad + (\text{route_sA2,top}) \cdot P_B1 \\
&\quad + (\text{route_sA1,top}) \cdot P_B1 \\
&\quad + (\text{route_sA0,top}) \cdot P_B1 \\
P'_B2 &= (c,r_3) \cdot P_B2 + (\text{route_sA3,top}) \cdot P'_B1 \\
&\quad + (\text{route_sA2,top}) \cdot P'_B1 \\
&\quad + (\text{route_sA1,top}) \cdot P'_B1 \\
&\quad + (\text{route_sA0,top}) \cdot P'_B1 \\
\\
P_A1 &= (a_A,r_1) \cdot P'_A1 + (b_A,r_2) \cdot P_A1 \\
&\quad + (\text{route_2_sB0,top}) \cdot P_B1 \\
&\quad + (\text{route_2_sB1,top}) \cdot P_B2 \\
P'_A1 &= (b_A,r_2) \cdot P_A1 + (c,r_3) \cdot P_A1 \\
&\quad + (\text{route_2_sB0,top}) \cdot P'_B1 \\
&\quad + (\text{route_2_sB1,top}) \cdot P'_B2
\end{aligned}$$

$$\begin{aligned} P_{B1} &= (a_{B,r_1}) \cdot P'_{B1} + (b_{B,r_2}) \cdot P_{B1} \\ &\quad + (\text{route_sA0,top}) \cdot P_{A1} \\ &\quad + (\text{route_sA1,top}) \cdot P_{A2} \\ &\quad + (\text{route_sA2,top}) \cdot P_{A3} \\ &\quad + (\text{route_sA3,top}) \cdot P_{A4} \\ P'_{B1} &= (b_{B,r_2}) \cdot P_{B1} + (c,r_3) \cdot P_{B1} \\ &\quad + (\text{route_sA0,top}) \cdot P'_{A1} \\ &\quad + (\text{route_sA1,top}) \cdot P'_{A2} \\ &\quad + (\text{route_sA2,top}) \cdot P'_{A3} \\ &\quad + (\text{route_sA3,top}) \cdot P'_{A4} \end{aligned}$$