Probabilistic Model Checking for Games of imperfect information

P. Ballarini, M. Fisher, M. Wooldridge

University of Liverpool
What is this work about

Uncertainty is relevant for a specific class of Games (game of imperfect information)

Q: Can we apply probabilistic model checking for analysing games in which players’ behaviour is characterised by uncertainty?
Motivation

Model checking for Multi-Agent Systems (MAS)

LTL model checking of BDI MAS (Bordini)
  AgentSpeak $\rightarrow$ Promela (SPIN)
  AgentSpeak $\rightarrow$ Java (Java-PathFinder)

can we extend it to probabilistic model checking so that uncertain behaviour can be accounted for?

We need a new language for uncertain MAS (Probmela)

Can we use any existing Probabilistic Modelling Framework (PRISM?) to reason about uncertain MAS?
Outline

• Games, strategies, equilibria
  • strategic games, equilibria
  • extensive games (perfect/imperfect information)
    • Alternating offers negotiation game

• Markovian model of the Alternating offers game

• Analysis through Model Checking

• Conclusion
Strategic Games

the outcome of the game is achieved in one-shot

- set of players: $N=\{1,..,n\}$
- players actions: $A_i=\{a_1, a_2,\ldots, a_k\}$
- players preferences: a relation over outcome utilities $\succeq_i$

$$G = \langle N, (A_i), (\succeq_i) \rangle$$

an action profile is combination of actions: $a = (a^1, a^2, \ldots, a^n)$
the outcome of an action profile is denoted: $O(a^1, a^2, \ldots, a^n)$
Example - Battle of Sexes

- two people wish to go out together to a concert of music by either the “Red Hot Chili Peppers” or “Bach”

- their main concern is to go out together but one prefers the “Peppers” and the other one “Bach”

- individual’s preferences are represented by payoff functions

<table>
<thead>
<tr>
<th></th>
<th>Jane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peppers</td>
<td>(2,1)</td>
</tr>
<tr>
<td>Bach</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Stephen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peppers</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Bach</td>
<td>(1,2)</td>
</tr>
</tbody>
</table>
Example- Battle of Sexes

Stephen

<table>
<thead>
<tr>
<th></th>
<th>Jane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peppers</td>
<td>(2,1)</td>
</tr>
<tr>
<td>Bach</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Jane

<table>
<thead>
<tr>
<th></th>
<th>Stephen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peppers</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Bach</td>
<td>(1,2)</td>
</tr>
</tbody>
</table>

Stephen’s preferences

(Peppers,Peppers) \succeq_S (Bach,Bach) \succeq_S (Peppers,Bach) \sim_S (Bach,Peppers)

Jane’s preferences

(Bach,Bach) \succeq_J (Peppers,Peppers) \succeq_J (Peppers,Bach) \sim_J (Bach,Peppers)
Nash Equilibria

- A profile of actions is a Nash Equilibria iff no player has interest in adopting another strategy assuming the other player sticks to his one.

The Battle of Sexes has 2 Equilibria: (Peppers, Peppers), (Bach, Bach)

i.e. togetherness rules
Extensive Games

They are sequential strategic games (the decision problem is iterated over time)

- set of players $N = \{1, \ldots, n\}$
- set of histories $H$
- preferences over histories (rather than over action profiles)
- a player function: $P(h)$ is the player who takes an action of history $h$

$$G = \langle N, H, P, (\succeq_i) \rangle$$
Extensive Games as Trees

Ext. Game example: two people propose different allocations for 2 indivisible items
Perfect information: strategies

A strategy in an Ext. Game of perfect information is a function that assigns an action to each non-terminal history.

(Perf.Inf. assumption: players are completely informed on past actions)

Strategies examples

1- \( s_1(e) = (2, 0) \quad s_2((2, 0)) = y \quad s_2((1, 1)) = n \quad \rightarrow \quad (2, 0) \)

2- \( s_1(e) = (2, 0) \quad s_2((2, 0)) = n \quad s_2((1, 1)) = y \quad \rightarrow \quad (?) \)
perfect information: equilibria

A Nash Equilibria of an Ext. Game of perfect information is a strategy profile \( s = (s_1, s_2, \ldots, s_n) \) such that no player would get a better outcome by choosing a different strategy assuming all other players are sticking with their ones.

Formally: a profile \( s^* = (s_1^*, \ldots, s_n^*) \) is a Nash Equilibria iff

\[
O(s^*_i, s_i) \succeq_i O(s^*_i, s_i) \quad \text{for all strategy } s_i \text{ of player } i
\]

\[
O(s^*) : \text{outcome for } s^* = (s_1^*, \ldots, s_n^*)
\]
Alternating offers game
(Rubinstein)

two players aim to split a pie (or bargain over an item)

players alternatively propose agreements in the set:

\[ X = \{(x_1, x_2) | x_i \geq 0 \text{ and } x_1 + x_2 = 1\} \]

D: disagreement

players either accept (Y) or Reject (N) the most recent offer they receive
\[ t = 0 \]

\[ (x^0, 0) \]

\[ t = 1 \]

\[ (x^1, 1) \]
Alternating offers game
(Rubinstein)

formally an Alt. Offers Game is given by:

\[ G = \langle \{1, 2\}, X \cup \{D\}, (\succeq_i) \rangle \]

where preferences are time-dependent

\[ \succeq_i \text{ is defined over } (X \times T) \cup \{D\} \]

histories are of type

\[ (x^0, N, x^1, N \ldots, X^t) \quad \text{non-terminal} \]

\[ (x^0, N, x^1, N \ldots, X^t, Y) \quad \text{terminal} \]
Alternating offers preferences

\( \succeq_i \) must fulfills some “basic” constraints

i- disagreement is the worst possible outcome

\[(x \times t) \succeq_i D\]

ii- pie is desirable

\[(x \times t) \succ_i (y \times t) \iff x_i > y_i\]

iii- time is valuable

\[(x \times t) \succ_i (x \times s) \text{ if } t < s\]
Alternating offers: equilibria

Given an Alter. Offers game

\[ G = \langle \{1, 2\}, X \cup \{D\}, (\succeq_i) \rangle \]

PROPERTY: there are infinite Nash Equilibria
Equilibria example

strategy: players keep asking the whole pie until time $t=n$ then they ask $x^*$ and each player will accept only $x^*$
Preferences: more constraints

iv- stationarity

\[(x \times t) \succ_i (y \times t+1) \iff (x \times 0) \succ_i (y \times 1)\]

v- increasing loss to delay

\[x_i - v_i(x^i, 1) \text{ increasing function of } x_i\]
THEOREM: if $\succeq_i$ fulfils all constraints i-v then there exists a unique strategy profile $(\sigma^*, \delta^*)$ which is a Nash Equilibria

$(x_1^*, x_2^*)$ is depends on both $\succeq_1$ and $\succeq_2$
Imperfect information: strategies

Imperf.Inf. assumption: players may have only partial info on past actions.

as a result some actions are determined by chance

$$G = \langle N, H, P, f_c, (I_i)(\succeq_i) \rangle$$

$$P(h) = c$$ the next action for history $h$ is determined by the lottery $f_c(h)$

a strategy in an Ext. Game of imperfect information is a function that assign to each non-terminal history a lottery over possible actions

preferences are over (induced) lotteries on the set of terminal histories
Markovian model of Negotiation

Markov processes are suitable for modelling past-independent behaviours (hence imperfect-information games)

we consider the imperfect-information variant of the alternating offers game

which is: we assume players actions being state-dependent, rather than path-dependent
the imperfect-info alternating offer game can be naturally encoded as a DTMC (players decision is a lottery over the possible actions)
Markovian model of Negotiation

players’ strategies depend on 2 parameters

i) - the Offer proposal function $p_{a \rightarrow \hat{a}(t)}$

- $IP^b$ initial price proposed by player b
- $RP^b$ reserved price of player b
- $T^b$ time-deadline of player b

ii) - the Acceptance Probability function

- $S_{AP}(x)$ for the Seller
- $B_{AP}(x)$ for the Buyer
Offer Function families

Conceder: player concedes a lot in early stage of negotiation
Boulware: player concedes a lot only close to deadline

\[
p_{a\to \hat{a}}(t) = \begin{cases} 
    IP^a + \phi^a(t)(RP^a - IP^a) & \text{for } a = b \text{ buyer}, \\
    RP^a + (1 - \phi^a(t))(IP^a - RP^a) & \text{for } a = s \text{ seller}
\end{cases}
\]

\[
\phi^a(t) = k^a + (1 - k^a)(\frac{t}{T_a})^{\frac{1}{\nu}}
\]
Offer Function approximation

with the PRISM model-checker we are forced to use two-segments linear approximation of non-linear Offer Functions.
Acceptance Probability functions

\[ S_{AP}(x,t) = \begin{cases} 
0 & \text{if } (x \leq S_{RP}) \land (t < T^s) \\
1 - \frac{S_{RP}}{x} & \text{if } (x > S_{RP}) \land (t < T^s) \\
1 & \text{if } (t \geq T^s) 
\end{cases} \]

\[ B_{AP}(x,t) = \begin{cases} 
1 & \text{if } (x \leq 0) \lor (t \geq T^b) \\
1 + \frac{S_{RP}}{x-(B_{RP}+S_{RP})} & \text{if } (S_{RP} < x < B_{RP}) \land (t < T^b) \\
0 & \text{if } (x > B_{RP}) \land (t < T^b) 
\end{cases} \]
PCTL Model-Checking

probabilistic extension of CTL for referring to Discrete Time Markov Chains

\[ \phi ::= tt | a | \phi \land \phi | \neg \phi | P_{\leq p}(\varphi) \]

\[ \varphi ::= \phi U^I \varphi \]
PCTL Model-Checking

\[ \phi_1 \equiv P_{\geq 0.8}[\Diamond (agreed = 100)] \]

\[ \phi_x \equiv P_{\exists}[\Diamond (agreed = x)] \]
Model Verification

by verifying \( \phi_x \equiv P?\left[\diamond(\text{agreed} = x)\right] \)
we devise the distribution of probability over the set of possible agreements, hence the expected utility

by comparing a number of strategy profiles we devise how strategy parameters affect the expected outcome of negotiation
One (fairly trivial) indication

The less a player concedes the higher his expected utility is going to be
Conceder-Conceder

Seller Offer Function

Cumulative Acceptance Prob
Conclusion

we have shown that:

under certain assumption a game of imperfect information can be encoded into a discrete-time Markovian model

PCTL model-checking can be used to verify such a model

model-checking allows for comparing of strategy profiles

such an approach differ from both classical game-theory analysis and from simulative analysis

issue:

can we perform a deeper analysis through model-checking? how about Nash-Equilibria analysis through model-checking?