Density dependent transmission from process algebra models of disease spread

Introduction

- Traditional differential equation SIR models take into account population level behaviours. E.g. Kermack-McKendrick

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \alpha I$$

$$\frac{dR}{dt} = \alpha I$$

WSCCS models

- Consider individual behaviours
- Define individuals in terms of behaviours and interactions
- Build population as a number of individuals in parallel
- Sumpter developed heuristic method for deriving difference equations which describe the average behaviour

WSCCS models

```
bs S1 1.t:S2
bs I1 pr.t:R2 + pa.t:T2
   + (1-pr-pa).t:I2
bs R1 1.t:R2
bs S2 1@1.infect^1:I1 + 1.t:S1
bs I2 1@1.infect^1:I1 + 1.t:I1
bpa T2 I2|Trans
bs Trans 1@1.infect^-1:T + 1.t:T
bs R2 1@1.infect^1:R1 + 1.t:R1
basi L t
btr Population S1|S1|S1|I1/L
```

Deriving equations

- Applying algorithm to the model gives the system of equations

$$S_{t+1} = S_t - \frac{p_a S_t I_t}{S_t + I_t + R_t}$$

$$I_{t+1} = (1 - p_r) I_t + \frac{p_a S_t I_t}{S_t + I_t + R_t}$$

$$R_{t+1} = R_t + p_r I_t$$

Transmission terms

- Kermack-Mckendrick has transmission term βSI – density dependent transmission

- WSCCS model has transmission term of the form $\beta' SI/N$ – frequency dependent transmission

Transmission terms

Frequency Dependent

- Sexually transmitted diseases
- Vector borne diseases

Density Dependent

- Colds and flus
- Measles, mumps, rubella, chicken pox

Density dependent transmission

- Is it possible to produce a WSCCS model which would lead to the transmission term βSI ?
- Does such a model have realistic rules of behaviour?
- Does a realistic density dependent model lead to a transmission term which closely fits to βSI ? Or other suggested terms?

Alternative transmission terms

- Hochberg

$$Transmission = \beta\left(S^{p} I^{q}\right)SI$$

- Briggs and Godfray

$$Transmission = \left[k \ln\left(1 + \frac{\beta I}{k}\right)\right]S$$

Density dependent transmission

- To achieve density dependent transmission the contact rate must change with the density of the population

- Individuals must be able to make multiple contacts per timestep – several ways to achieve this

Density dependent transmission

Parallel agents

```
bs S1 1@1.infect^1:SI2 + 1.t:S2
bpa I1 T1|Trans|Trans|Trans
bs T1 1@1.infect^1:I2 + 1.t:I2
bs Trans 1@1.infect^-1:T + 1.t:T
bs R1 1@1.infect^1:R2 + 1.t:R2
```

Density dependent transmission

Parallel agents

$$S_{t+1} = (1 - p_d)S_t - \frac{mp_a S_t I_t}{S_t + I_t + R_t} + F$$

$$I_{t+1} = (1 - p_r - p_d)I_t + \frac{mp_a S_t I_t}{S_t + I_t + R_t}$$

$$R_{t+1} = (1 - p_d)R_t + p_r I_t$$

Density dependent transmission

- By choosing

$$m = k \times N_t$$

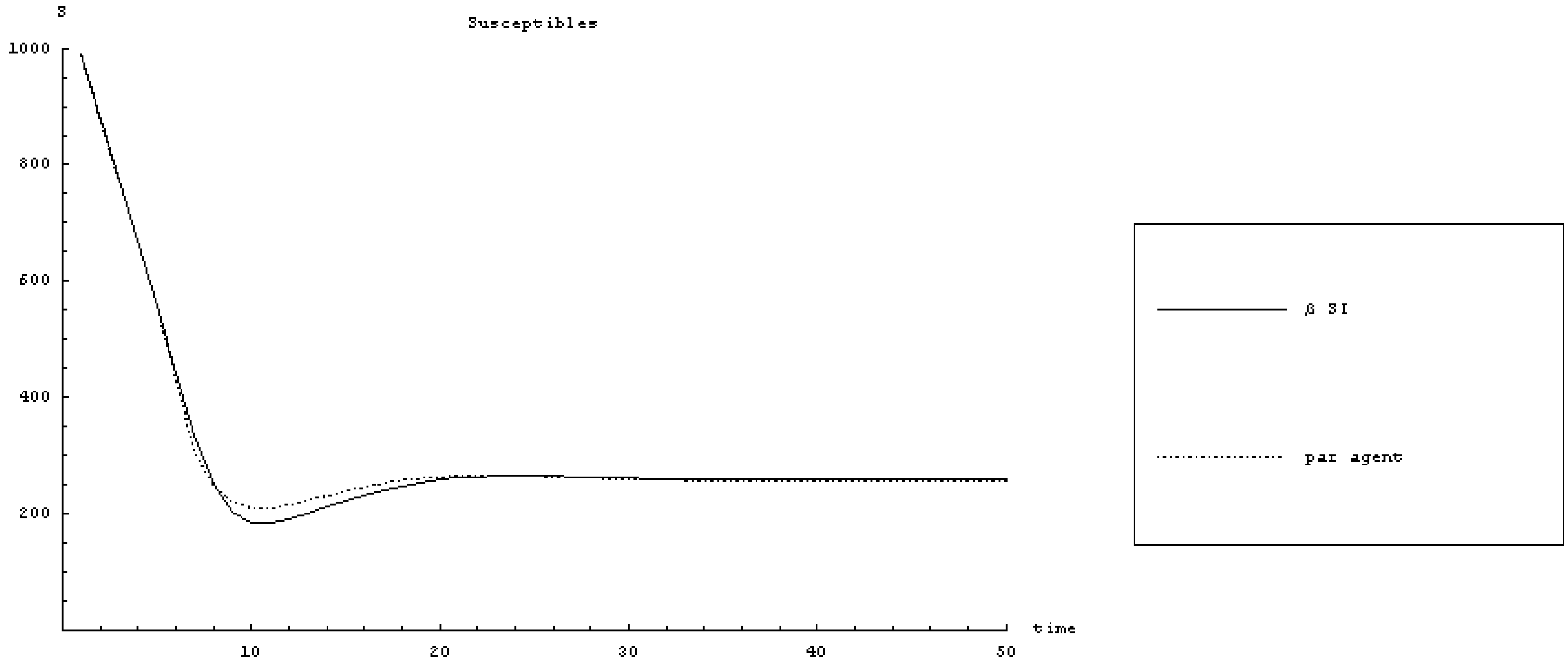
transmission term would be $\beta S_t I_t$ *with* $\beta = k\, p_a$

- m must be an integer therefore we have

$$Transmission = Round\left[k \times N_t\right] \frac{p_a S_t I_t}{N_t}$$

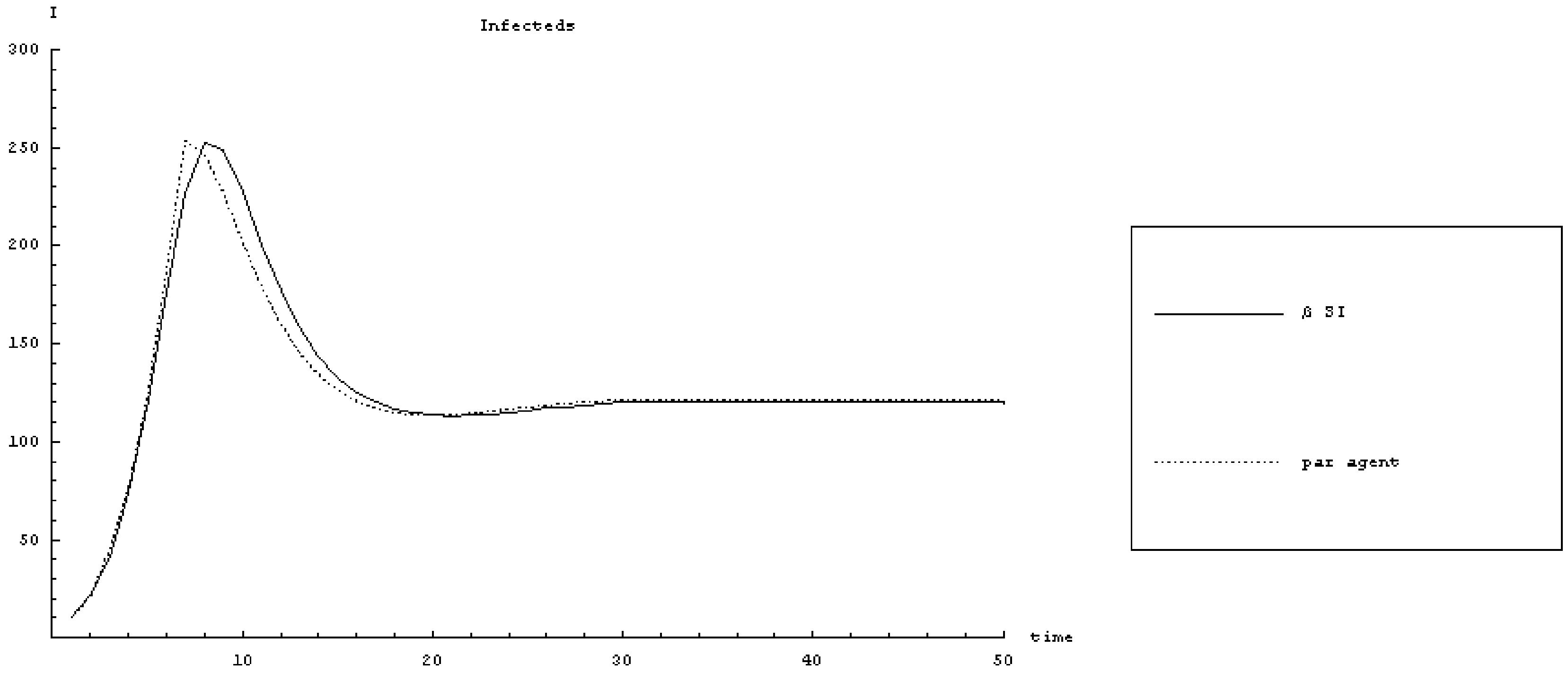
Density dependent transmission

Parallel action model - Susceptibles



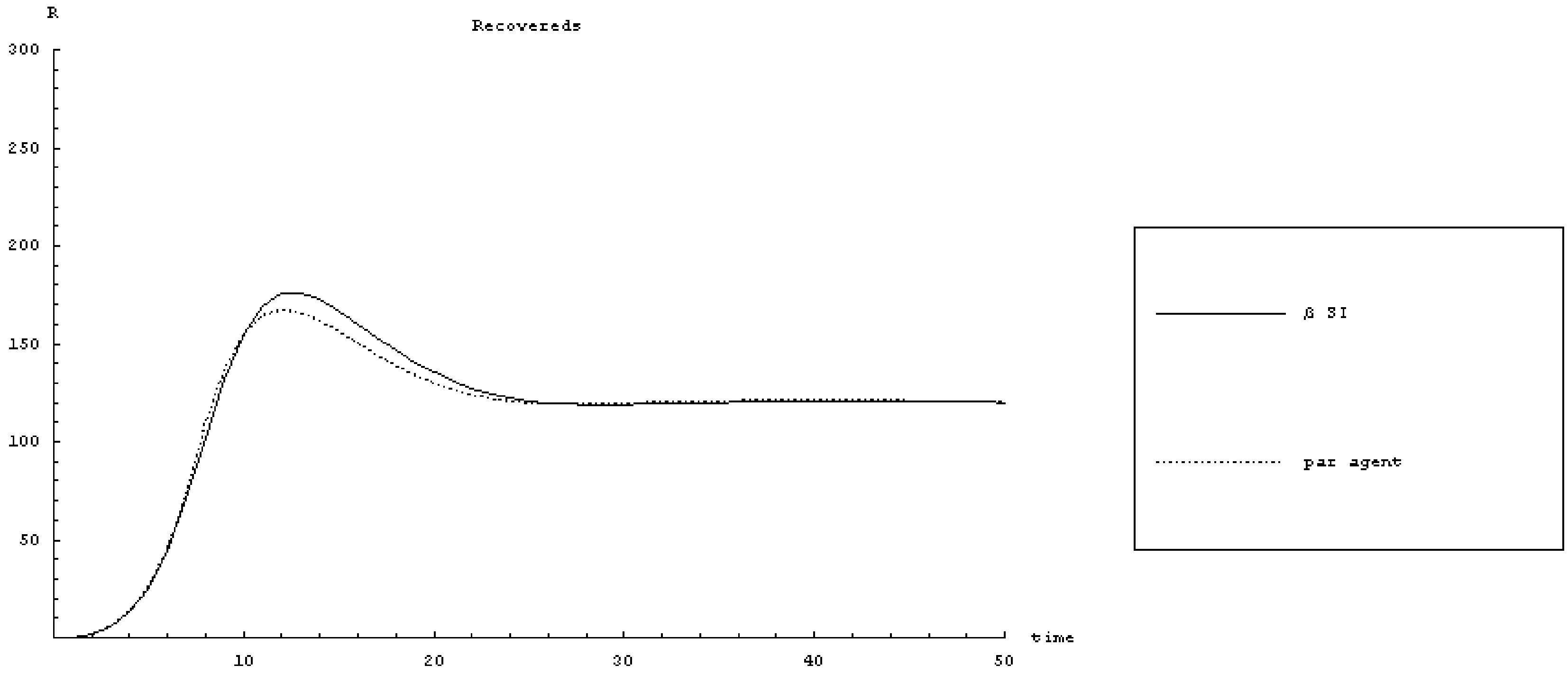
Density dependent transmission

Parallel action model - Infecteds



Density dependent transmission

Parallel action model - Recovereds



Density dependent transmission

Timesteps

```
bs S1 1.infect1:SI12 + 1.t:S12
bs S12 1.infect2:SI13 + 1.t:S13
bs S13 1.infect3:SI2 + 1.t:S2

bs SI12 1.infect2:SI13 + 1.t:SI13
bs SI13 1.infect3:SI2 + 1.t:SI2
```

Density dependent transmission

Timesteps

```
bs I1 1.infect1^-1:I12 + 1.t:I12
bs I12 1.infect2^-1:I13 + 1.t:I13
bs I13 1.infect3^-1:I2 + 1.t:I2

bs R1 1.infect1:R12 + 1.t:R12
bs R12 1.infect2:R13 + 1.t:R13
bs R13 1.infect3:R2 + 1.t:R2
```

Density dependent transmission

Timesteps

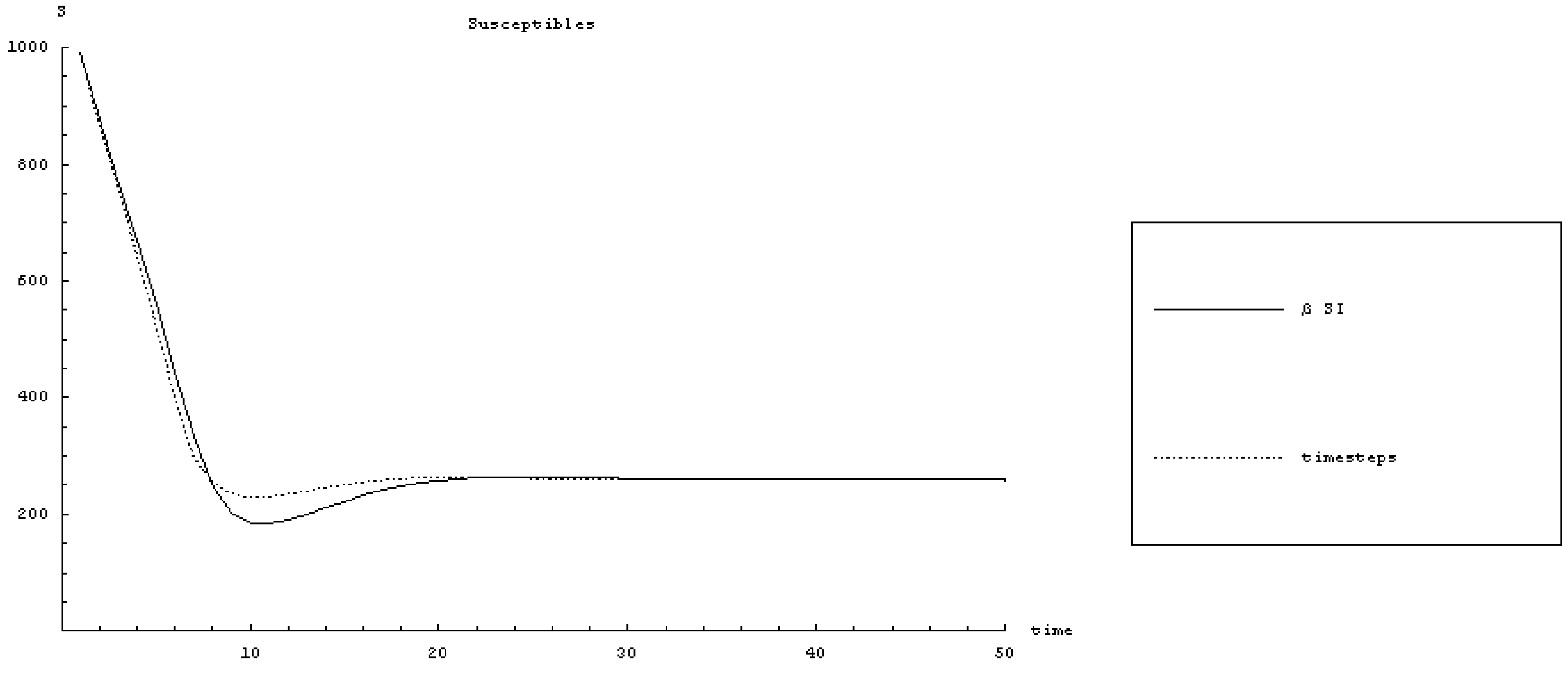
$$S_{t+1} = (1 - p_d)S_t - \sum_{j=1}^{m} \binom{m}{j} \frac{p_a S_t I_t^j}{N_t^j} + F$$

$$I_{t+1} = (1 - p_r - p_d)I_t + \sum_{j=1}^{m} \binom{m}{j} \frac{p_a S_t I_t^j}{N_t^j}$$

$$R_{t+1} = (1 - p_d)R_t + p_r I_t$$

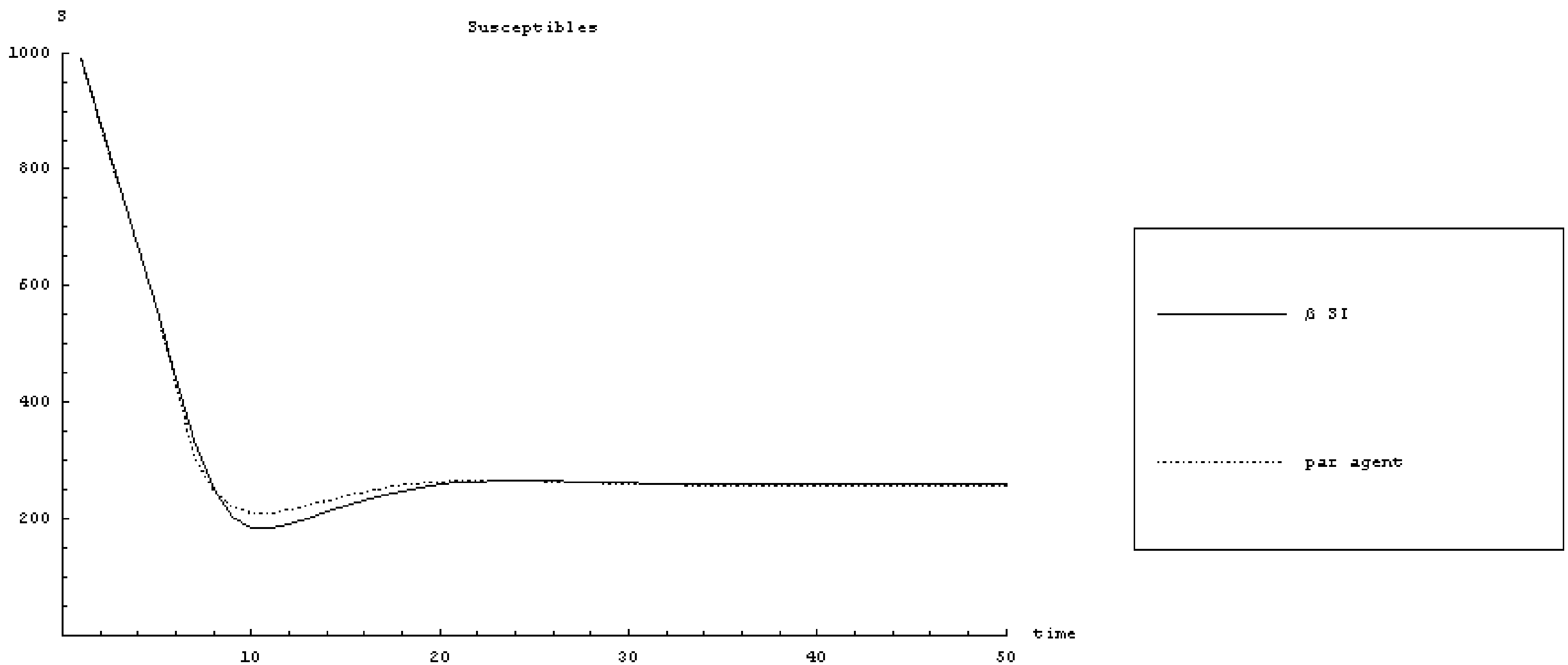
Density dependent transmission

Timesteps model - Susceptibles



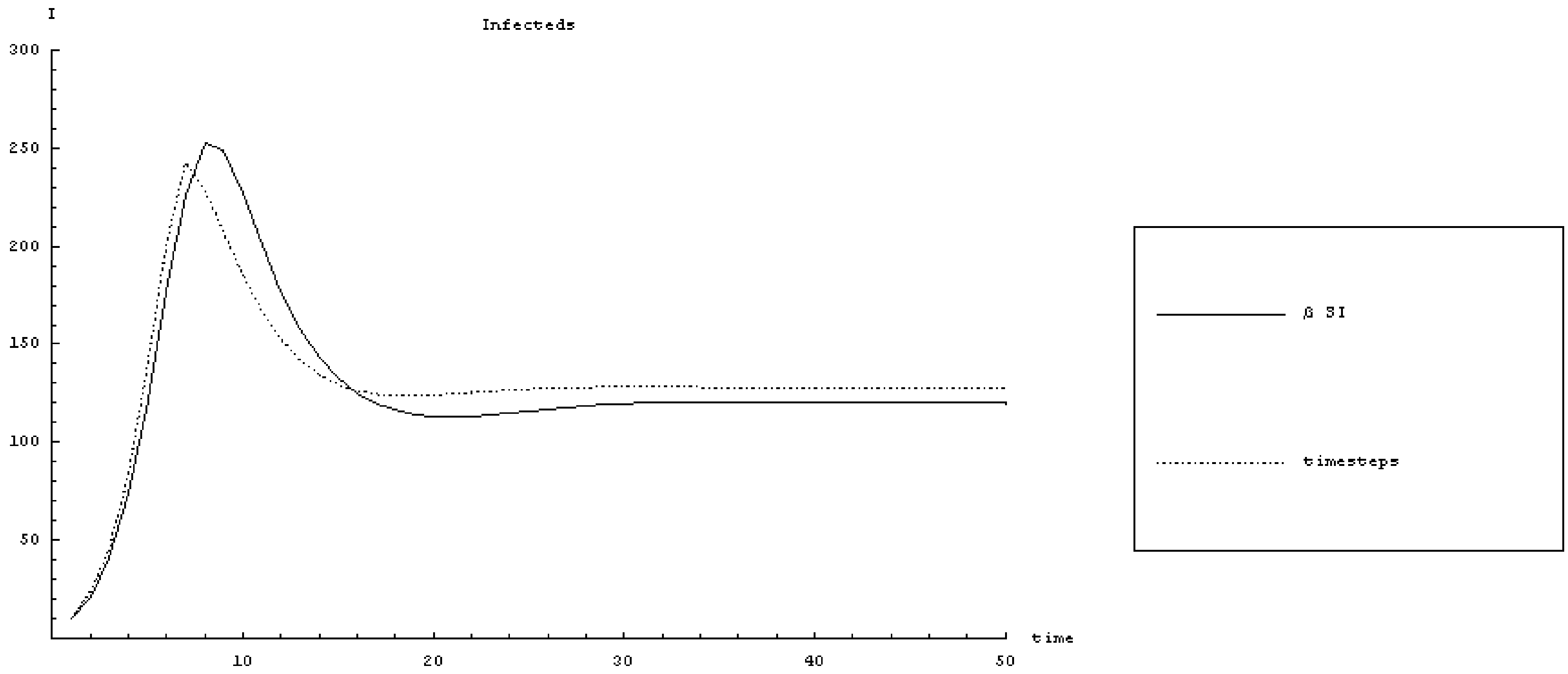
Density dependent transmission

Parallel action model - Susceptibles



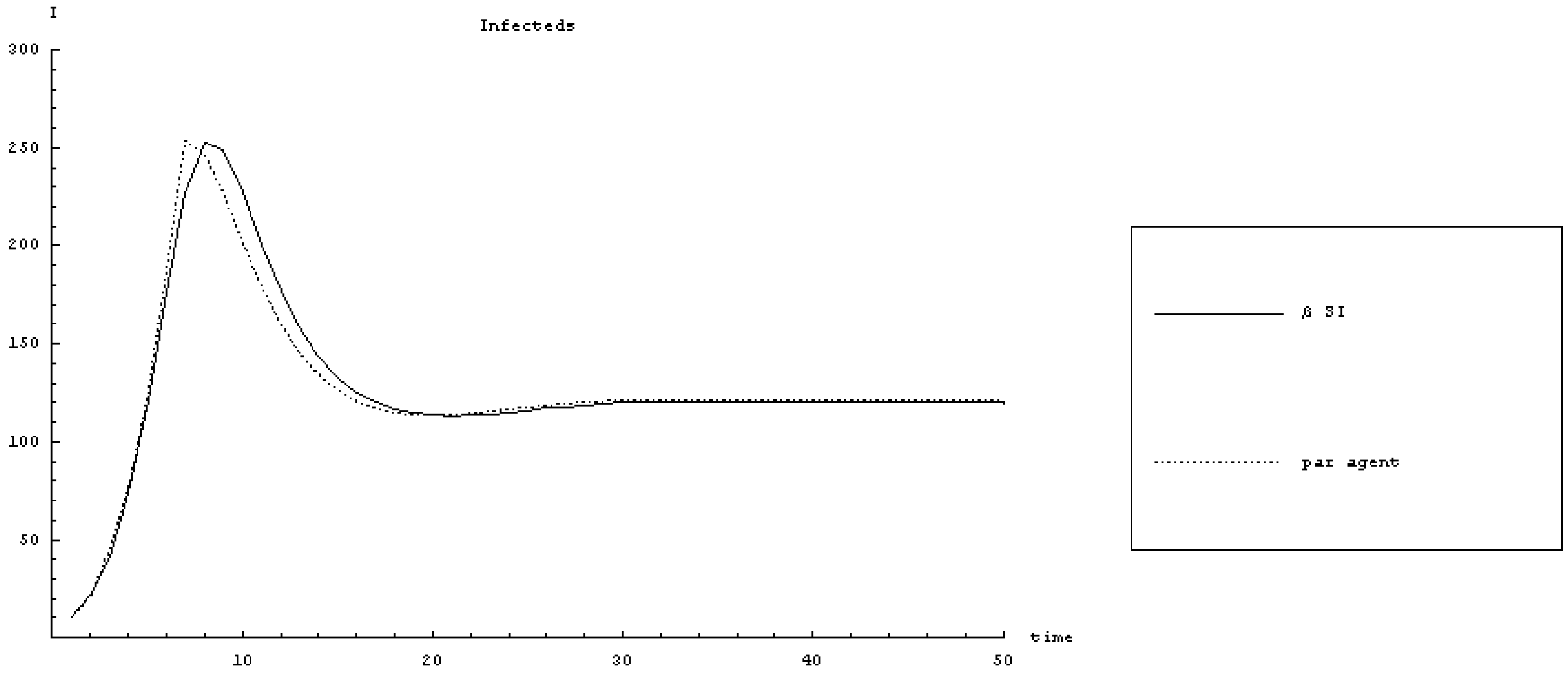
Density dependent transmission

Timesteps model - Infecteds



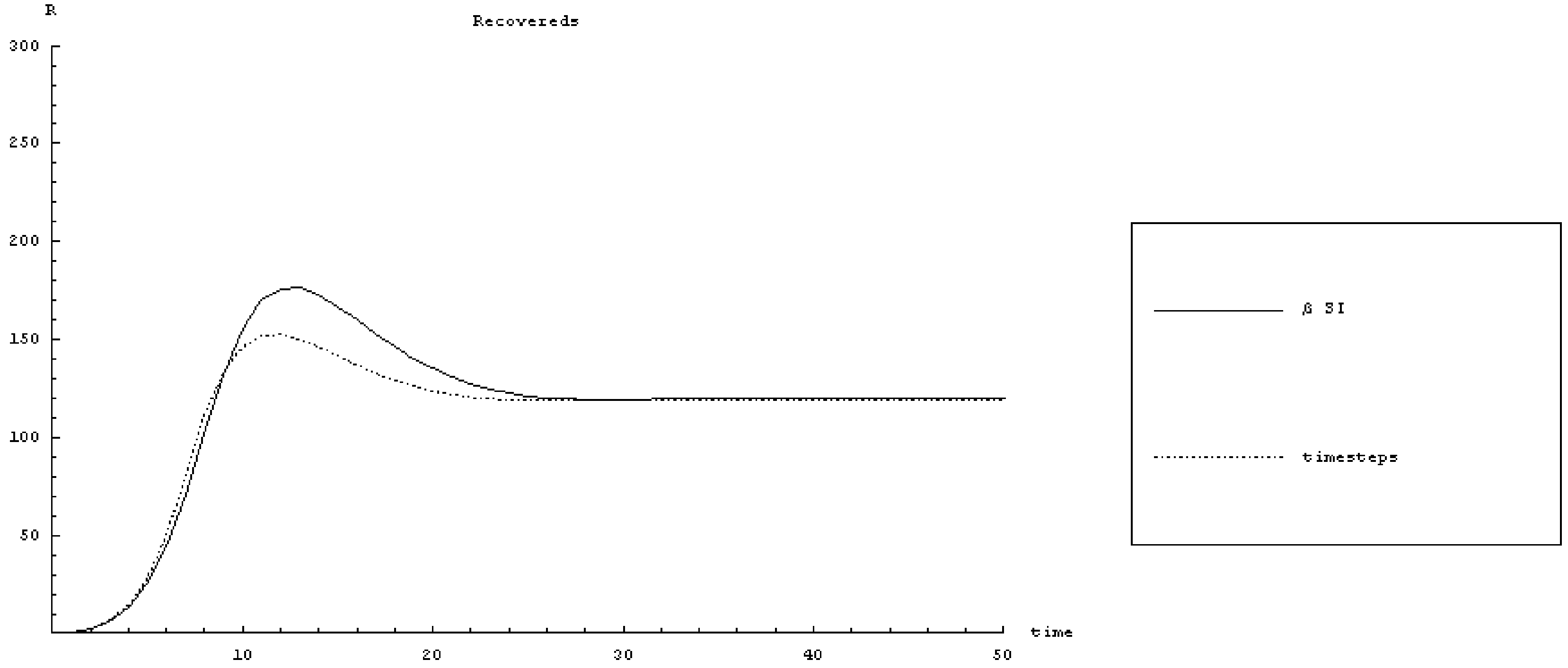
Density dependent transmission

Parallel action model - Infecteds



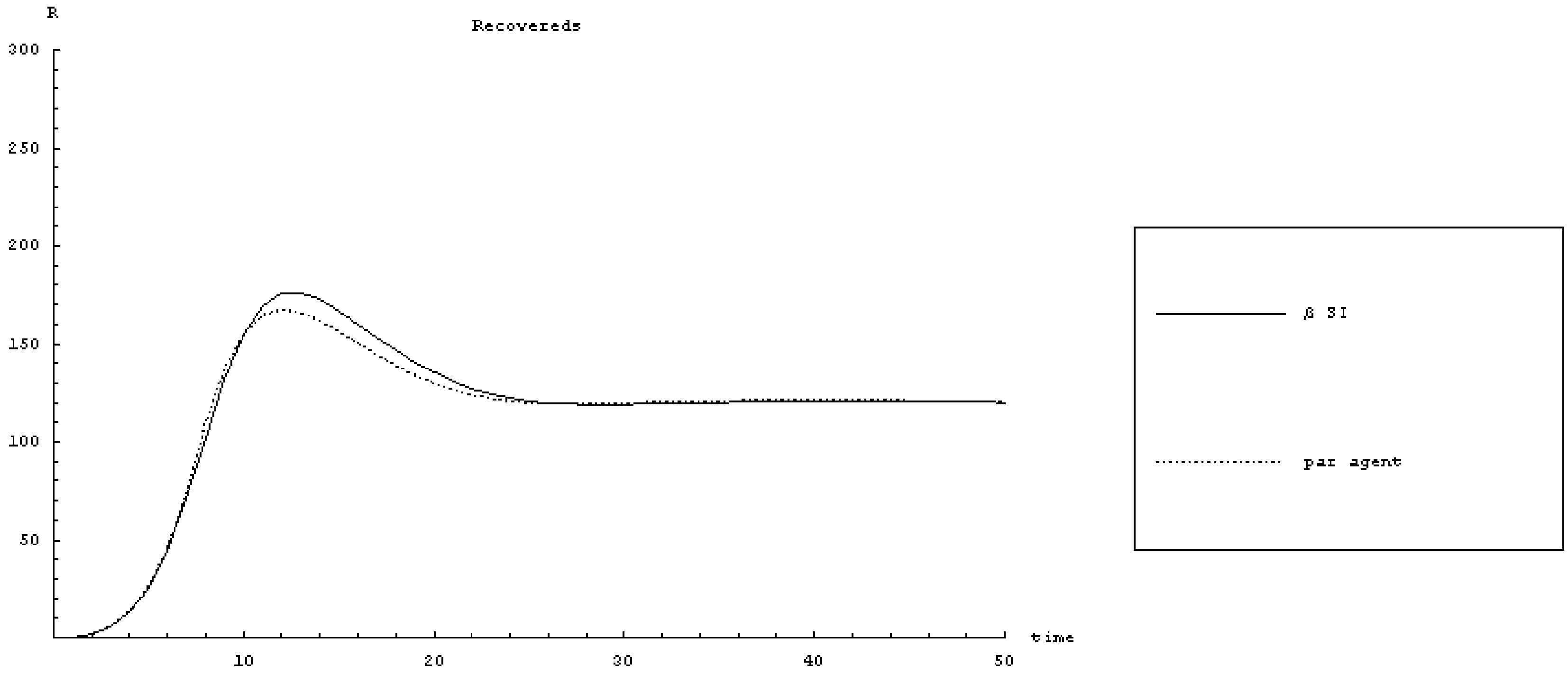
Density dependent transmission

Timesteps model - Recovereds



Density dependent transmission

Parallel action model - Recovereds



Future work

- Other methods for making multiple contacts in WSCCS model
- Compare terms from WSCCS models to other proposed transmission terms

Density dependent transmission

Parallel actions

```
bs S1 1.infect^1:SI2 + 1.t:S2
bs I1 1.infect^-3:I2 +
  1.infect^-2:I2 + 1.infect^-
  1:I2 + 1.t:I2
bs R1 1.infect^1:R2 + 1.t:R2
```

Density dependent transmission

- Leads to complex transmission term

$$Transmission = p_a S_t \frac{\sum_{r=0}^{I_t} \binom{I_t}{r} \sum_{k=r}^{mr} \binom{S_t + I_t - 1}{k - 1}}{\sum_{r=0}^{I_t} \binom{I_t}{r} \sum_{k=r}^{mr} \binom{S_t + I_t}{k - 1}}$$