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# A Theory of Impedance Control based on Internal Model Uncertainty

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Efficient human motor control is characterised by an extensive use of joint impedance modulation, which to a large extent is achieved by co-contracting antagonistic muscle pairs in a way that is beneficial to the specific task. Studies in single and multi joint limb reaching movements revealed that joint impedance is increased with faster movements [1] as well as with higher positional accuracy demands [2]. A large body of experimental work has investigated the motor learning processes in tasks with changing dynamics conditions (e.g., [3]) and it has been shown that subjects generously make use of impedance control to counteract destabilising external force fields (FF). In the early stage of dynamics learning humans tend to increase co-contraction. As learning progresses in consecutive reaching trials, a reduction in co-contraction with a parallel reduction of the reaching errors made can be observed. While there is much experimental evidence available for the use of impedance control in the CNS, no generally-valid computational model of impedance control derived from first principles have been proposed so far. Many of the proposed computational models have either focused on the biomechanical aspects of impedance control [4] or have proposed simple low level mechanisms to try to account for observed human co-activation patterns [3]. However these models are of a rather descriptive nature and do not provide us with a general and principled theory of impedance control in the nervous system.

Here we develop a powerful computational model for impedance control, which describes muscle co-activation in human arm reaching tasks as an emerging mechanism from first principles of optimality. We hypothesise that, in conjunction with an appropriate antagonistic arm and *motor variability* model, impedance control emerges from an optimisation process that minimises prediction uncertainties of the internal model. Our model is formalized within the theory of *stochastic Optimal Feedback Control (OFC)* [5], in which an actor's goal is expressed as a solution to an optimisation process that typically minimises energy consumption and reaching error. Unlike previous OFC models, which required analytic dynamics model formulations, here we postulate that the dynamics model is acquired as a motor learning process based on continuous sensorimotor feedback. Because this *stochastic OFC with learned dynamics (OFC-LD*) updates the internal dynamics model from plant data during control, it can be used to model adaptation processes due to changing dynamics conditions. This allows us to investigate human FF adaptation paradigms within the powerful framework of optimality.

The human sensorimotor system is known to exhibit a highly stochastic behaviour and a complete motor control theory must be able to deal with the detrimental effects of *signal dependent noise (SDN)*[1]. Previously proposed stochastic optimal control models [6] assumed *naive SDN* (Fig. 1B), which ignored the impedance to noise characteristics of the musculoskeletal system and therefore failed to model co-contraction. Here however we model an *extended* type of SDN (Fig. 1C), the magnitude of which realistically decreases with higher co-activation levels [7]. The learner interprets this motor variability as *prediction uncertainty*, which is given algorithmically in form of heteroscedastic (i.e., locally valid) prediction variances. With these ingredients we formulate a *minimum-uncertainty* optimal control model that introduces this stochastic information into OFC-LD. Due to the realistic nature of the motor variability our system exhibits, the OFC-LD solutions will, while still trying to reduce energetic costs and endpoint error, favour co-contraction in order to reduce the negative effects of the SDN. In summary, this normative model of impedance control for antagonistic limb systems is based on the quality of the learned internal model and therefore leads to the intuitive requirement that impedance will be increased in cases where the actor is uncertain about his model predictions. This is of special importance during adaptation tasks, where prediction uncertainty also increases, leading to similar increases in co-contraction.

We evaluated our model in several simulation experiments with stationary dynamics (Fig. 2) as well as in adaptation tasks (Fig. 3). The results show that our computational model is able to predict many well-known impedance control phenomena from the first principles of optimality, and that the minimum-uncertainty approach can conceptually explain the origins of co-activation in volitional human reaching tasks. We believe that our model will generalise well to more complex (multi-joint) plant models.

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[1] Here we use the terms "noise" and "motor variability" interchangeably.

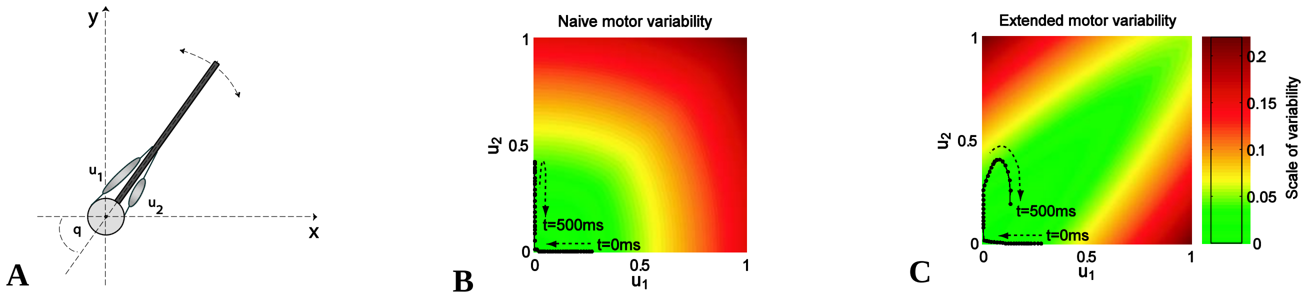


**Figure 1:** **A** - Elbow model with two muscles used in a point-to-point reaching task. Its stochastic dynamics are defined as $dx = f(x,u)\,dt + F(u)\,d\omega$, where $x = [q, \dot{q}]$ is the arm state and $d\omega$ is Brownian motion noise. **B & C** – The coloured regions represent two different control-dependent noise functions F(u) used in OFC. **C-** The proposed extended motor variability favours co-contraction, which is defined here as the minimum of two antagonistic muscle signals $\min(u_1, u_2)$. The black lines show the obtained optimal muscle activation sequence (500ms duration) where each dot represents a discrete time step starting at *t* = 0ms.

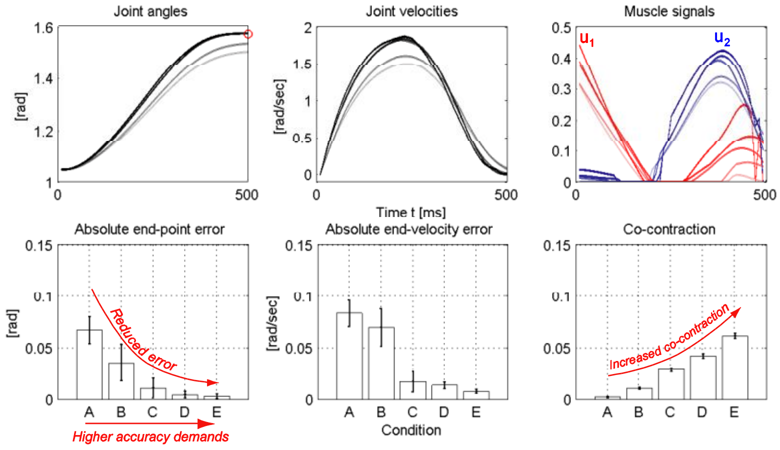
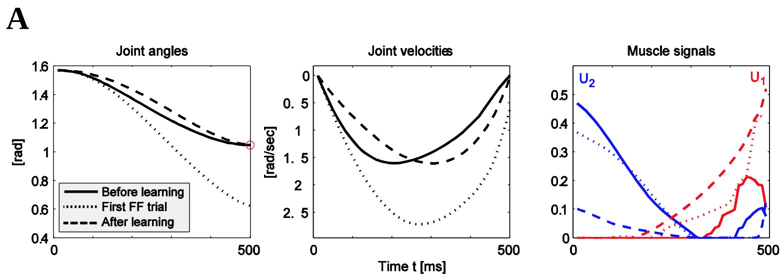


**Figure 2:** Experimental results from our OFC-LD model for different accuracy demands. We ran OFC-LD for 5 different conditions *A* to *E* with increasing accuracy demands, which were formalized in the OFC cost function. The first row shows the averaged joint angles (left), the averaged joint velocities (middle) and the averaged muscle signals (right) over 20 trials for conditions *A* to *E*. The darkness of the lines indicates the level of accuracy; the brightest line indicates condition *A* and the darkest condition *E*. An inverse relationship can be observed (second row) between accuracy and co-contraction [1]. We obtained similar results (not shown here) for reaches with varying velocities, where faster reaches induced higher co-contraction [2]. Both results emerge from the fact that the system tries to reduce motor variability by increasing co-contraction.



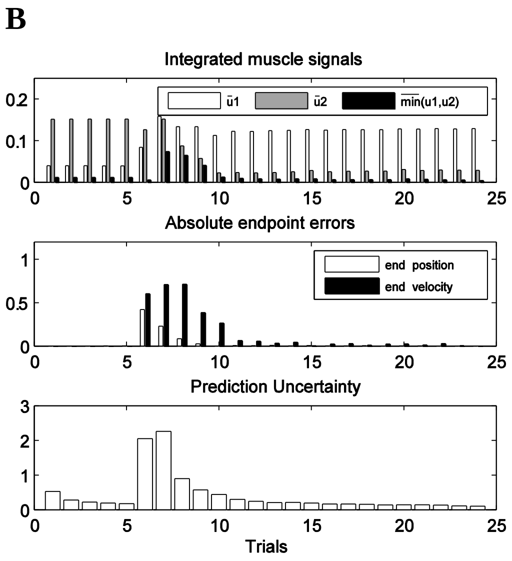


**Figure 3:** **A** - Experimental results from the stochastic OFC-LD during adaptation towards a constant FF. The solid line represents the reaching with OFC-LD in the null field (NF) condition, the dotted line shows the first FF trial and the dashed line the trajectories after adaptation. **B** - Accumulated statistics during 25 adaptation trials using stochastic OFC-LD. Trials 1 to 5 are performed in the NF condition. Top: Mean muscle signals and co-contraction integrated during 500ms reaches. Middle: Absolute joint errors and velocity errors at final time *T* = 500*ms*. Bottom: Integrated prediction uncertainties along the current optimal trajectory. The co-contraction is strongly correlated to the prediction uncertainty exhibited during adaptation (trials 5 to 10).

## References

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