Continuous Estimation of Mean and Uncertainty

Ian Saunders $(P)^1$, and Sethu Vijayakumar¹

¹ Institute of Perception, Action and Behaviour, School of Informatics, University of Edinburgh, UK E-mail: i.saunders@sms.ed.ac.uk, sethu.vijayakumar@ed.ac.uk

Abstract— Sensory uncertainty affects our perception and motor actions, but the mechanisms by which we estimate uncertainty are largely unexplored. We introduce a novel experimental paradigm that requires subjects to continuously report their (evolving) sufficient statistics of visual cues over time. We show that subjects rapidly accumulate evidence over the course of a trial to form an accurate estimate of the mean that equally weights all seen cues. Moreover, subjects have knowledge of their continuous objective uncertainty, although it is estimated with a conservative safety margin.

Keywords—Continuous Estimation, Uncertainty

1. Introduction

The uncertainty of sensory information influences perception when we reach for noisy targets [1] and we combine multiple sensory modalities by weighting sensory information according to their reliabilities [2]. A growing body of psychophysical experiments supports the proposition that perception is statistically-optimal. However, despite its fundamental importance to the theory, the question of how humans gather the relevant statistical information to make optimal decisions remains largely unexplored. Recently it was shown that subjective estimation of sensory uncertainty is related to objective uncertainty [3] which may be achieved by accumulating sensory evidence over time [4].

We have developed a novel task that requires subjects to explicitly track the statistical properties of noise-perturbed visual cues. We ask subjects to continuously (i) track the mean of the cues; and (ii) indicate the range in which they believe the mean of the cues to lie. We modulate the underlying distribution of the cues by adjusting the cue variance from trial-to-trial and by inducing cue perturbations within trials. This allows us to measure the contribution of each visual cue to the formation of mean and confidence estimates and to probe the mechanisms of temporal cue integration.

2. Methods

The experimental setup involves using a projection

arrangement (Fig.1A, see [2]) and subjects control a variable sized "net" with the forefinger and thumb of their right hand. The visual cues consisted of a sequence of points drawn from a prescribed mean and variance during a single trial (Fig.1B) and distributed in time rather than space -- this allows us to probe how the perception of mean and uncertainty evolve as evidence arrives. On each trial the standard deviation of the cues is varied, $\sigma \in \{50, 120, 200\}$ pixels. Each 4-second trial comprises 15 cues, separated into three blocks, $b \in \{1, 2, 3\}$, one of which is perturbed by a fixed amount $a \in \{-0.3\sigma, 0, 0.3\sigma\}$ pixels. On each trial σ , a and b are chosen at random in a fully counterbalanced design. These manipulations were crucial to enable us to calculate the influence of each cue on the behavioural decision. Subjects are required to indicate the range of values in which they believe the mean of the cues to lie, using a variable-aperture cursor (Fig. 1C), controlled by their thumb and forefinger - defining a confidence window. This approach has previously been used to measure subjective uncertainty of a random walking stimulus [4]. Finger positions are tracked and recorded using a Polhemus Liberty 240Hz motion tracking system. Fourteen naive, right-handed healthy volunteers completed a calibration experiment to measure baseline performance in each condition -- same as the main experiment described above, except subjects controlled a fixed-size cursor. The resultant localisation accuracy defined the objective uncertainty for each experimental condition. In the main experiment, the task was deemed successful on a given trial if the mean of the cues lies within the aperture of their net: points were awarded in such cases. The subjects were encouraged to match this objective uncertainty, by awarding maximal points if subjects were successful and chose an aperture less than or equal to their expected objective uncertainty and fewer points for larger apertures.

Trajectories were recorded and separated into a mean estimate trajectory and a confidence estimate trajectory. To compute the contribution of each visual cue to the mean estimate, a multiple linear regression was performed at each

Figure 1. Experimental Setup. (A) Subjects observe visual cues visual display through a projection setup (see [2]). time (B) During a trial a sequence of noisy visual cues are generated around a randomised target location perturbation µ, sampled from a distribution with mirror standard deviation σ . On each trial, σ is varied and subsets of cues are perturbed. (C) Subjects indicate the range of values in which the mean С visual of the cues is believed to lie, using cursor a visual cursor controlled by their right hand. grasp aperture





Figure 2. Results. (A) Typical trajectories showing the mean estimate (solid red and blue lines) and confidence estimate (dashed red and blue lines) as a result of cue sequence (black dots). An early-onset rightward perturbation (red) and middle-onset leftward perturbation (blue) are shown. (B) Weights assigned to the cues seen so far (solid lines) \pm SEM across subjects (shaded regions), computed by least-squares regression at each time step (see methods). Coloured arrows indicate the time step at which the correspondingly coloured integration window is observed. (C) Weights assigned to each cue over time \pm SEM across subjects. Coloured arrows indicate the cues presented at each time step and the correspondingly coloured curve indicates the evolving weight assigned to the cue. (D) Confidence window \pm SEM across subjects for early, middle and late onset perturbations (grey shaded region). Within each plot, each curve corresponds to the average trajectory for different σ . The confidence window increases in response to larger σ and to perturbations (arrows). (E) Average confidence window for each σ , plotted against the expected objective error \pm SEM across subjects in each direction. The confidence window consistently overestimates the expected objective error.

time-step using a non-negative weight least-squares method [5]. This method assigned a weight to each cue as a measure of its contribution to the decision at each time step.

3. Results

Performance in the calibration experiment was compared to the main experiment. An ANOVA (within-subjects) of mean endpoint error found no significant difference between the two ($F_{2,12}$ =0.022, *p*=.86), justifying use of the calibration data to shape the rewards.

Typical trajectories in the main experiment are shown in Fig.2A. The effect of cue perturbations and cue variance were analysed. Early perturbations resulted in larger trajectory deviations than later perturbations and larger cue variance resulted in greater trajectory variability and increased endpoint errors. These findings were robust across subjects (data not shown).

We computed the contribution of each cue to the mean estimate (see methods). This revealed that cues were weighted according to an integration window that assigns approximately equal weight to all cues (prior to a fixed delay, see Fig. 2B and caption). The integration window is flat at each time point, indicating that subjects integrate new evidence and reweight old evidence continuously. The weight assigned to each cue rises as it is observed but then decays to remain equally weighted with the other cues (Fig. 2C). These findings (and the one related to cue uncertainty) are consistent with an ideal-observer model with simple sensory and motor constraints that we have fitted (omitted due to space constraints). Fig. 2D illustrates an increase in confidence over time as more cues arrive, represented by a gradual reduction in the *confidence window*. Across subjects the confidence window increases as a function of cue variance and the onset of perturbations (see Fig. 2D and captions).

An ANOVA (within-subjects) on the objective uncertainty revealed significant main effects of σ (F_{2,12}=261, *p*<.001), and *p* (F_{2,12}=110, *p*<.001), and likewise an ANOVA on confidence window revealed significant main effects of σ (F_{2,12}=29.5, *p*<.001) and *p* (F_{2,12}=37.6, *p*<.001). However, the confidence window does not match the objective (Fig. 2E) consistent with [3] - subjects tend to be conservative on the confidence window necessary to achieve the maximum score.

References

- [1] Tassinari, Hudson and Landy (2006) Combining priors and noisy visual cues in a rapid pointing task. *J Neurosci* 26: 10154–10163.
- [2] Ernst and Banks (2002) Humans integrate visual and haptic information in a statistically optimal fashion. *Nature 415: 429–433.*
- [3] Barthelmé and Mamassian (2009) Evaluation of objective uncertainty in the visual system. *PLoS Comput Biol 5:* e1000504.
- [4] Graf, Warren and Maloney (2005) Explicit estimation of visual uncertainty in human motion processing. *Vision Res* 45: 3050–3059.
- [5] Lawson and Hanson (1974) Solving least squares problems. Prentice-Hall, Chapter 23, p. 161