Volume Illumination, Contouring

Computer Animation and Visualisation – Lecture 10
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Overview

- Volume Illumination

- Contouring
  - Problem statement
  - Tracking
  - Marching squares
  - Ambiguity problems
  - Marching cubes
  - Dividing squares
Light Propagation in Volumes

• Lighting in volume
  - Up to now we only considered transmission and emission
  - Volume can also:
    - reflect light
    - scatter light into different directions
Volume Illumination

• Why do we want to illuminate volumes?

• illumination helps us to better understand 3D structure
  – displays visual cues to surface orientation
  – highlight significant gradients within volume
Phong Illumination Model

- Simple 3 parameter model
- The sum of 3 illumination terms:
  - **Diffuse**: non-shiny illumination and shadows
  - **Specular**: bright, shiny reflections
  - **Ambient**: 'background' illumination

\[
\text{Diffuse (directional)} + \text{Specular (highlights)} + \text{Ambient (color)} = \text{Rc}
\]
Diffuse Reflection

\[ I = I_p k_d \cos \theta \]

- \( I_p \): Light Intensity
- \( \theta \): the angle between the normal vector direction towards the light and the light source
- \( k_d \): diffuse reflectivity

Example: a sphere (lit from left), no dependence on camera angle!
Direct reflections of light source off shiny object

- specular intensity $n = $ shiny reflectance of object
- Result: specular highlight on object

$$I = I_p k_s (\cos \alpha)^n$$

No dependence on object color.
Ambient Lighting

Simple approximation to complex 'real-world' process

Result: globally uniform color for object

\[ I = \text{resulting intensity} \]
\[ I_a = \text{light intensity} \]
\[ k_a = \text{reflectance} \]

\[ I = k_a I_a \]

Example: sphere
Combining Diffuse and Specular Reflections
Illumination of Volumes

For every voxel ray intersects, need to consider:

- Light absorbed.
- Light emitted.
- Light scattered out of the ray.
- Light scattered into the ray.
Example: multiple scattering
Single scattering

Multiple scattering is expensive
Here we only do single scattering
Shading an Embedded iso-surface

• Classify volume with a step function
• For calculating the colour of every voxel, use regular specular / diffuse surface shading
• **Remember** lighting requires
  • illumination direction
    - camera model (position)
    - surface orientation
    - **need** to calculate and store **surface normal**
Estimating the surface normal from the depth

• Use distance map to the iso-surface value

1. Determine the threshold value
2. Determine the surface voxels based on the threshold
3. Compute the normal vectors based on centred difference method

For example, if we sample the centre of the voxels,

\[
\delta z_x = \frac{1}{2} (z(x+1, y) - z(x-1, y))
\]

\[
\delta z_y = \frac{1}{2} (z(x, y+1) - z(x, y-1))
\]

\[
N = \left( \begin{array}{ccc}
-\delta z_x / (\delta z_x^2 + \delta z_y^2 + 1) & -\delta z_y / (\delta z_x^2 + \delta z_y^2 + 1) & 1 / (\delta z_x^2 + \delta z_y^2 + 1)
\end{array} \right)
\]
Result: illuminated iso-surface

- Surface normals recovered from depth map of surface
Estimating Opacity Gradient

- Use 3D centred difference operator

\[
\nabla I = (I_x, I_y, I_z) = (\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y}, \frac{\delta I}{\delta z})
\]

\[
\frac{\delta I}{\delta x} = \frac{I(x+1, y, z) - I(x-1, y, z)}{2}
\]

We can extract the normal vectors of the region where the scalar values are changing significantly, i.e. boundary of tissues.
Illuminating Opacity (Scalar) Gradient

- Illuminate “scalar gradient” instead of iso-surface
  - requirement: estimate and store gradient at every voxel

Composite

Shaded opacity gradient (shades changes in opacity)
Illumination: storing normal vectors

- Visualisation is **interactive**
  - compute normal vectors for surface/gradient once
  - store normal
  - perform interactive shading calculations

- **Storage**:
  - $256^3$ data set of 1-byte scalars $\sim 16\text{Mb}$
  - normal vector (stored as floating point(4-byte)) $\sim 200\text{Mb}$!
  - **Solution**: quantise direction & magnitude as small number of bits
Illumination: storing normal vectors

- Quantize vector direction into one of N directions on a sub-divided sphere

Subdivide an octahedron into a sphere.
Number the vertices.
Encode the direction according to the nearest vertex that the vector passes through.
For infinite light sources, only need to calculate the shading values once and store these in a table.
Overview

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  - Tracking
  - Marching squares
  - Ambiguity problems
  - Marching cubes
  - Dividing squares
Contouring

• Contours explicitly *construct* the **boundary** between regions with values

• Boundaries correspond to:
  - lines in 2D
  - surfaces in 3D (known as isosurfaces)
  - of constant scalar value
Example: contours

- lines of constant pressure on a weather map (isobars)
- surfaces of constant density in medical scan (isosurface)
  - “iso” roughly means equal / similar / same as
Contouring

- **Input**: 2D or 3D grid with scalar values at the nodes
- **Output**: Contours (polylines, polygons) that connect the vertices with the same scalar value
2D Contour

- **Input Data:** 2D structured grid of scalar values

```
0  1  1  3  2
1  3  6  6  3
3  7  9  7  3
2  7  8  6  2
1  2  3  4  3
```

- Difficult to visualise transitions in data
  - use **contour** at specific scalar value to highlight **transition**

- What is the contour of 5?
Methods of Contour Line Generation

• Approach 1: Tracking
  - find contour intersection with an edge
  - track it through the cell boundaries
    - if it enters a cell then it must exit via one of the boundaries
    - track until it connects back onto itself or exits dataset boundary
  - If it is known to be only one contour, stop
  - otherwise
    - Check every edge

• Approach 2: Marching Squares Algorithm
Marching Squares Algorithm

• Focus: intersection of contour and cell edges
  - how the contour passes through the cell

• Assumption: a contour can pass through a cell in only a finite number of ways
  - A vertex is inside contour if scalar value > contour
  - outside contour if scalar value < contour
  - 4 vertices, 2 states (in or out)
Marching Squares

- $2^4 = 16$ possible cases for each square
  - small number so just treat each one separately
MS Algorithm Overview

• Main algorithm
  1. Select a cell
  2. Calculate inside/outside state for each vertex
  3. Look up topological state of cell in state table
     determine which edge must be intersected (i.e. which of the 16 cases)
  4. Calculate contour location for each intersected edge
  5. Move (or march) onto next cell
     • until all cells are visited GOTO 2
MS Algorithm - notes

• Intersections for each cell must be merged to form a complete contour
  - cells processed independently
  - further “merging” computation required
  - disadvantage over tracking (continuous tracked contour)

• Easy to implement (also to extend to 3D)
• Easy to parallelise
MS : Dealing with ambiguity?

Split

Join

• Choice independent of other choices
  - either valid: both give continuous and closed contour
Example: Contour Line Generation

- 3 main steps for each cell
  - here using simplified summary model of cases
Step 1: classify vertices

- Decide whether each vertex is inside or outside contour

- No intersection.
- Contour intersects 1 edge
- Contour intersects 2 edges
- Ambiguous case.
Step 2: identify cases

- Classify each cell as one of the cases

- No intersection.
- Contour intersects 1 edge
- Contour intersects 2 edges
- Ambiguous case.

Contour value = 5
Step 3: interpolate contour intersections

- Determine the edges that are intersected
  - compute contour intersection with each of these edges
Ambiguous contour

- Finally: resolve any ambiguity
  - here choosing “join” (example only)
MS : Dealing with ambiguity?
One solution

- Calculate the value at the middle of the square by interpolation
- Check if it is under or above the threshold value
- Choose the pattern that matches
Ambiguous contour

- No intersection.
- Contour intersects 1 edge
- Contour intersects 2 edges
- Ambiguous case.
Step 3: interpolate contour intersections

- No intersection.
- Contour intersects 1 edge
- Contour intersects 2 edges
- Ambiguous case.

Split

4.875
2D : Example contour

A slice through the head

A Quadric function.

(with colour mapping added)
3D surfaces : marching cubes

• Extension of Marching Squares to 3D
  - data : 3D regular grid of scalar values
  - result : 3D surface boundary instead of 2D line boundary
  - 3D cube has 8 vertices → $2^8 = 256$ cases to consider
    - use symmetry to reduce to 15

• Problem : ambiguous cases
  - cannot simply choose arbitrarily as choice is determined by neighbours
  - poor choice may leave hole artefact in surface
Marching Cubes - cases

- Ambiguous cases
  - 3, 6, 10, 12, 13 – split or join ?

Figure 4. Cube Numbering.

Figure 3. Triangulated Cubes.
Example of bad choices

- The dark dots are the interior
- There are edges which are not shared by both cubes
- Need to make sure there is no contradiction with the neighbors
Cracks eliminated
Other two possible triangulations

- Need to decide how the faces are intersected by the contours
Adding more patterns

- Adding more patterns for 3, 6, 10, 12, 13 [Neilson '91]
- Compute the values at the middle of the faces and the cubes
- Selecting the pattern that matches
Rendering Implicit Surfaces

The marching cubes algorithm is useful for rendering implicit surfaces where $F(x,y,z) = 0$

Inside: $F(x,y,z) > 0$
Outside: $F(x,y,z) < 0$
Marching Cubes by CUDA

- http://www.youtube.com/watch?v=Y5URxpX8q8U
Dividing Cubes Algorithm

• Marching cubes : Problem
  - often produces more polygons than pixels for given rendering scale
  - Problem : causes high rendering overhead

• Solution : Dividing Cubes Algorithm
  - Draw points instead of polygons (faster rendering)
  - Need 1: efficient method to find points on surface
    2: method to shade points
Example: 2D divided squares for 2D lines

Find pixels that intersect contour
- Subdivide them
2D “divided squares” for lines

Find pixels that intersect line
- **Subdivide them** (usually in 2x2)
- Repeat recursively
2D “divided squares” for lines

Find pixels that intersect line
- Subdivide them
- Repeat recursively
until screen resolution reached
- Fill in the pixel with the color of the line
Extension to 3D

- Find **voxels** which intersect **surface**
- Recursively subdivide the voxels that intersect the contour
  - Until the voxel fits within a pixel
- Calculate **mid-points of voxels**
- Calculate the color of the pixel by shading
Drawing divided cubes surfaces

- **surface normal** for lighting calculations
  - interpolate from voxel corner points

\[
\left( \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \approx \left( \frac{F_{x+\Delta x} - F_{x-\Delta x}}{2\Delta x}, \frac{F_{y+\Delta y} - F_{y-\Delta y}}{2\Delta y}, \frac{F_{z+\Delta z} - F_{z-\Delta z}}{2\Delta z} \right)
\]

- **problem with camera zoom**
  - ideally dynamically re-calculate points
  - not always computationally possible
Dividing Cubes: Example

50,000 points

when sampling less than screen resolution structure of surface can be seen
Summary

• Contouring Theory
  - 2D: Marching Squares Algorithm
  - 3D: Marching Cubes Algorithm [Lorensen '87]
    - marching tetrahedra, ambiguity resolution
    - limited to regular structured grids
  - 3D Rendering: Dividing Cubes Algorithm [Cline '88]

Readings

• G.M. Nielson, B Hamann, “The Asymptotic Decider: Resolving the Ambiguity in Marching Cubes”
• W.E. Lorensen, H.E. Cline, “Marching Cubes: A high resolution 3D surface construction algorithm”
• H.E. Cline, W.E. Lorensen and S. Ludke, “Two algorithms for the three-dimensional reconstruction of tomograms”