Contouring

Computer Animation and Visualisation – Lecture 13

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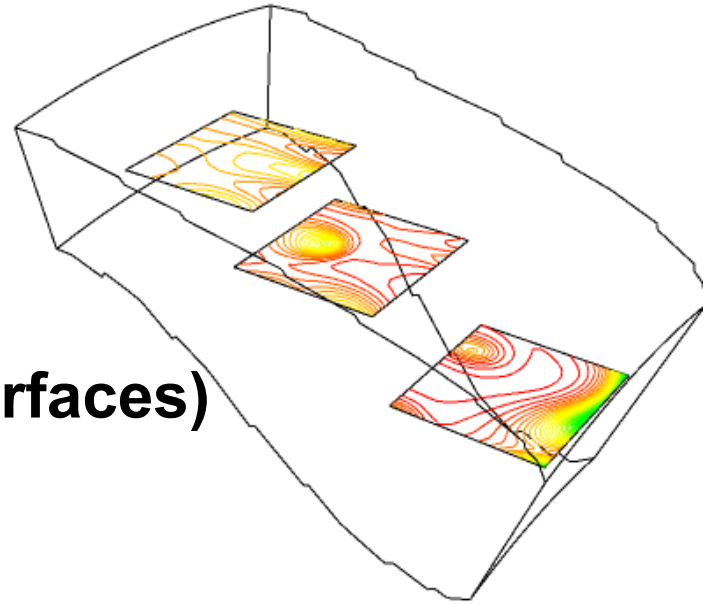
Overview

- Contouring
  - Problem statement
  - Tracking
  - Marching squares
  - Ambiguity problems
  - Marching cubes
  - Dividing squares

Contouring

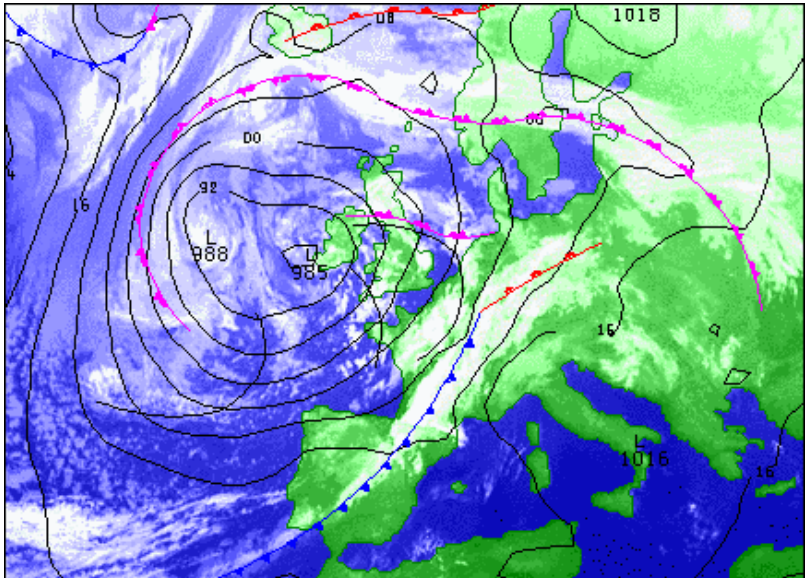
- Contours explicitly *construct* the **boundary between regions with values**

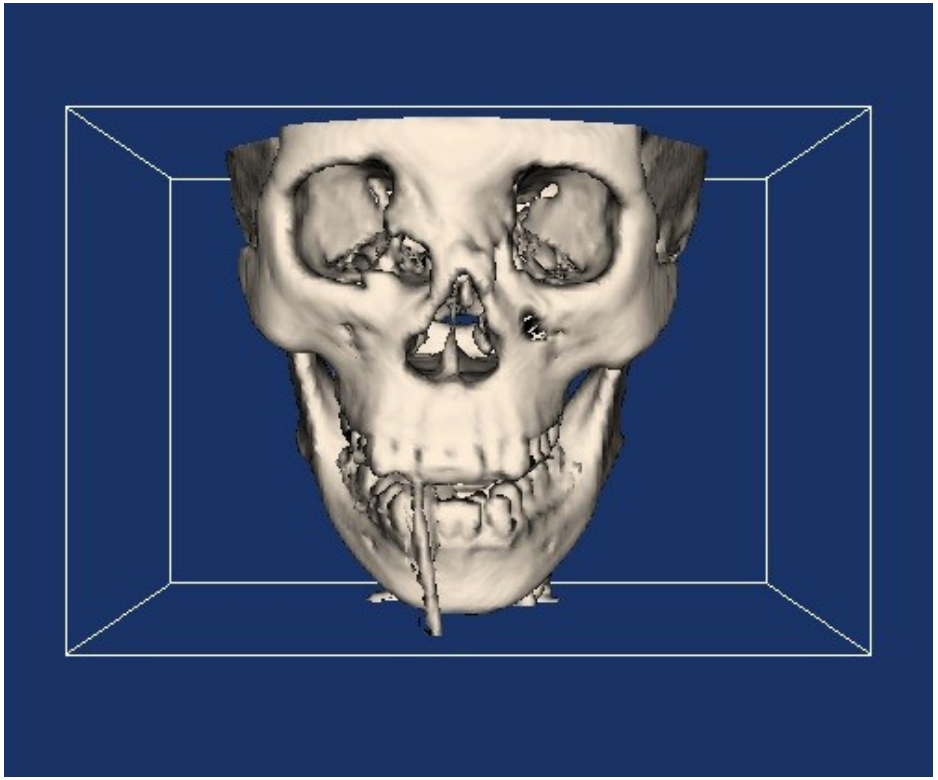


- Boundaries correspond to:
  - **lines in 2D (known as isobars)**
  - **surfaces in 3D (known as isosurfaces)**

  - **of constant scalar value**

Example : contours

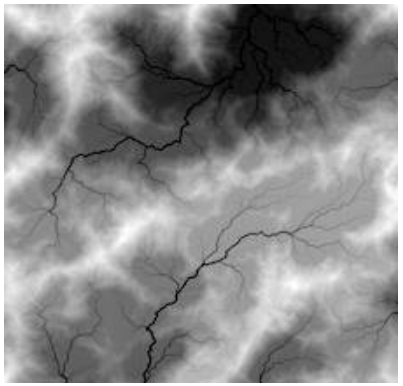


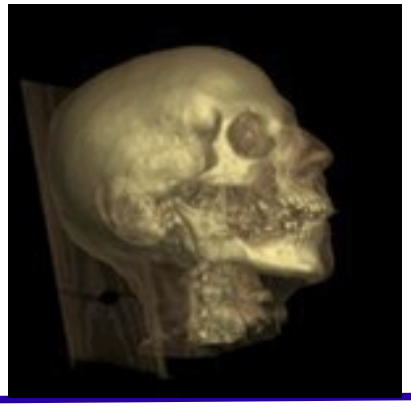


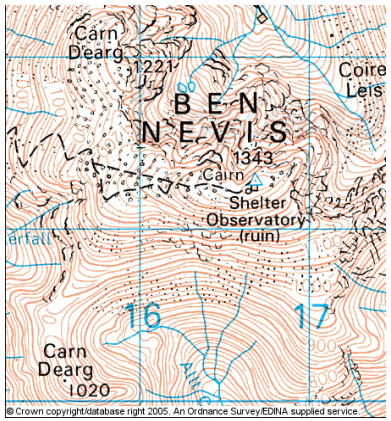
- lines of constant pressure on a weather map (isobars)
- surfaces of constant density in medical scan (isosurface)
  - “iso” roughly means equal / similar / same as

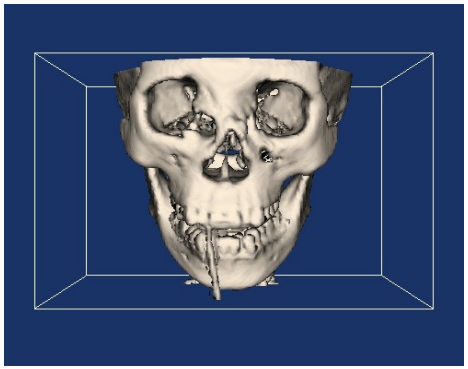
Contouring

- Input : 2D or 3D grid with scalar values at the nodes
- Output : Contours (polylines, polygons) that connect the vertices with the same scalar value









2D Contour

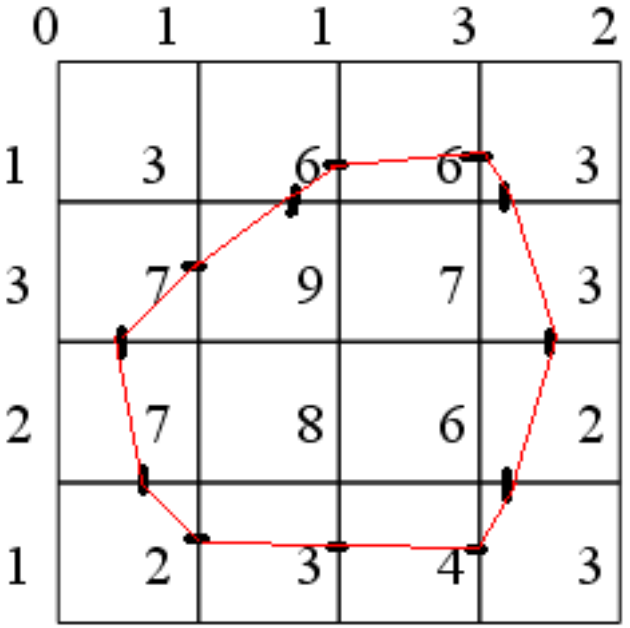
- **Input Data:** 2D structured grid of scalar values

	0	1	1	3	2
1	3	6	6	3	
3	7	9	7	3	
2	7	8	6	2	
1	2	3	4	3	

- Difficult to visualise transitions in data
  - use **contour** at specific scalar value to highlight **transition**
- What is the contour of 5**?**

Methods of Contour Line Generation

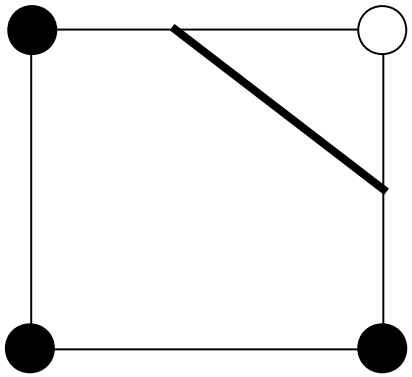
- **Approach 1 : Tracking**
  - find contour intersection with an edge
  - track it through the cell boundaries
    - if it enters a cell then it must exit via one of the boundaries
    - track until it connects back onto itself or exits dataset boundary
  - If it is known to be only one contour, stop
  - *otherwise*
  - Check every edge
- **Approach 2 : Marching Squares Algorithm**



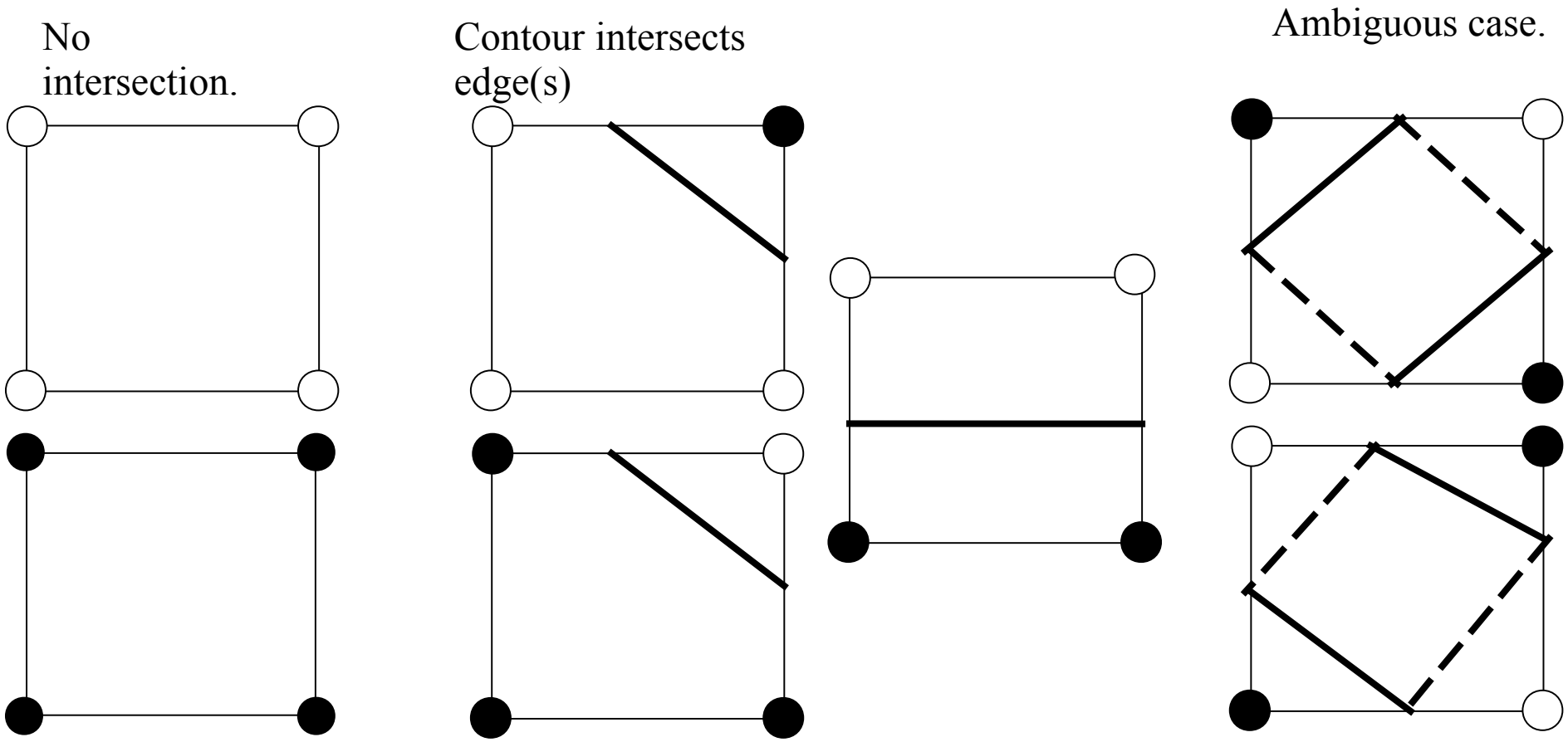
Marching Squares Algorithm

- **Focus : intersection of contour and cell edges**
  - **how the contour passes through the cell**
- **Assumption: a contour can pass through a cell in only a finite number of ways**
  - **A vertex is inside contour if scalar value > contour**

    outside contour if scalar value < contour
  - **4 vertices, 2 states (in or out)**

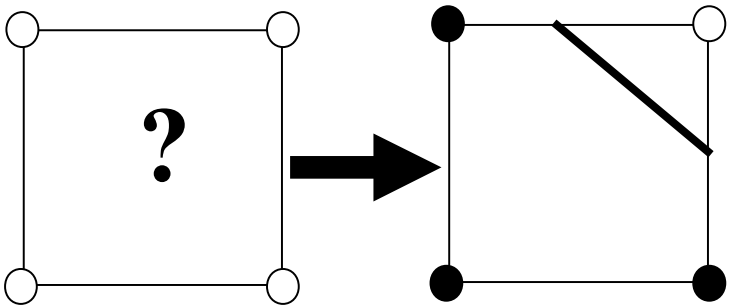


Marching Squares



- 2^4 = 16 possible cases for each square
  - small number so just treat each one separately

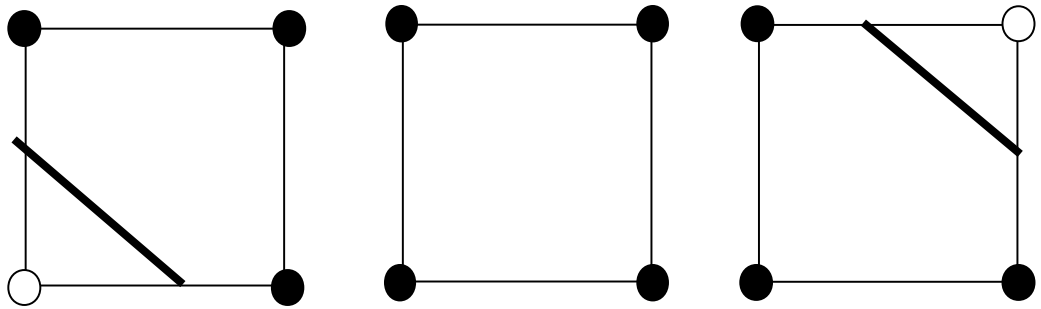
MS Algorithm Overview



- **Main algorithm**
  1. Select a cell
  2. Calculate inside/outside state for each vertex
  3. Look up topological state of cell in state table

     determine which edge must be intersected (i.e. which of the 16 cases)
  4. Calculate contour location for each intersected edge
  5. Move (or **march) onto next cell**
     - **until all cells are visited GOTO 2**

MS Algorithm - notes

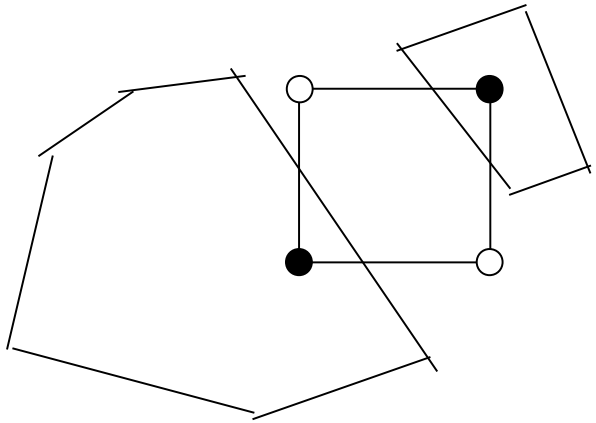


- Intersections for each cell must be merged to form a complete contour
  - cells processed independently
  - further **“merging” computation required**
  - disadvantage over tracking (continuous tracked contour)

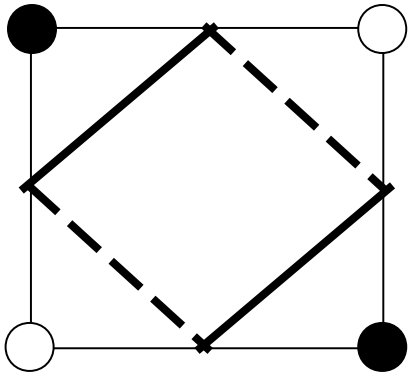
- Easy to implement (also to extend to 3D)
- Easy to parallelise

MS : Dealing with ambiguity ?

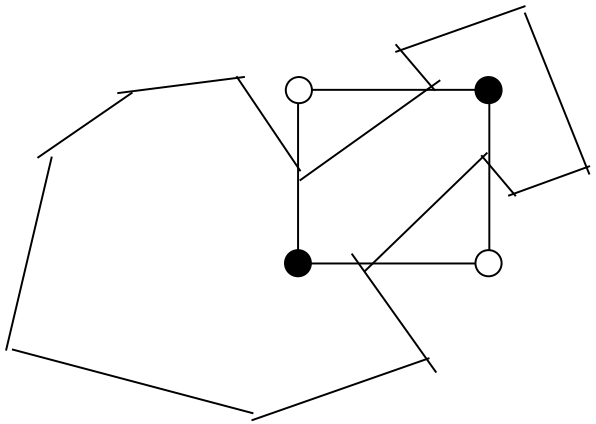
Split

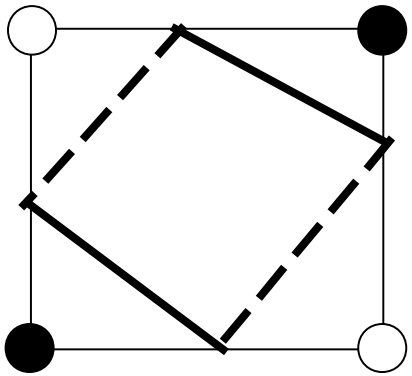


Ambiguous case.



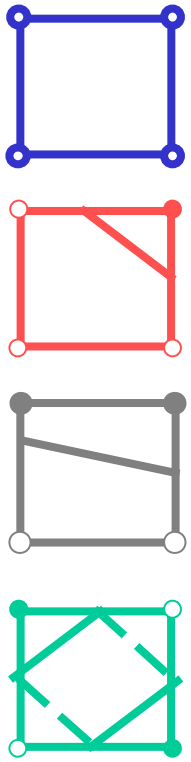
Join





- Choice independent of other choices
  - either valid : both give continuous and closed contour

Example : Contour Line Generation

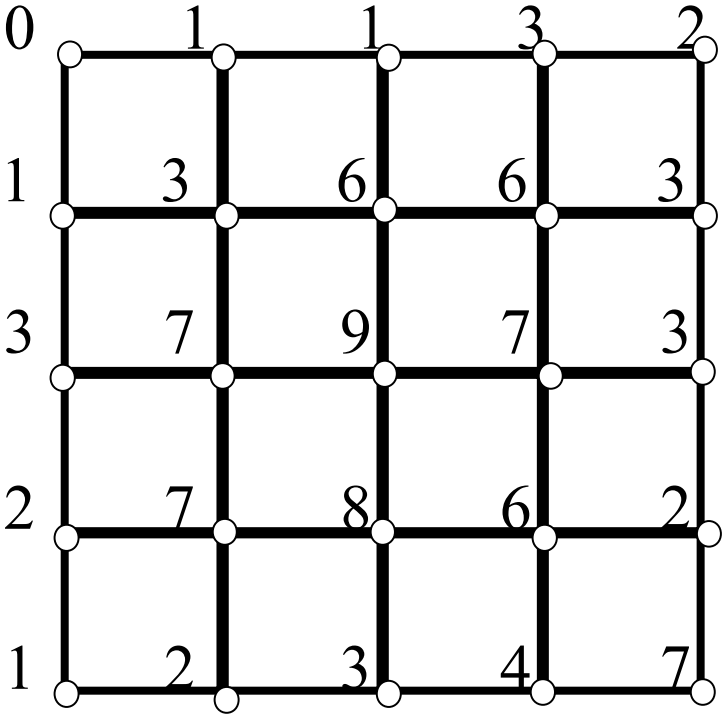


No intersection.

Contour intersects 1 edge

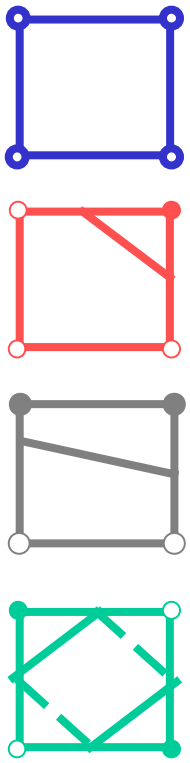
Contour intersects 2 edges

Ambiguous case.



- 3 main steps for each cell
  - here using simplified summary model of cases

Step 1 : classify vertices



No intersection.

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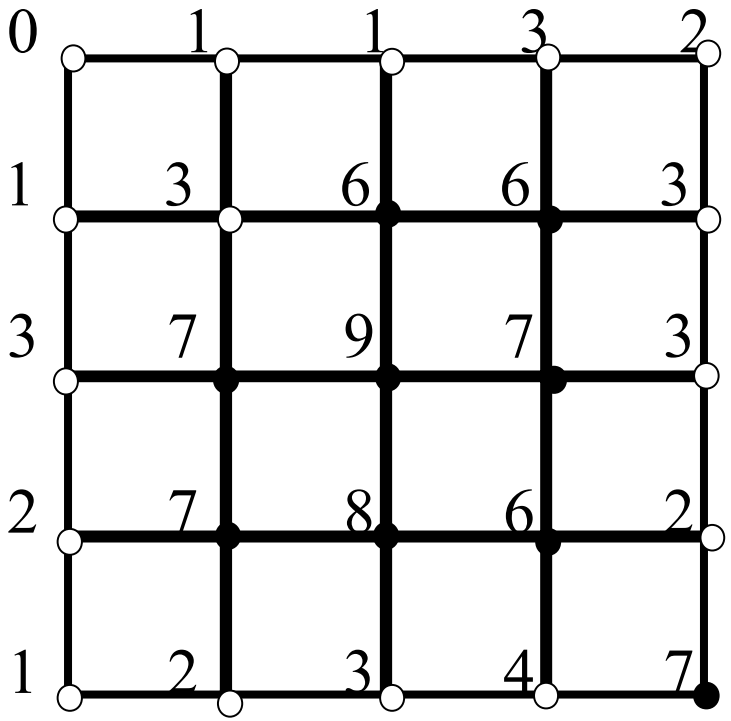
Contour intersects 2 edges

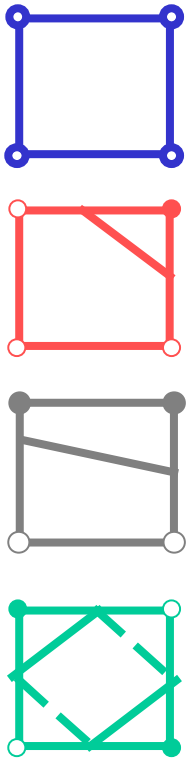
Ambiguous case.



Contour value =5

- Decide whether each vertex is inside or outside contour

Step 2 : identify cases



No intersection.

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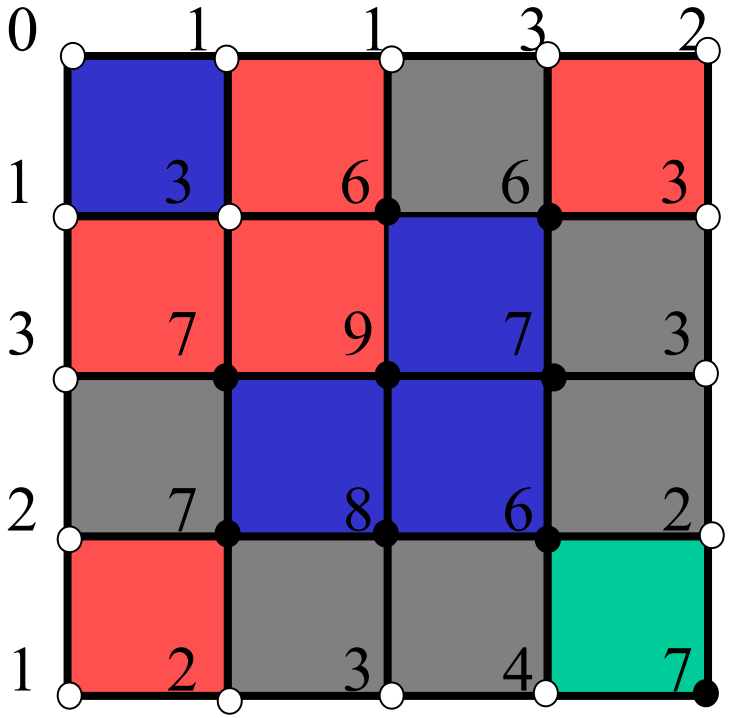
Contour intersects 2 edges

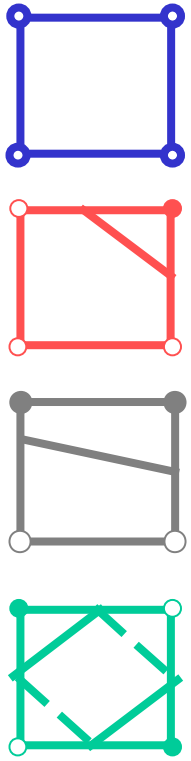
Ambiguous case.



Contour value =5

- Classify each cell as one of the cases

Step 3 : interpolate contour intersections

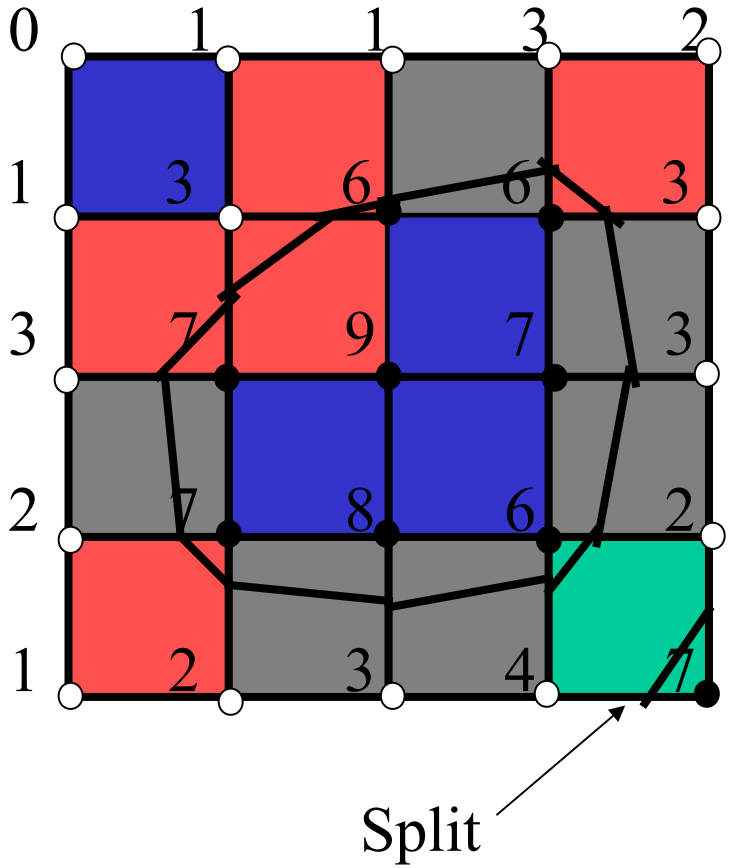


No intersection.

Contour intersects 1 edge

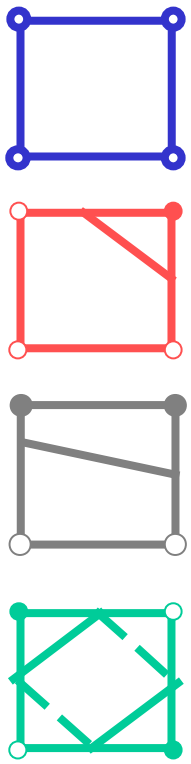
Contour intersects 2 edges

Ambiguous case.



- Determine the edges that are intersected
  - compute contour intersection with each of these edges

Ambiguous contour

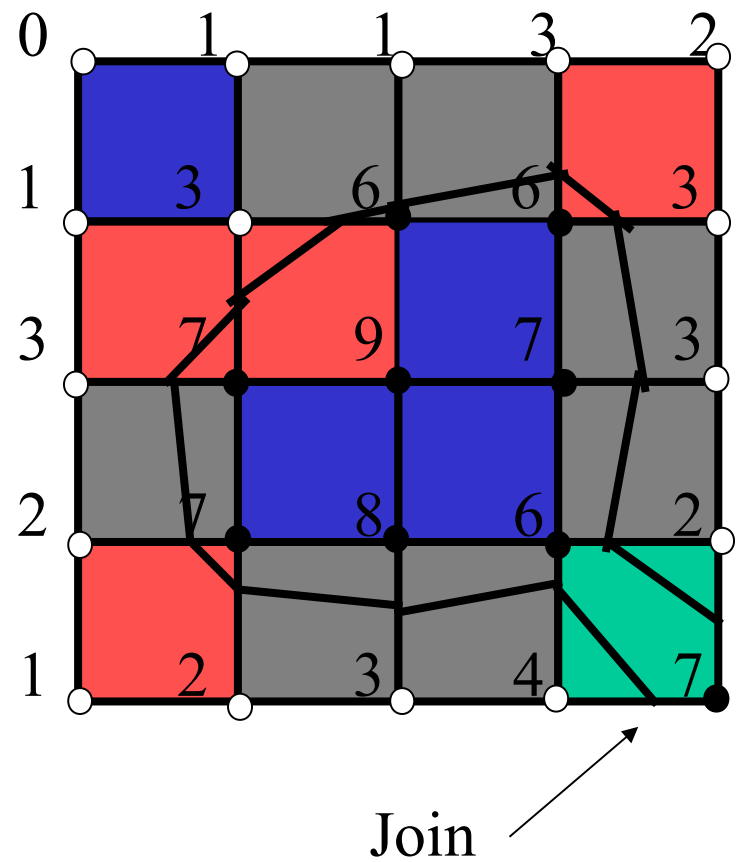


No intersection.

Contour intersects 1 edge

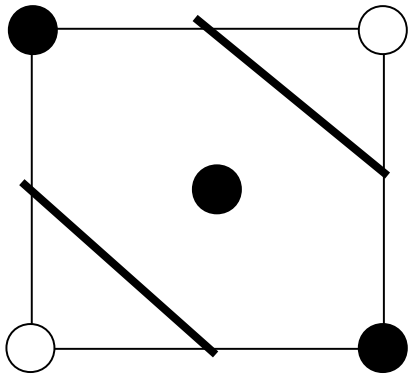
Contour intersects 2 edges

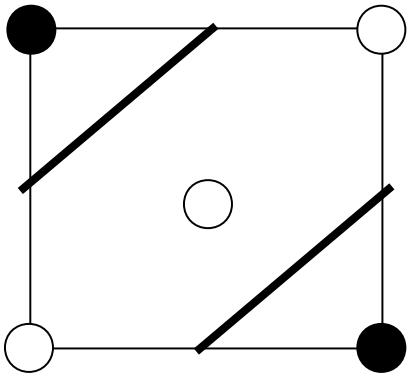
Ambiguous case.



- Finally : resolve any ambiguity
  - here choosing “join” (example only)

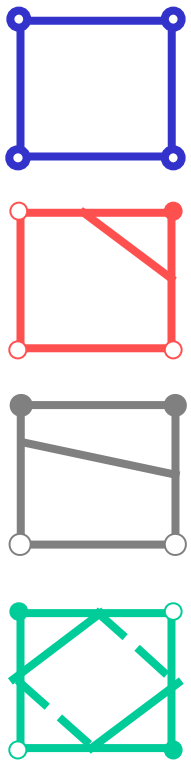
MS : Dealing with ambiguity ? One solution





- Calculate the value at the middle of the square by interpolation
- Check if it is under or above the threshold value
- Choose the pattern that matches

Ambiguous contour



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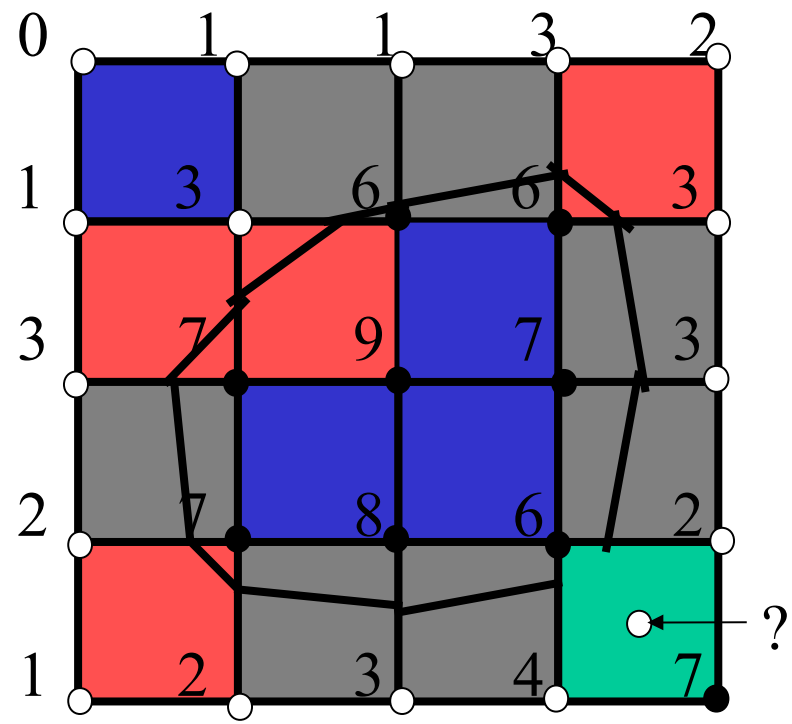
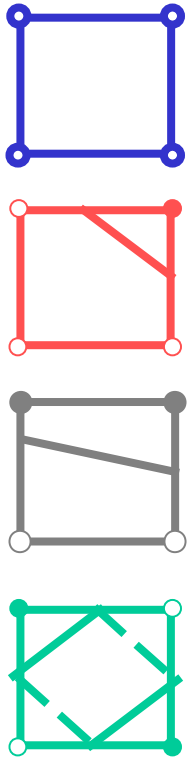
Ambiguous case.



Step 3 : interpolate contour intersections



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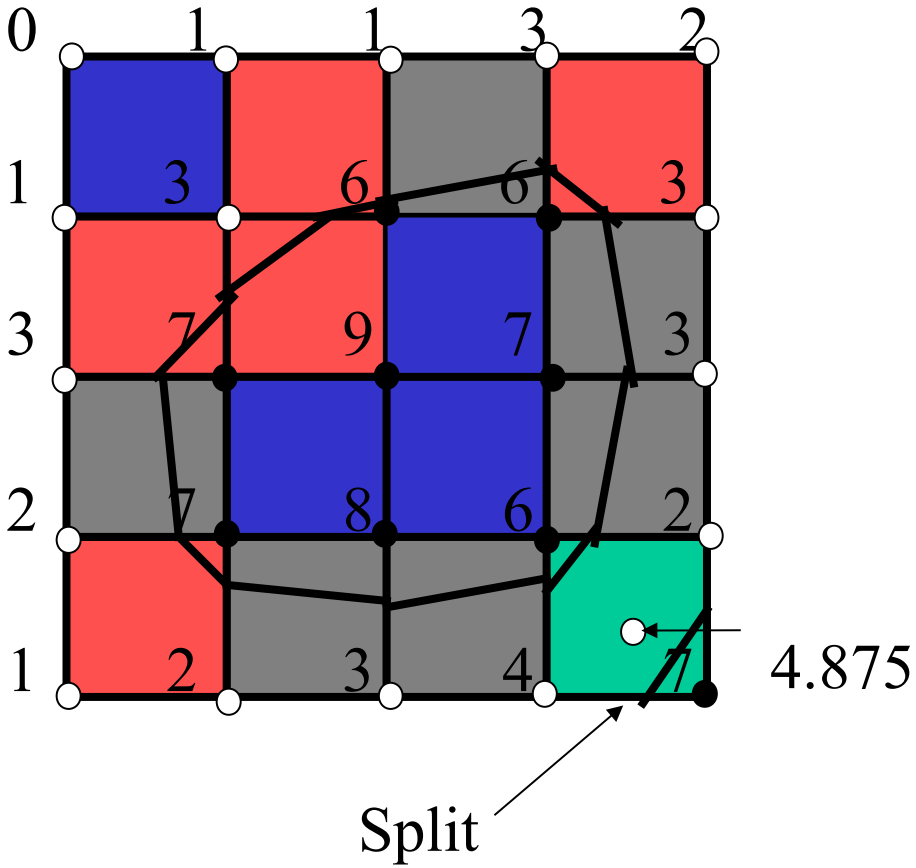
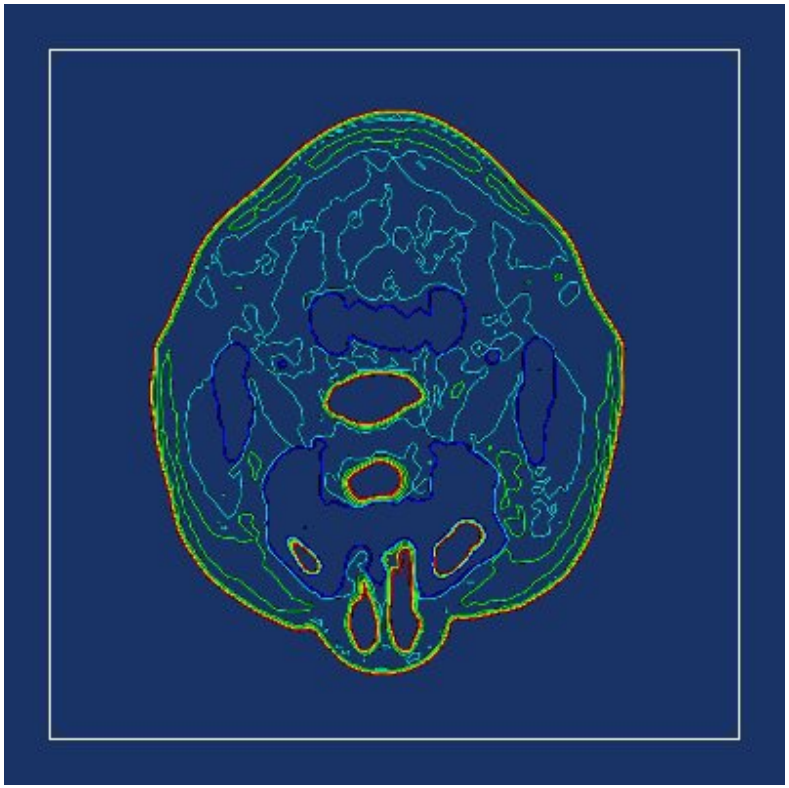
Ambiguous case.



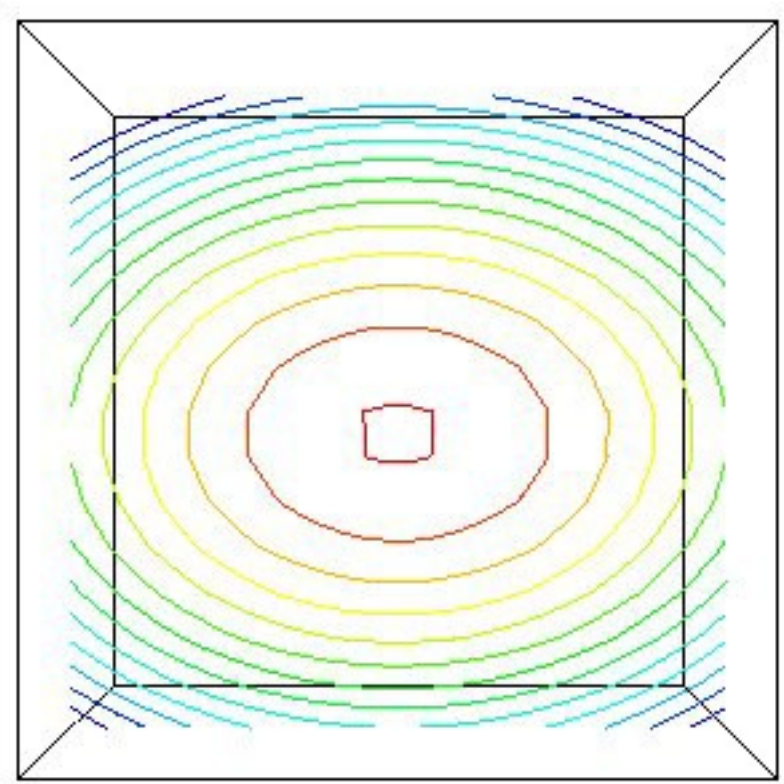
2D : Example contour

A slice through the head



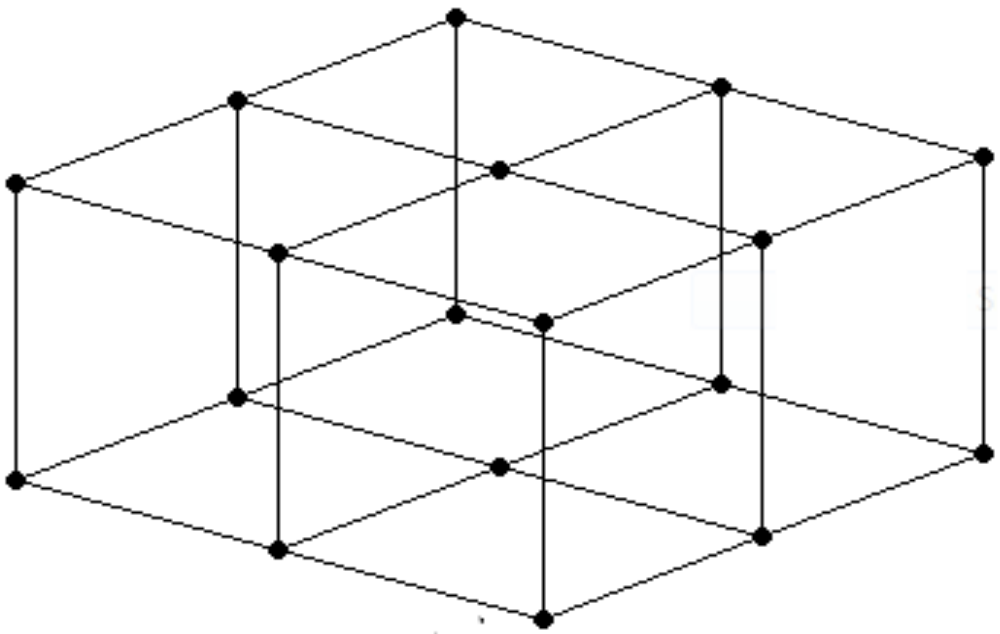
(with colour mapping added)

A Quadric function.



3D surfaces : marching cubes

- Extension of Marching Squares to **3D**
  - **data** : 3D regular grid of scalar values
  - **result :** 3D surface boundary instead of 2D line boundary



3D surfaces : marching cubes

- Extension of Marching Squares to **3D**
  - **data** : 3D regular grid of scalar values
  - **result :** 3D surface boundary instead of 2D line boundary
  - 3D cube has 8 vertices → 2^8 = 256 cases to consider
    - use symmetry to reduce to 15

- **Problem : ambiguous cases**
  - cannot simply choose arbitrarily as choice is determined by neighbours
  - poor choice may leave hole artifact in surface

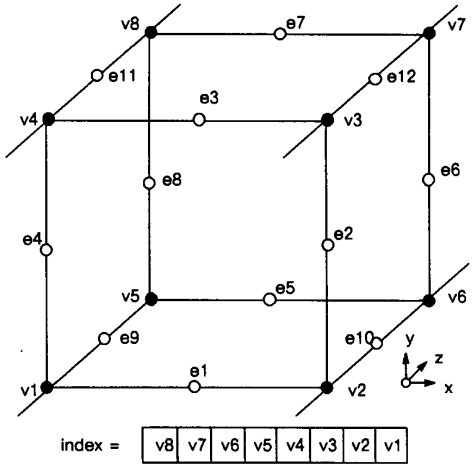


Figure 4. Cube Numbering.

Marching Cubes - cases

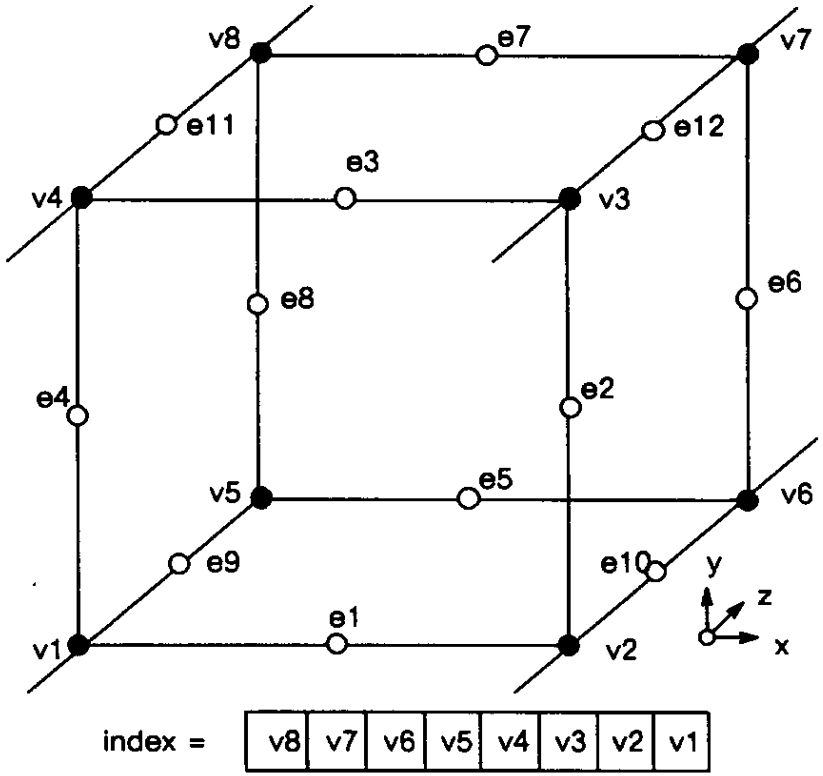


Figure 4. Cube Numbering.

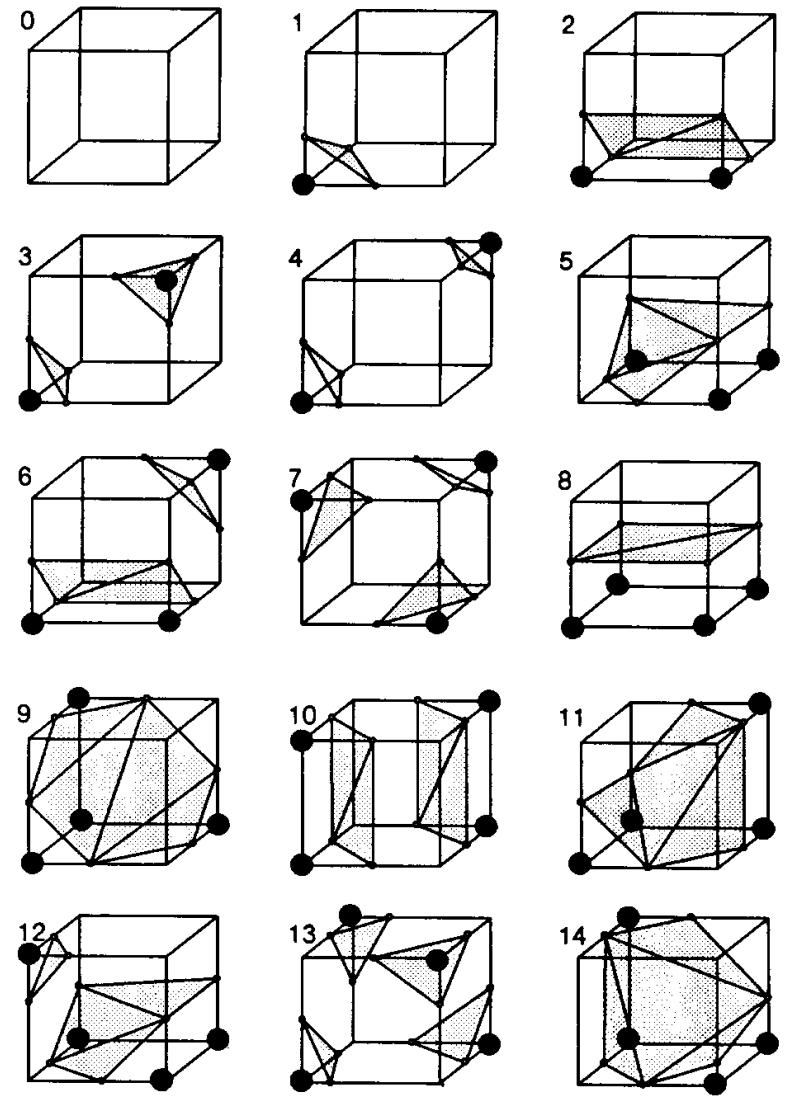
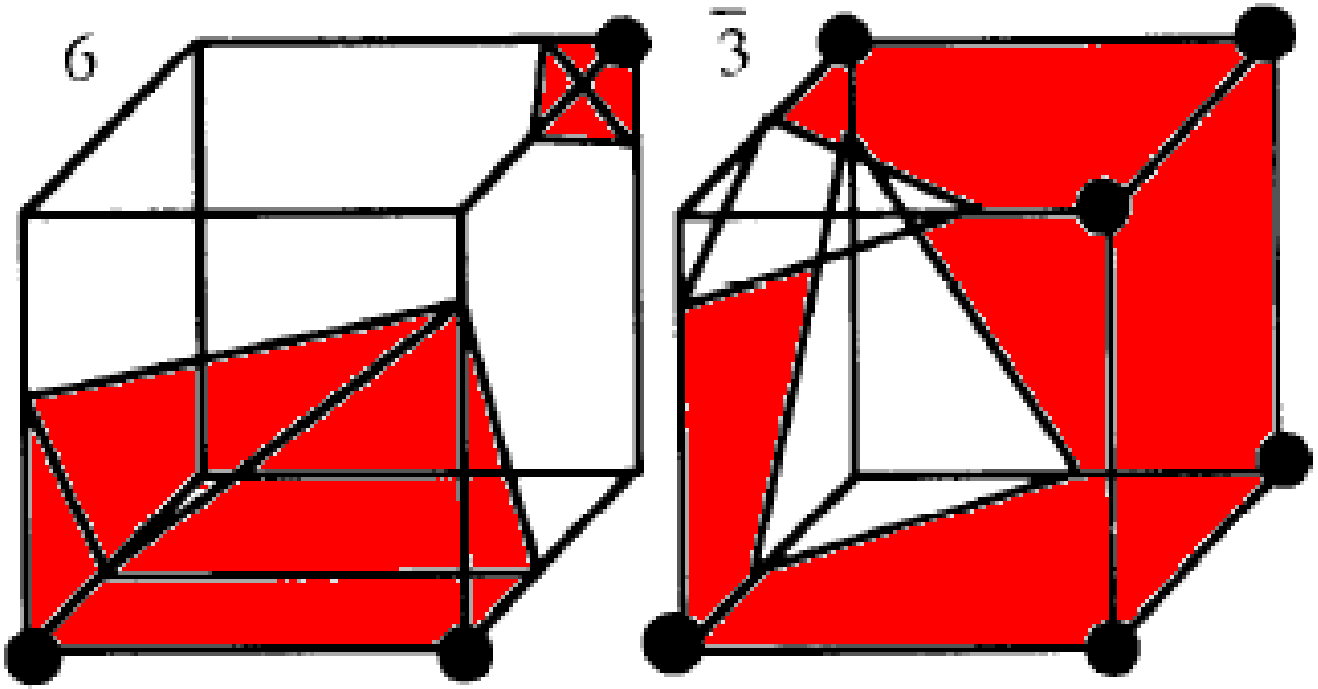


Figure 3. Triangulated Cubes.

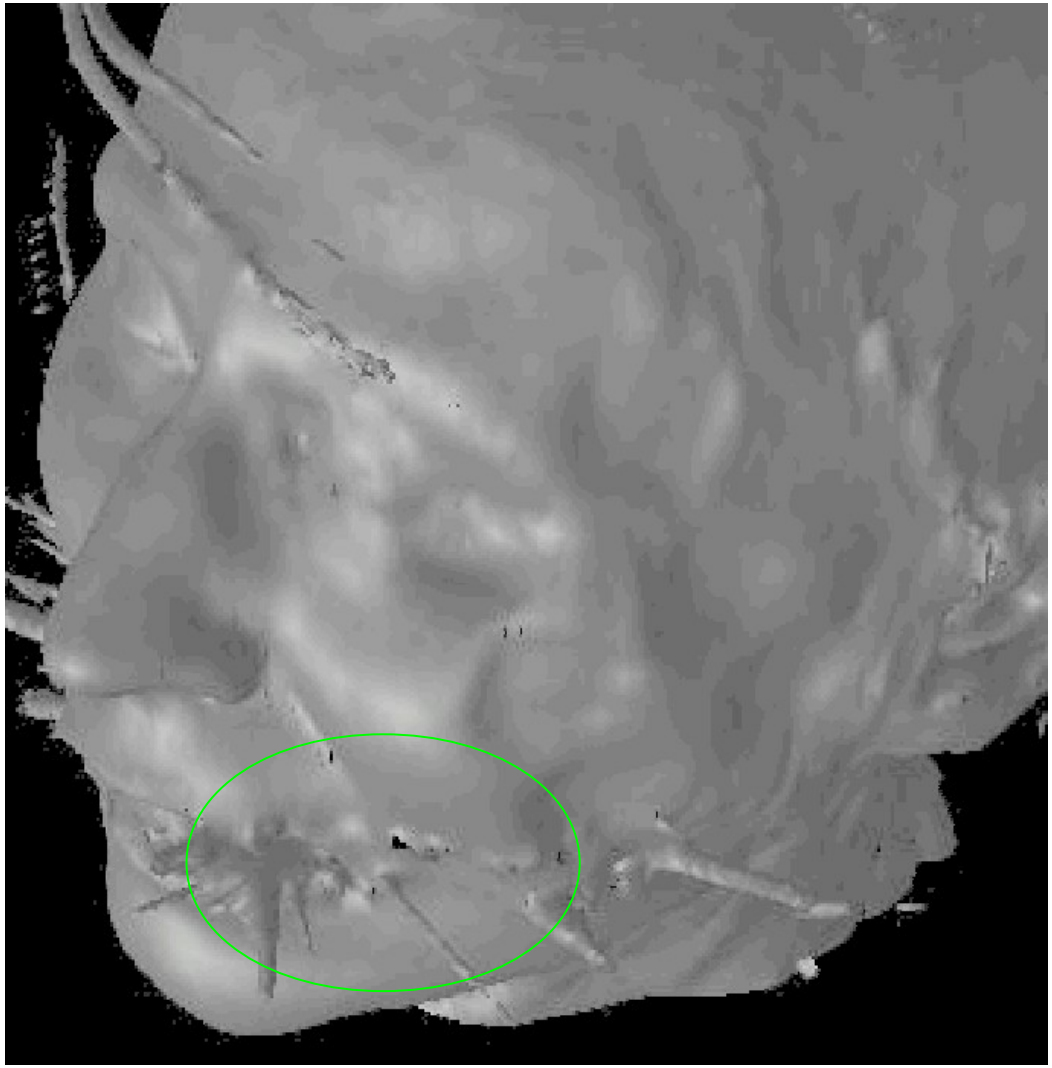
- Ambiguous cases
  - e.g., 3,6,10,12,13 – split or join ?

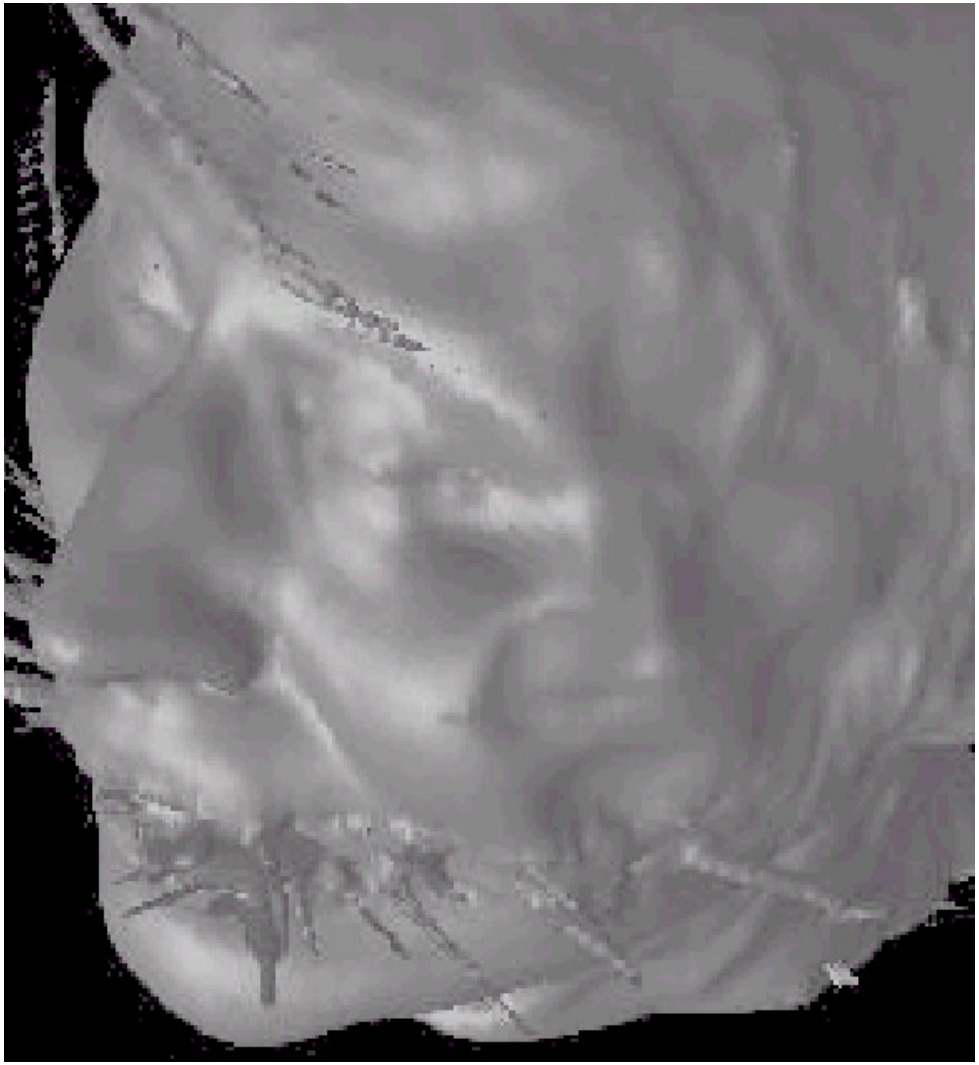
Example of bad choices



- The dark dots are the interior
- There are edges which are not shared by both cubes
- Need to make sure there is no contradiction with the neighbors

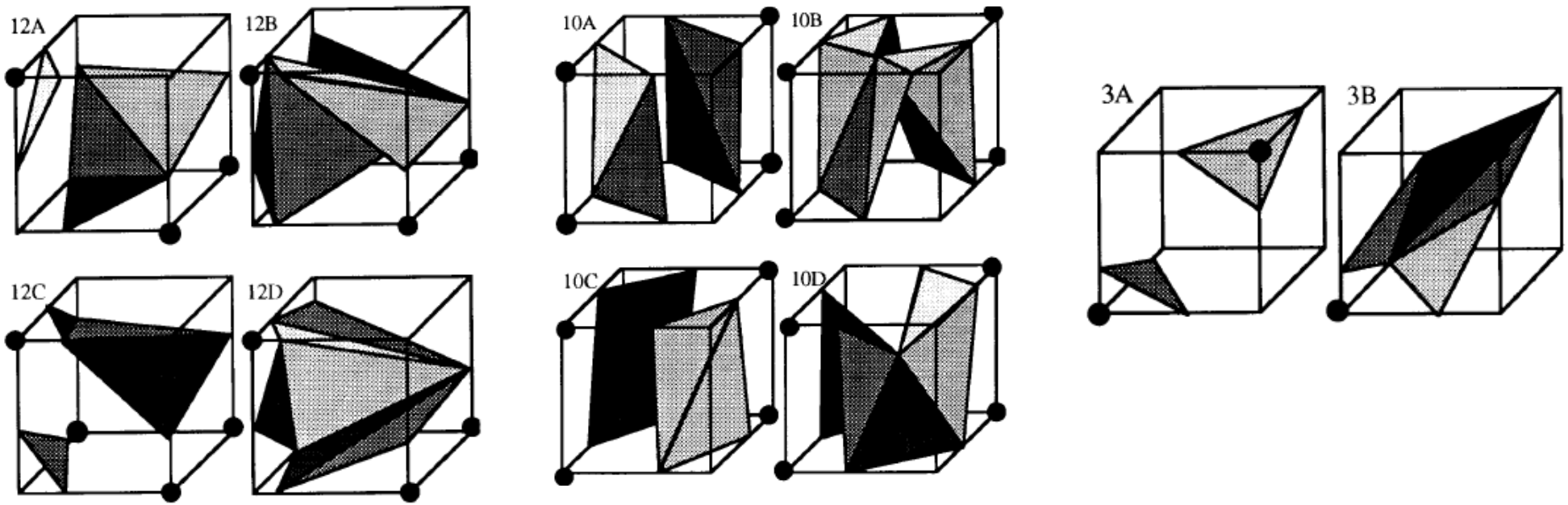
Cracks eliminated



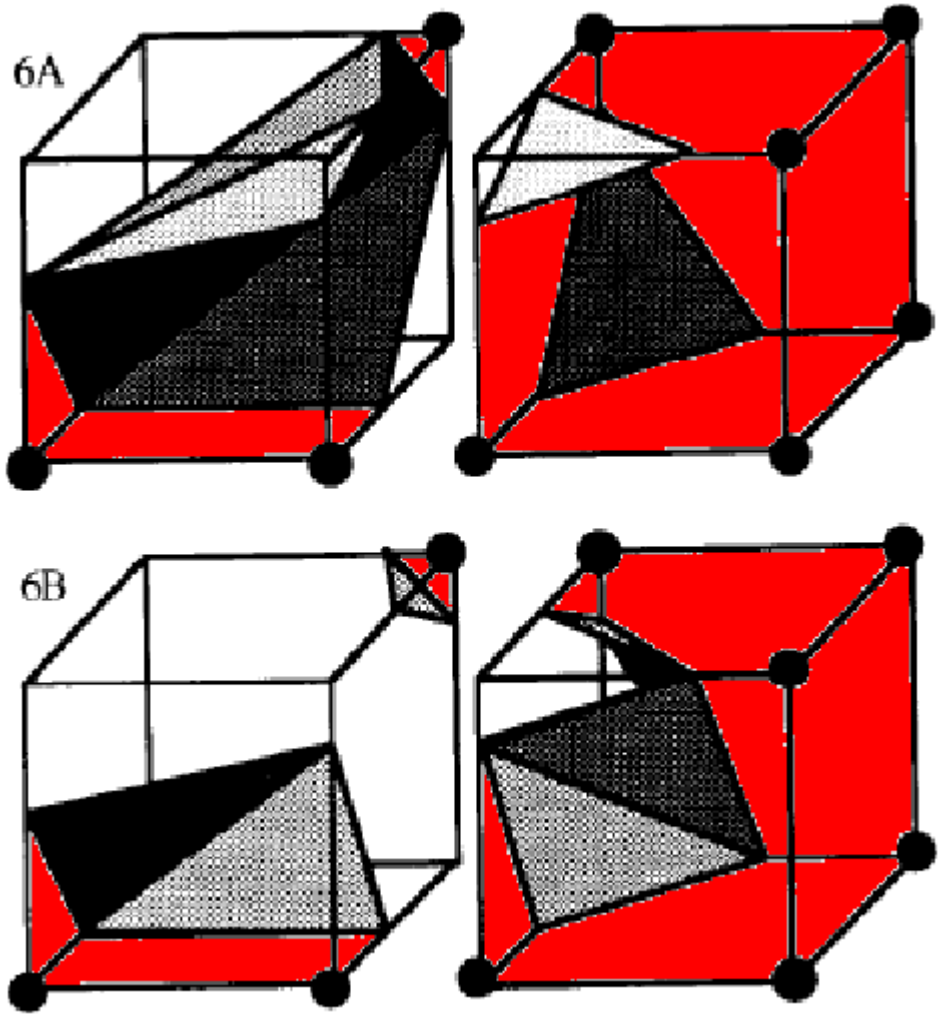


Adding more patterns

- Adding more patterns for 3,6,10,12,13 [Neilson '91]
- Compute the values at the middle of the faces and the cubes
- Selecting the pattern that matches



Other two possible triangulations



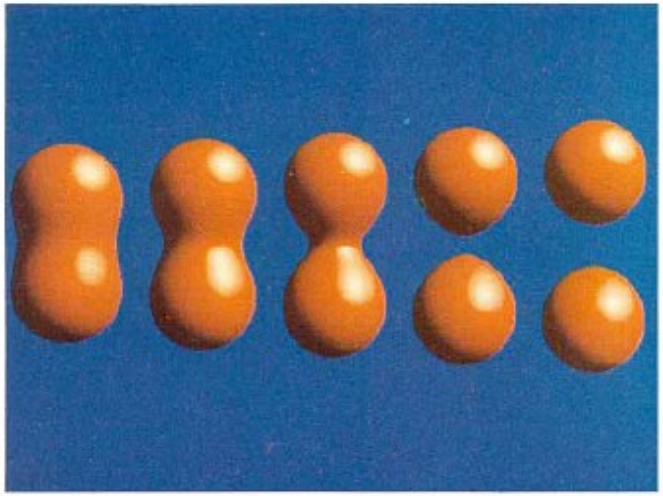
- Need to decide how the faces are intersected by the contours

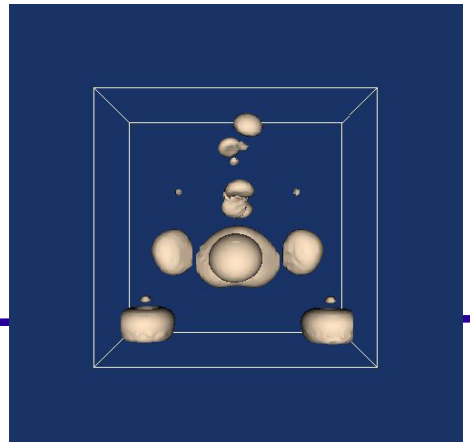
Rendering Implicit Surfaces

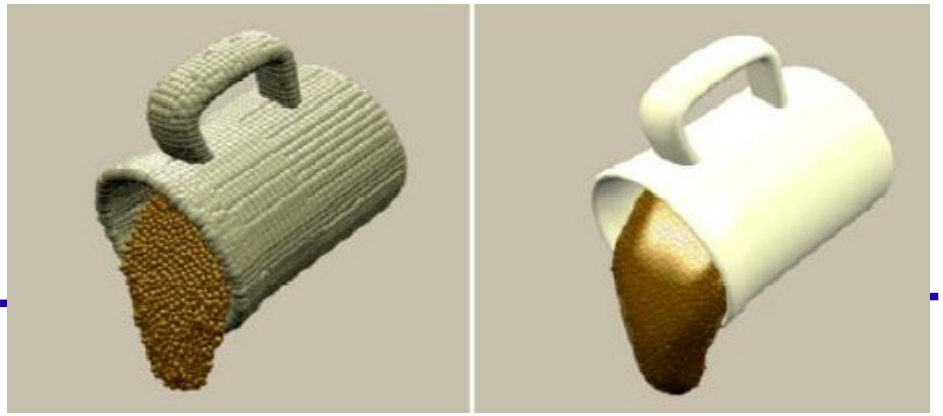
The marching cubes algorithm is useful for rendering implicit surfaces where F(x,y,z) = 0

Inside : F(x,y,z) > 0

Outside: F(x,y,z) < 0







Marching Cubes by CUDA

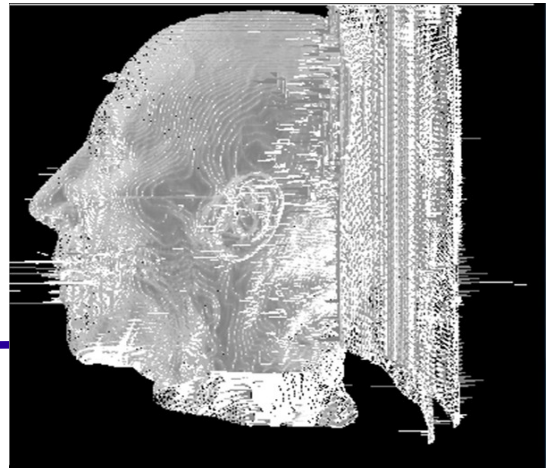
- http://www.youtube.com/watch?v=Y5URxpX8q8U

Dividing Cubes Algorithm

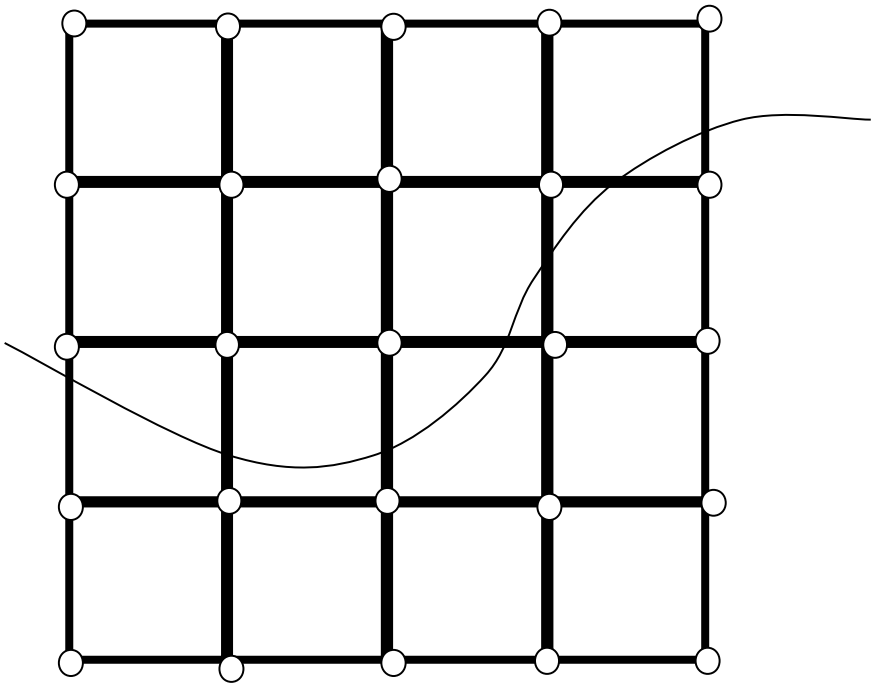
- **Marching cubes : Problem**
  - Often **produces more polygons than pixels** for given rendering scale
  - **Problem :** causes **high rendering overhead**

- **Solution : Dividing Cubes Algorithm**
  - **Draw points instead of polygons** ***(faster rendering)***
  - **Need 1: efficient method to find points on surface**

    2: method to shade points



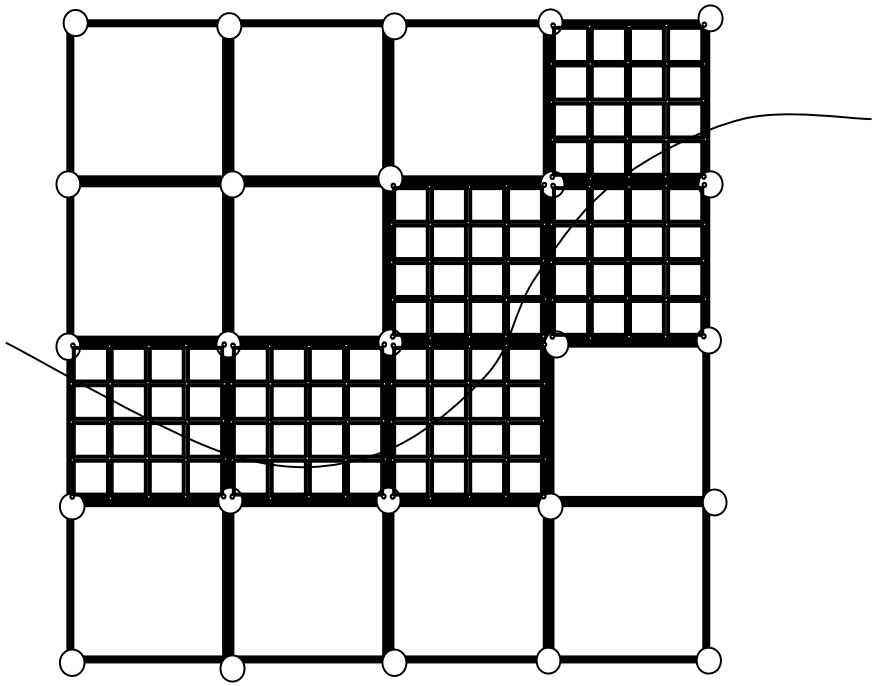
Example : 2D divided squares for 2D lines



Find pixels that intersect contour

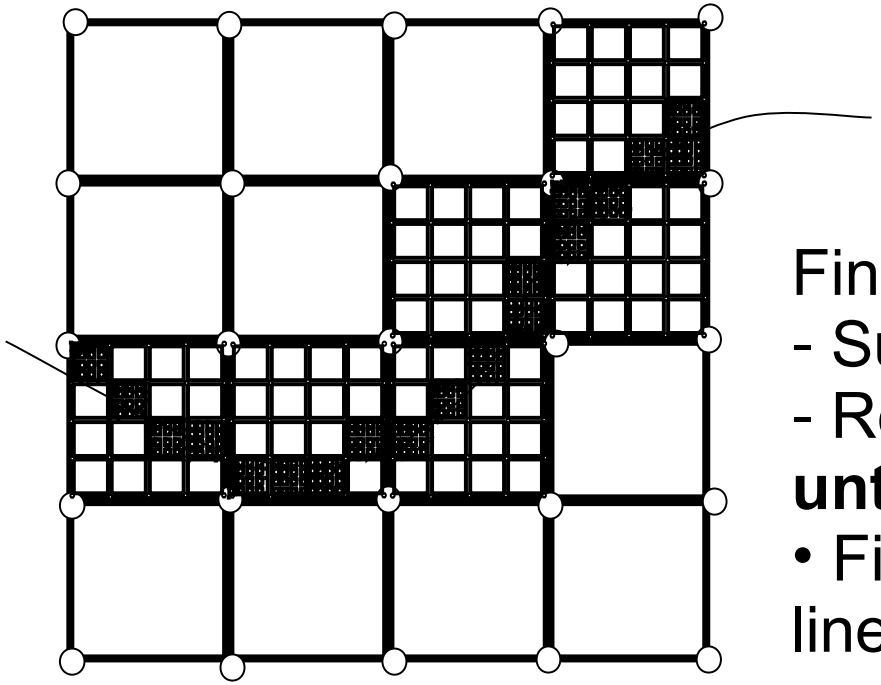
- Subdivide them

2D “divided squares” for lines



Find pixels that intersect line
- **Subdivide them** (usually in 2x2)
- **Repeat recursively**

2D “divided squares” for lines



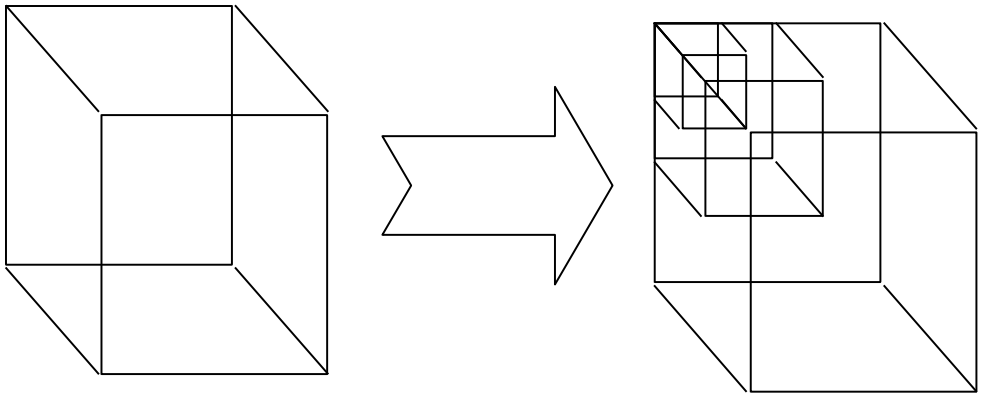
Find pixels that intersect line
- Subdivide them
- Repeat recursively

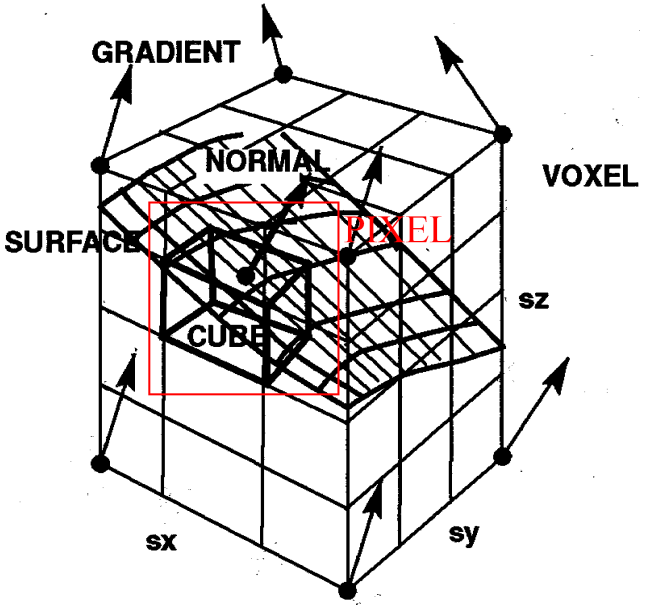
until screen resolution reached

• Fill in the pixel with the color of the line

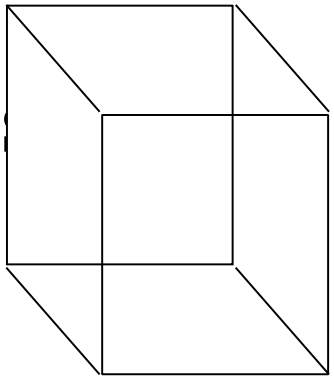
Extension to 3D

- Find **voxels** which intersect **surface**
- Recursively subdivide the voxels that intersect the contour
  - Until the voxel fits within a pixel
- Calculate **mid-points of voxels**
- Calculate the color of the pixel by shading





Drawing divided cubes surfaces



- **surface normal** for lighting calculation
  - interpolate **from voxel corner points**

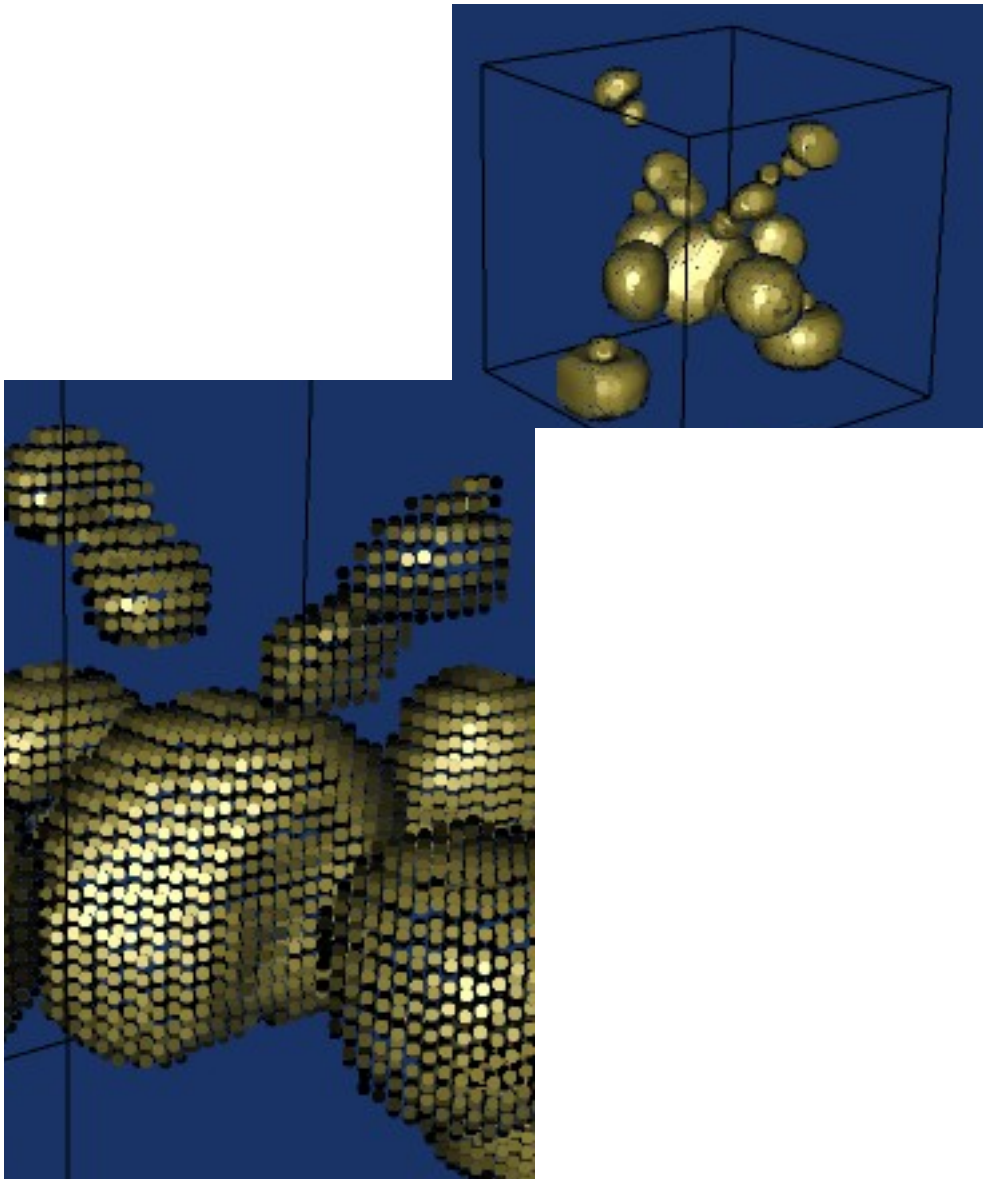
$$\left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) \approx \left(\frac{F^{x+\Delta x} - F^{x-\Delta x}}{2\Delta \mathrm{x}}, \frac{F^{y+\Delta y} - F^{y-\Delta y}}{2\Delta \mathrm{y}}, \frac{F^{z+\Delta z} - F^{z-\Delta z}}{2\Delta \mathrm{z}}\right)$$

- **problem with camera zoom**
  - ideally dynamically re-calculate points
  - not always computationally possible

Dividing Cubes : Example

50,000 points

when sampling less than screen resolution structure of surface can be seen



Summary

- **Contouring Theory**
  - 2D : **Marching Squares Algorithm**
  - 3D : **Marching Cubes Algorithm** [Lorensen '87]
    - marching tetrahedra, ambiguity resolution
    - limited to regular structured grids
  - 3D Rendering : **Dividing Cubes Algorithm** [Cline '88]

Readings

- G.M. Nielson, B Hamann, “The Asymptotic Decider: Resolving the Ambiguity in Marching Cubes”
- W.E. Lorensen, H.E. Cline, “Marching Cubes: A high resolution 3D surface construction algorithm”
- H.E. Cline, W.E. Lorensen and S. Ludke, “Two algorithms for the three-dimensional reconstruction of tomograms”