Tensor Visualisation

Computer Animation and Visualisation
Lecture 15

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Overview

• Tensor Visualisation
  - What is tensor
  - Methods of visualization
    - 3D glyphs
    - vector and scalar field
    - hyper-streamlines
    - LIC in 3D volumes
Reminder : Attribute Data Types

• Scalar
  - colour mapping, contouring

• Vector
  - lines, glyphs, stream {lines | ribbons | surfaces}

• Tensor
  - complex problem
  - today: **simple techniques for tensor visualisation**
What is a tensor?

- A tensor is a table of rank $k$ defined in $n$-dimensional space ($\mathbb{R}^n$)
  - generalisation of vectors and matrices in $\mathbb{R}^n$
    - Rank 0 is a scalar
    - Rank 1 is a vector
    - Rank 2 is a matrix
    - Rank 3 is a regular 3D array

- $k$: rank defines the **topological dimension** of the attribute
  - i.e. it can be indexed with $k$ separate indices
- $n$: defines the **geometrical dimension** of the attribute
  - i.e. $k$ indices each in range 0→(n-1)
Tensors in $\mathbb{R}^3$

• Here we limit discussion to tensors in $\mathbb{R}^3$
  
  - In $\mathbb{R}^3$ a tensor of rank $k$ requires $3^k$ numbers
    
    - A tensor of rank 0 is a scalar \((3^0 = 1)\)
    - A tensor of rank 1 is a vector \((3^1 = 3)\)
    - A tensor of rank 2 is a 3x3 matrix (9 numbers)
    - A tensor of rank 3 is a 3x3x3 cube (27 numbers)

\[
V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad T = \begin{bmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{bmatrix}
\]

• We will only treat rank 2 tensors – i.e. matrices
Where do tensors come from?

• Stress/strain tensors
  - analysis in engineering

• DT-MRI
  - molecular diffusion measurements

• These are represented by 3x3 matrices
  - Or three normalized eigenvectors and three corresponding eigenvalues
Stresses and Strain

• The stress tensor:

In the direction of
\begin{align*}
  x: & \quad \sigma_{xx} \quad \sigma_{xy} \quad \sigma_{xz} \\
  y: & \quad \sigma_{yx} \quad \sigma_{yy} \quad \sigma_{yz} \\
  z: & \quad \sigma_{zx} \quad \sigma_{zy} \quad \sigma_{zz}
\end{align*}

stress on the face normal to \(x\):
\[ \sigma_{xx} \quad \sigma_{xy} \quad \sigma_{xz} \]
stress on the face normal to \(y\):
\[ \sigma_{yx} \quad \sigma_{yy} \quad \sigma_{yz} \]
stress on the face normal to \(z\):
\[ \sigma_{zx} \quad \sigma_{zy} \quad \sigma_{zz} \]

- A ‘normal’ stress is a stress perpendicular (i.e. normal) to a specified surface
- A shear stress acts tangentially to the surface orientation
- Stress tensor: characterised by principle axes of tensor
  - Eigenvalues (scale) of normal stress along eigenvectors (direction)
  - Form 3D co-ordinate system (locally) with mutually perpendicular axes
MRI : diffusion tensor

- Water molecules have **anisotropic diffusion** in the body due to the cell shape and membrane properties
  - Neural fibers : long cylindrical cells filled with fluid
  - Water diffusion rate is fastest along the axis
  - Slowest in the two transverse directions
  - **Brain functional imaging** by detecting the anisotropy

![Diagram of diffusion tensor](image)
Computing Eigenvectors

- 3x3 matrix results in **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)
- form 3D coordinate system (locally) with mutually perpendicular axes
- ordering by eigenvector referred to as **major**, **medium** and **minor eigenvectors**

![Diagram of eigenvectors and stress tensor]
Tensors : Visualisation Methods

- 2 main techniques : glyphs & vector methods

- Glyphs
  - 3D ellipses particularly appropriate (3 modes of variation)

- Vector methods
  - a symmetric rank 2 tensor can be visualised as 3 orthogonal vector fields (i.e. using eigenvectors)
  - hyper-streamline
  - Noise filtering algorithms – LIC variant
Tensor Glyphs

- **Ellipses**
  - rotated into coordinate system defined by eigenvectors of tensor
  - axes are scaled by the eigenvalues
  - very suitable as 3 modes of variation
- **Classes of tensor:**
  - (a,b) - large major eigenvalue
    - ellipse approximates a line
  - (c,d) - large major and medium eigenvalue
    - ellipse approximates a plane
  - (e,f) - all similar - ellipse approximates a sphere
Diffusion Tensor Visualisation

Anisotropic tensors indicate nerve pathway in brain:

- **Blue shape** – tensor approximates a line.

- **Yellow shape** – tensor approximates a plane.

- **Yellow transparent shape** – ellipse approximates a sphere

Colours needed due to **ambiguity in 3D shape** – a line tensor viewed ‘end-on’ looks like a sphere.
Stress Ellipses

- Force applied to dense 3D solid – resulting **stress at 3D position in structure**
- Ellipses visualise the stress tensor
- Tensor Eigenvalues:
  - Large **major eigenvalue** indicates principle direction of stress
  - ‘Temperature’ **colourmap** indicates size of major eigenvalue (magnitude of stress)
Lines, Hedgehogs

- Using hedgehogs to draw the three eigenvectors
  The length is the stress value
- Good for simple cases as above
  - Applying forces to the box
  - Green represents positive, red negative
Streamlines for tensor visualisation

- Each eigenvector defines a vector field
- Using the eigenvector to create the streamline
  - We can use the Major vector, the medium and the minor vector to generate 3 streamlines

Figure 8. Hyperstreamlines for minor, intermediate and major principal stress for a point-load.
Streamlines for tensor visualisation

- Often major eigenvector is used, with medium and minor shown by other properties
  - **Major vector is relevant in the case of anisotropy** - indicates nerve pathways or stress directions.

http://www.cmiv.liu.se/
Hyper-streamlines [Delmarcelle et al. '93]

- Construct a **streamline from vector field of major eigenvector**

- **Form ellipse together with medium and minor eigenvectors**
  - both are orthogonal to streamline direction
  - use major eigenvector as surface normal (i.e. orientation)

- **Sweep ellipse along streamline**
  - **Hyper-Streamline** *(type of stream polygon)*
LIC algorithm for tensors

• Linear Integral Convolution – LIC
  
  - ‘blurs’ a noise pattern with a vector field
  - For tensors
    - can apply ‘blur’ consecutively for 3 vector field
directions (of eigenvectors)
    - using result from previous blur as input to next stage
    - use volume rendering with opacity = image intensity
      value for display

(Sigfridsson et al. ’02)
Scalar field Method for Tensors

- Scalarfield: Produce grayscale image intensity in relation to tensor class (or closeness too). *(scalar from tensors)*

Greyscale image shows how closely the tensor ellipses approximate a line.

Greyscale image shows how closely the tensor ellipses approximate a plane.

Greyscale image shows how closely the tensor ellipses approximate a sphere.
Reading

• Processing and Visualization of Diffusion Tensor MRI [Westin et al. '02]

• Tensor field visualisation using adaptive filtering of noise fields combined with glyph rendering [Sigfridsson et al. '02]


• Westin et al. ‘02, “Processing and visualization for diffusion tensor MRI”