Tensor Visualisation and Information Visualisation

Computer Animation and Visualisation
Lecture 15

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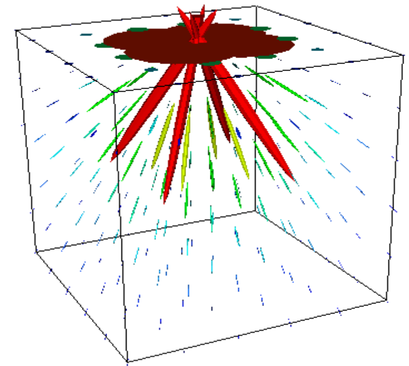
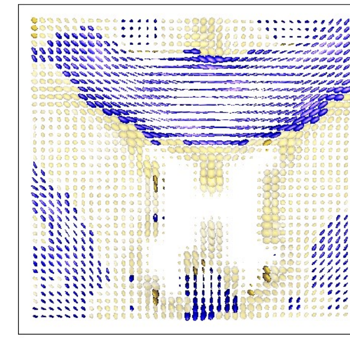
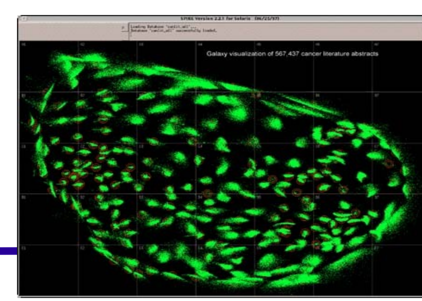
Overview

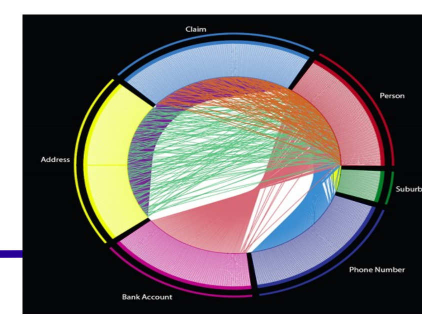
- **Tensor Visualisation**
  - **What is tensor**
  - **Methods of visualization**
    - **3D glyphs**
    - **vector and scalar field**
    - **hyper-streamlines**
    - **LIC** in 3D volumes



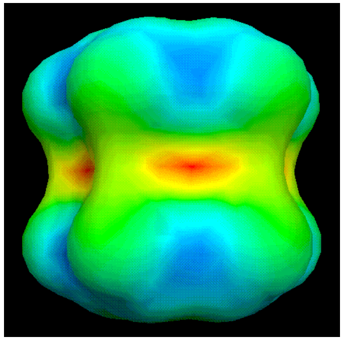
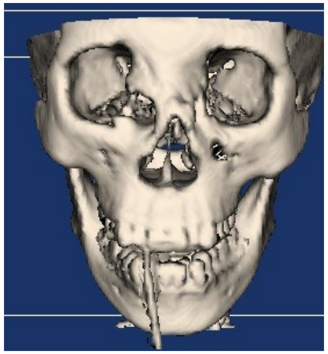


- Information Visualisation
  - Univariate, bivariate, trivariate, multi-variate data
  - Relations visualized by lines, tree visualization
  - Document visualization

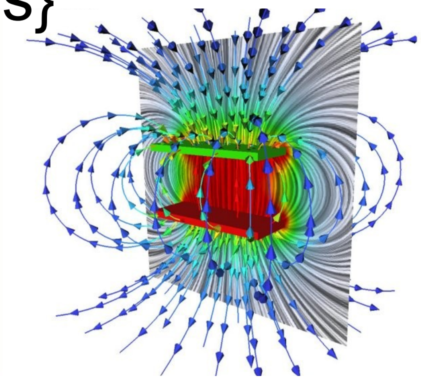




Reminder : Attribute Data Types

- **Scalar**
  - colour mapping, contouring
- **Vector**
  - lines, glyphs, stream {lines | ribbons | surfaces}



- **Tensor**
  - complex problem
  - today : **simple techniques for tensor visualisation**

What is a tensor ?

- A tensor is a table of rank *k* defined in *n*-dimensional space ($\mathbb{R}^n$)
  - generalisation of vectors and matrices in $\mathbb{R}^n$
    - Rank 0 is a scalar
    - Rank 1 is a vector
    - Rank 2 is a matrix
    - Rank 3 is a regular 3D array
  - *k* : rank defines the **topological dimension** of the attribute
    - i.e. it can be indexed with *k* separate indices
  - *n* : defines the ***geometrical dimension*** of the attribute
    - i.e. k indices each in range $0 \rightarrow (n-1)$

Tensors in $\mathbb{R}^3$

- Here we limit discussion to tensors in $\mathbb{R}^3$
  - In $\mathbb{R}^3$ a tensor of rank *k* requires 3^k numbers
    - A tensor of rank 0 is a scalar *($3^0 = 1$)*
    - A tensor of rank 1 is a vector *($3^1 = 3$)*
    - A tensor of rank 2 is a 3x3 matrix (9 numbers)
    - A tensor of rank 3 is a 3x3x3 cube (27 numbers)

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad T = \begin{bmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{bmatrix}$$

- We will only treat rank 2 tensors – i.e. matrices

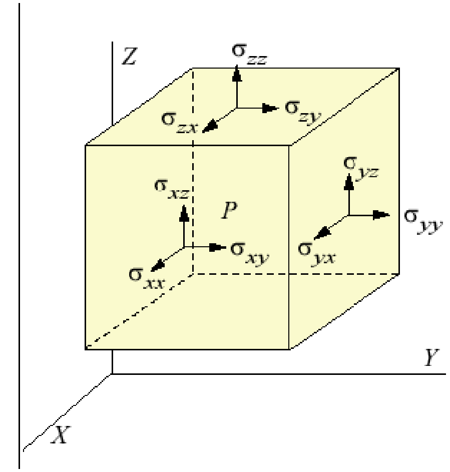
Where do tensors come from?

- **Stress/strain tensors**
  - analysis in engineering
- **DT-MRI**
  - molecular diffusion measurements
- *These are represented by 3x3 matrices*
  - *Or three normalized eigenvectors and three corresponding eigenvalues*

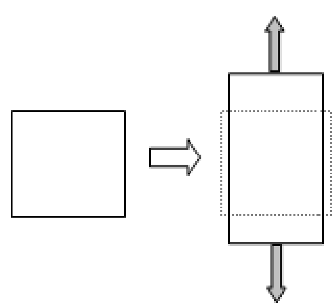
Stresses and Strain 1

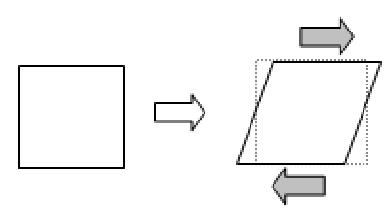
- The **stress tensor:**

	In the direction of x:	y:	z:
stress on the face normal to x:	σ_{xx}	σ_{xy}	σ_{xz}
stress on the face normal to y:	σ_{yx}	σ_{yy}	σ_{yz}
stress on the face normal to z:	σ_{zx}	σ_{zy}	σ_{zz}



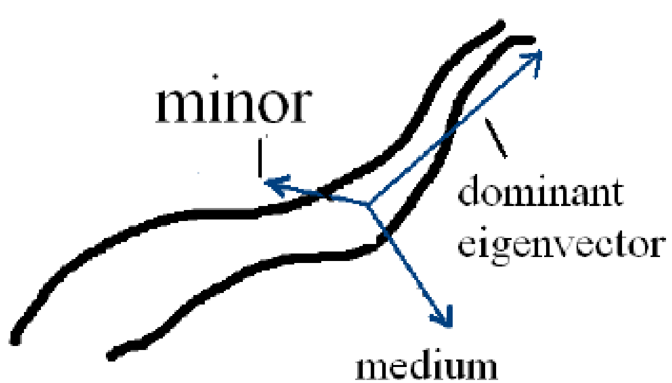
  - A **'normal' stress is a stress perpendicular (i.e. normal)** to a specified surface
  - A **shear stress acts tangentially** to the surface orientation
  - Stress tensor : characterised by **principle axes of tensor**
    - **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)
    - Form 3D co-ordinate system (locally) with mutually perpendicular axes





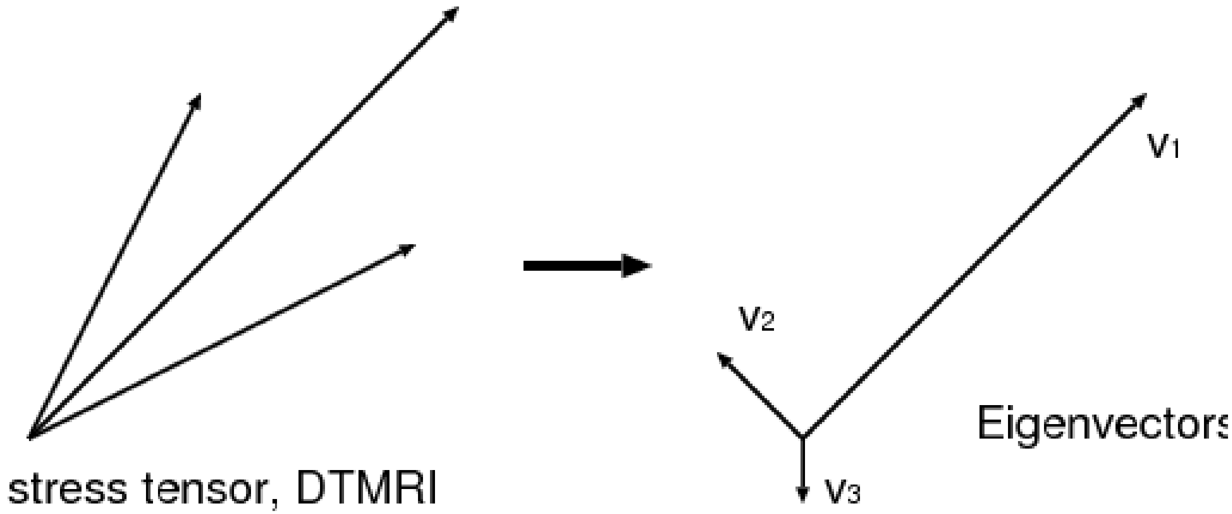
MRI : diffusion tensor

- Water molecules have **anisotropic diffusion in the body due to the cell shape and membrane properties**
  - Neural fibers : long cylindrical cells filled with fluid
  - Water diffusion rate is fastest along the axis
  - Slowest in the two transverse directions
  - brain functional imaging by detecting the anisotropy



Computing Eigenvectors

- 3x3 matrix results in **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)
- form 3D coordinate system (locally) with mutually perpendicular axes
- ordering by eigenvector referred to as **major, medium and minor eigenvectors**



Tensors : Visualisation Methods

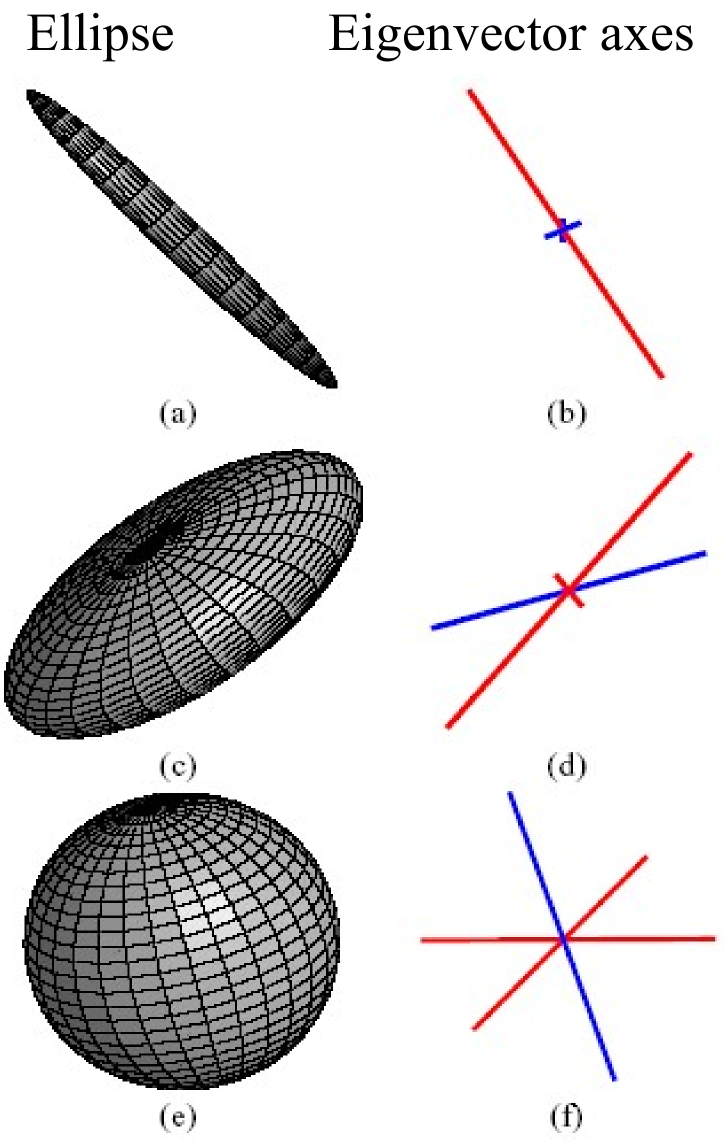
- 2 main techniques : glyphs & vector methods

- **Glyphs**
  - 3D ellipsoids particularly appropriate (3 modes of variation)

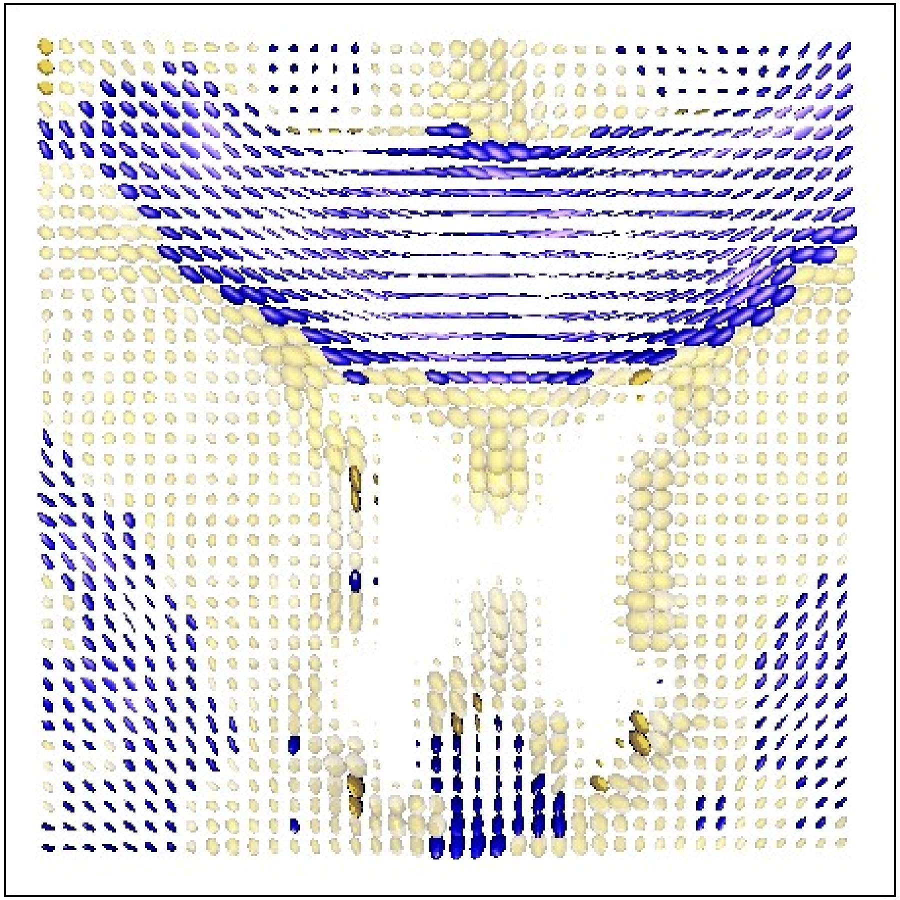
- **Vector methods**
  - **a symmetric rank 2 tensor can be visualised as 3 orthogonal vector fields** (i.e. using eigenvectors)
  - **hyper-streamline**
  - Noise filtering algorithms – **LIC variant**

Tensor Glyphs

- **Ellipsoids**
  - **rotated into coordinate system defined by eigenvectors** of tensor
  - axes are **scaled by the eigenvalues**
  - **very suitable as 3 modes of variation**
- **Classes of tensor:**
  - (a,b) - **large major eigenvalue**
    - ellipse approximates a **line**
  - (c,d) **- large major and medium eigenvalue**
    - ellipse approximates a **plane**
  - (e,f) **- all similar** - ellipse approximates a **sphere**



Diffusion Tensor Visualisation



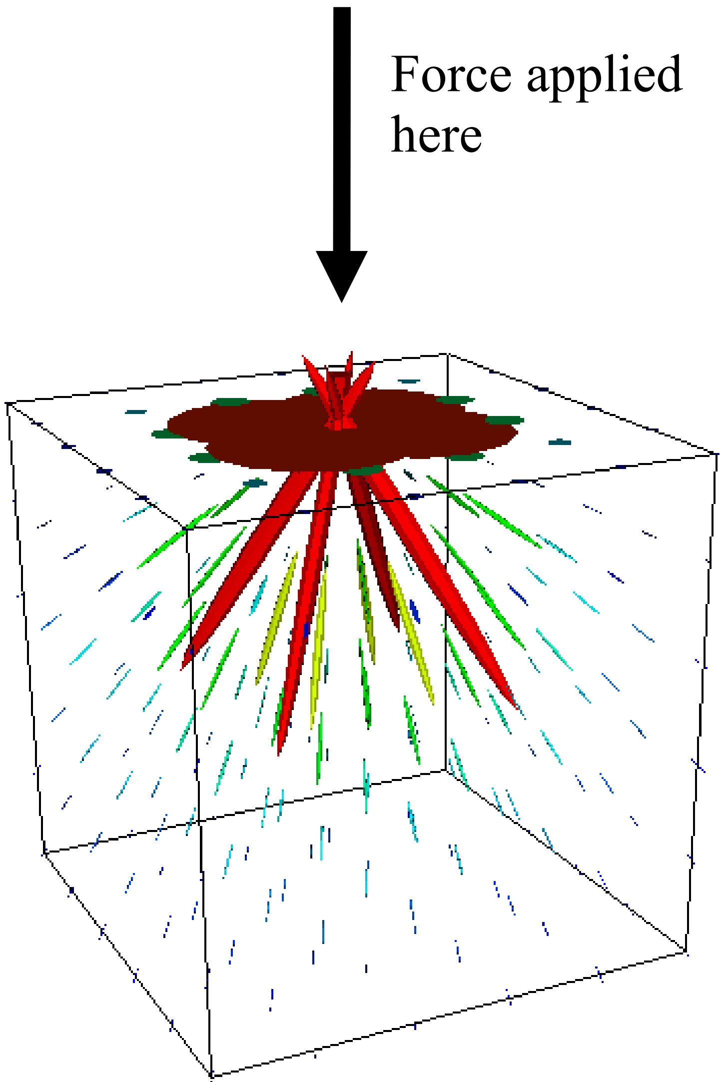
Baby's brain image (source: R.Sierra)

Anisotropic tensors indicate nerve pathway in brain:

- **Blue shape** – tensor approximates a line.
- **Yellow shape** – tensor approximates a plane.
- **Yellow transparent shape** – ellipsoids approximates a sphere

Colours needed due to **ambiguity in 3D shape** – a line tensor viewed 'end-on' looks like a sphere.

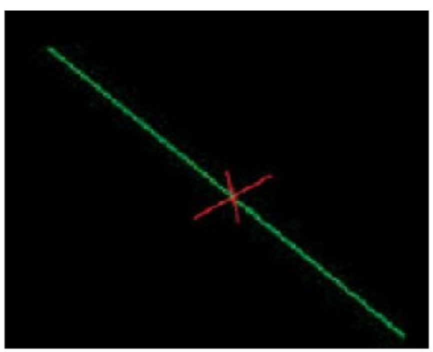
Stress Ellipses

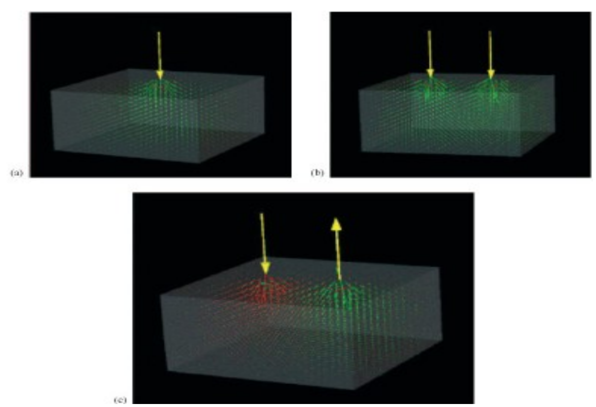


- Force applied to dense 3D solid – resulting **stress at 3D position in structure**
- Ellipsoids visualise the stress tensor
- Tensor Eigenvalues:
  - Large **major eigenvalue indicates principle direction of stress**
  - 'Temperature' **colormap indicates size of major eigenvalue (magnitude of stress)**

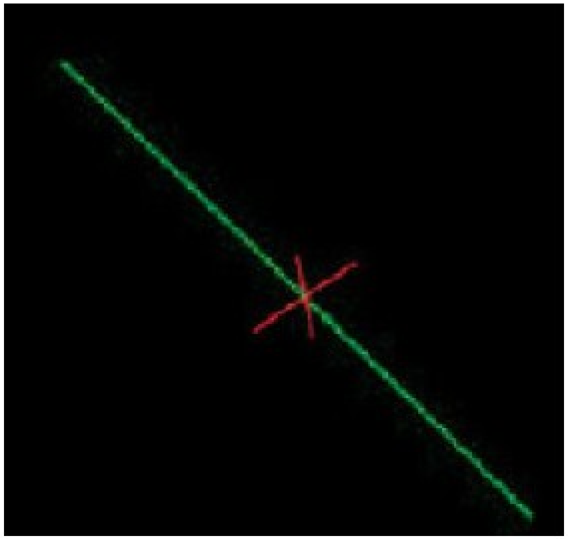
Tensor Visualisation as Vectors

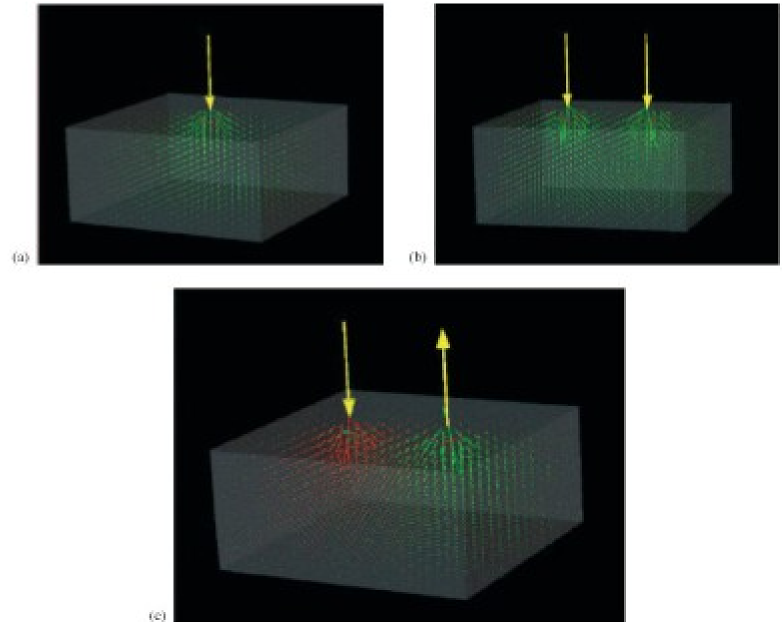
- Visualise just the major **eigenvectors as a vector field**
  - alternatively medium or minor eigenvector
  - use any of vector visualisation techniques from lecture 14





Lines, Hedgehogs





- Using hedgehogs to draw the three eigenvectors

  The length is the stress value

- Good for simple cases as above
  - Applying forces to the box
  - Green represents positive, red negative

Streamlines for tensor visualisation

- Often major eigenvector is used, with medium and minor shown by other properties
  - **Major vector is relevant in the case of anisotropy** - indicates nerve pathways or stress directions.

http://www.cmiv.liu.se/

Streamlines for tensor visualisation

- **Each eigenvector defines a vector field**
- Using the eigenvector to create the streamline
  - **We can use the Major vector, the medium and the minor vector to generate 3 streamlines**

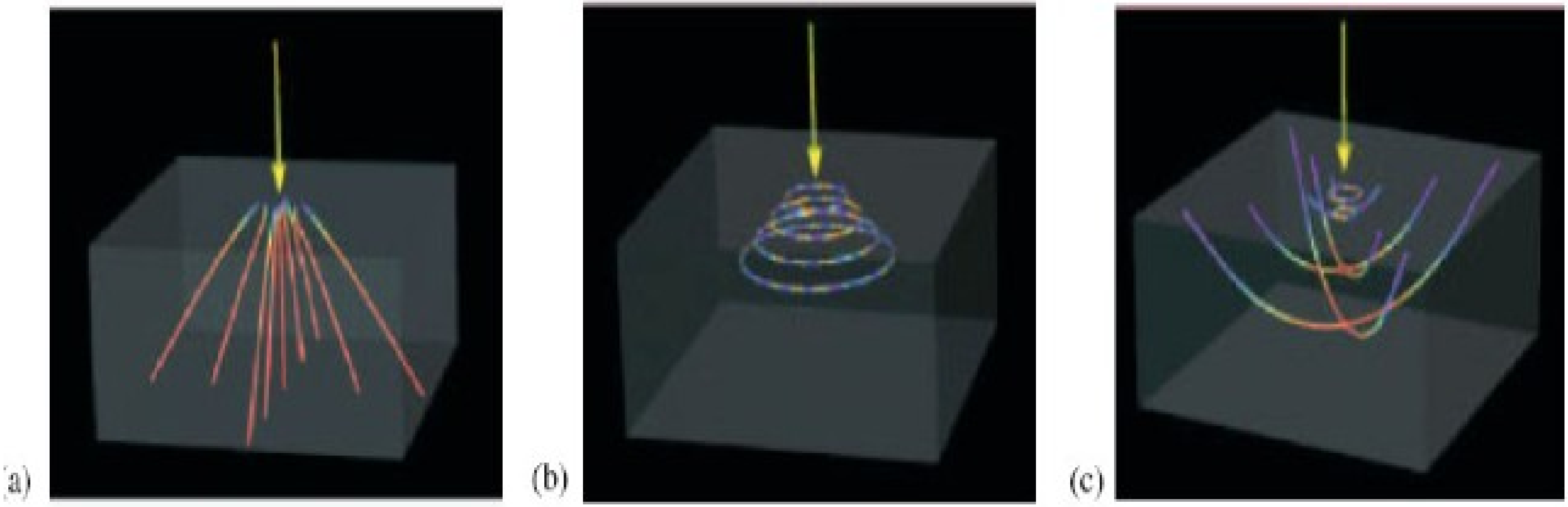
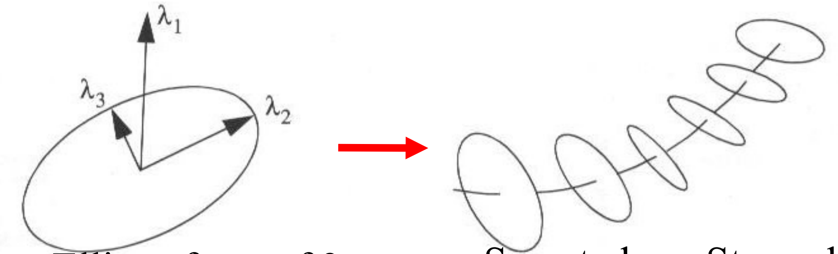


Figure 8. Hyperstreamlines for minor, intermediate and major principal stress for a point-load.

Hyper-streamlines [Delmarcelle et al. '93]

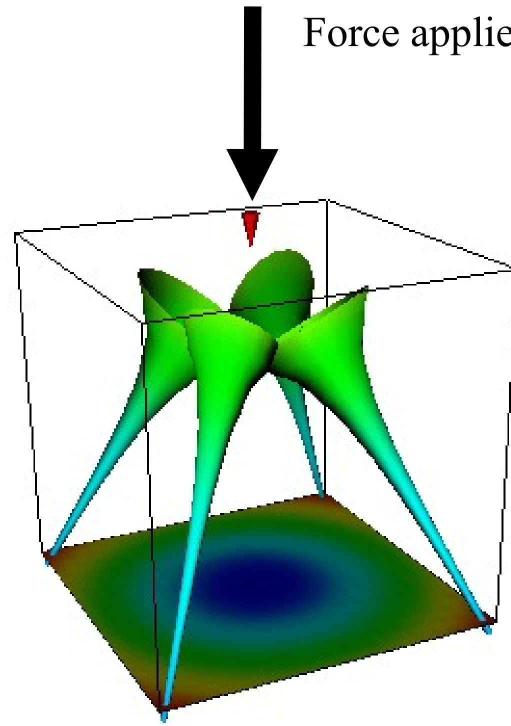
- Construct a **streamline from vector field of major eigenvector**



Ellipse from of 3 orthogonal eigenvectors

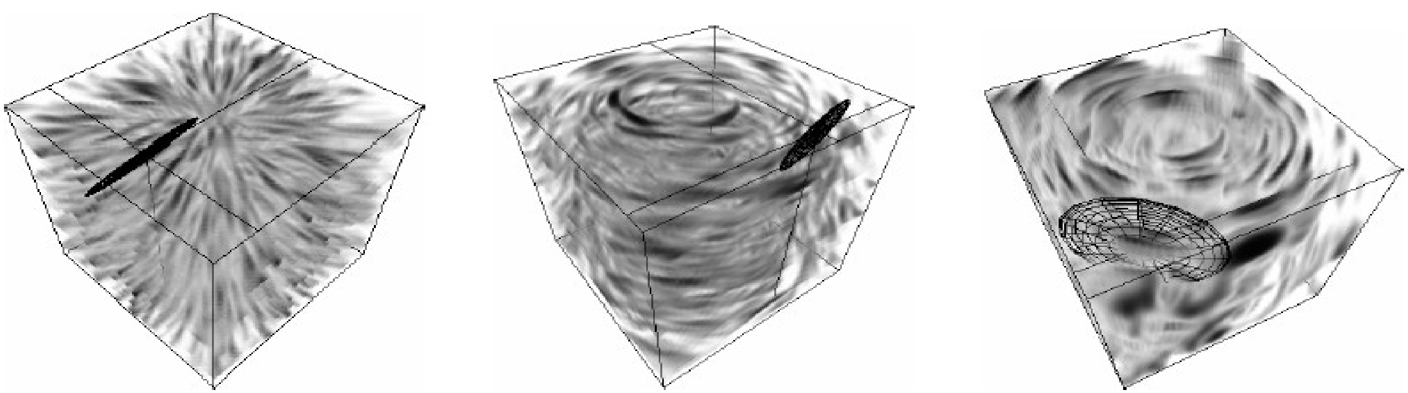
Swept along Streamline

- **Form ellipse together with medium and minor eigenvector**
  - both are orthogonal to streamline direction
  - use major eigenvector as surface normal (i.e. orientation)
- **Sweep ellipse** along streamline
  - ***Hyper-Streamline*** ***(type of stream polygon)***



LIC algorithm for tensors

- **Linear Integral Convolution – LIC**
  - **'blurs' a noise pattern with a vector field**
  - For **tensors**
    - can **apply 'blur' consecutively for 3 vector field directions** (of eigenvectors)
    - using result from previous blur as input to next stage
    - use **volume rendering** with opacity = image intensity value **for display**

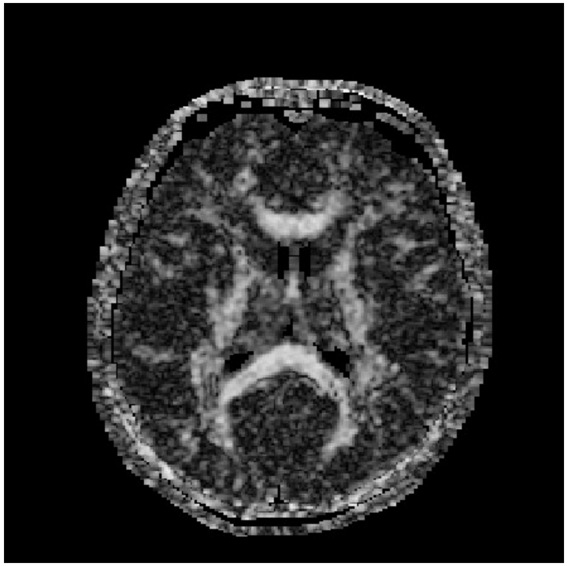


(a) (b) (c)

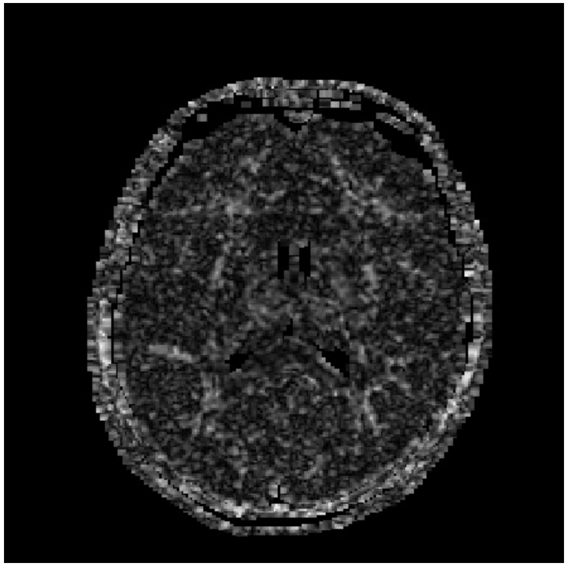
[*Sigfridsson et al. '02*]

Scalar field Method for Tensors

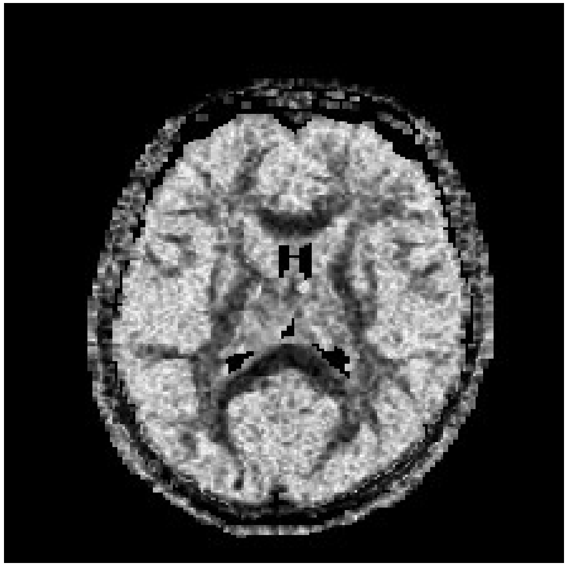
- **Scalarfield** : Produce grayscale image intensity in relation to tensor class (or closeness too). *(scalar from tensors)*



Greyscale image shows **how closely the tensor ellipsoids approximate a line.**



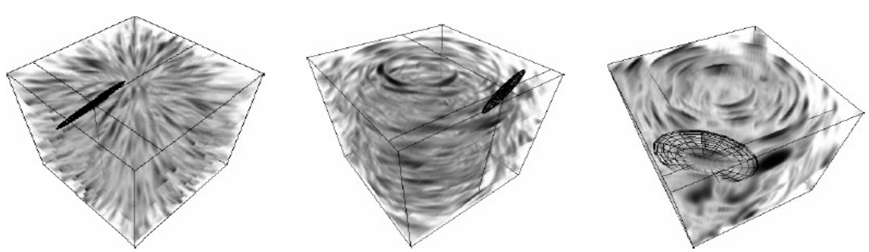
Greyscale image shows **how closely the tensor ellipsoids approximate a plane.**



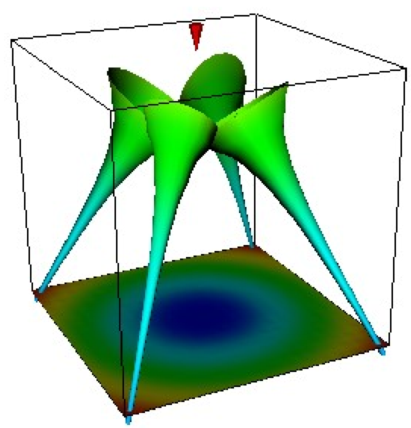
Greyscale image shows **how closely the tensor ellipsoids approximate a sphere.**

Summary

- **Tensor visualisation**
  - **Visualizing higher dimensional data at every point**
  - **Here we focused on 3x3 matrix**
  - **Computing the eigenvectors and visualising the eigenvectors**
  - **Apply vector field visualisation techniques to the three principal axes**



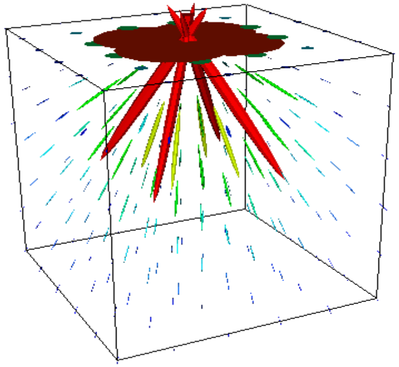
(a) (b) (c)

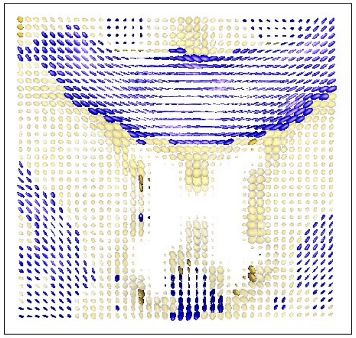


[*Sigfridsson et al. '02*]

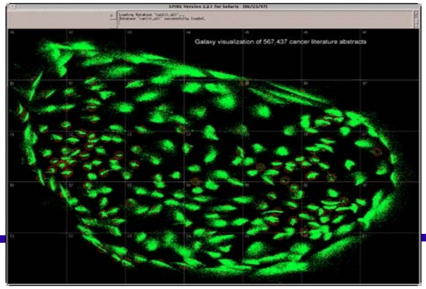
Overview

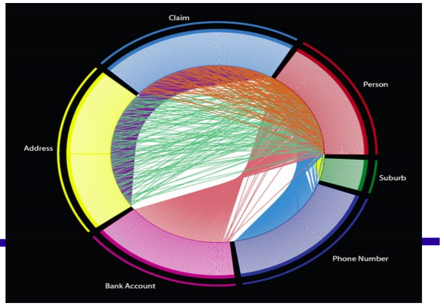
- Tensor Visualisation
  - What is tensor
  - Methods of visualization
    - 3D glyphs
    - vector and scalar field
    - hyper-streamlines
    - LIC in 3D volumes





- **Information Visualisation**
  - **Univariate, bivariate, trivariate, multi-variate data**
  - **Relations visualized by lines, tree visualization**
  - **Document visualization**

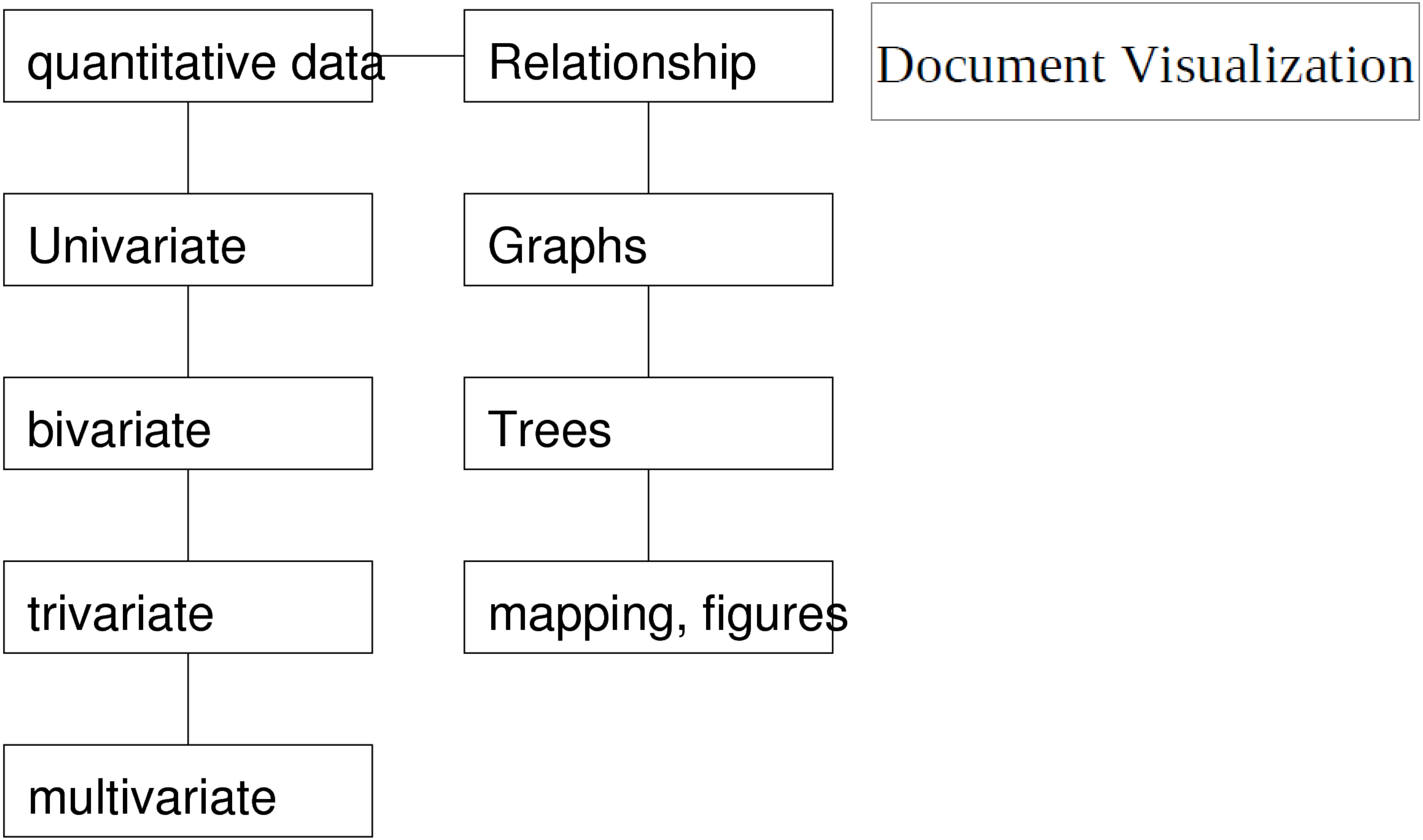




What is Information Visualisation ?

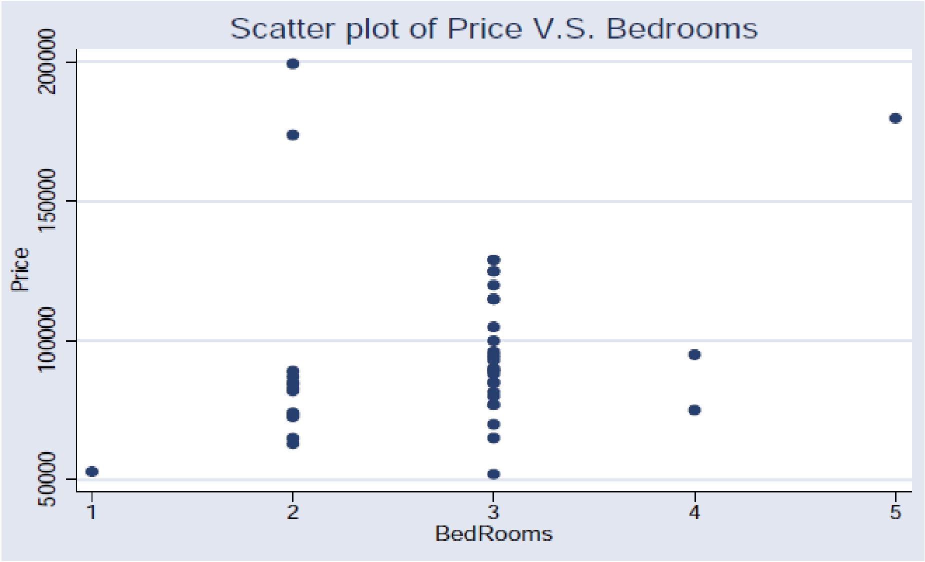
- Visualising ***discrete*** **data with no spatial information**
- Visualisation of important information contained in abstract data types
  - Needs to be intuitive
  - Such that people can easily and quickly understand
- *Tools for*
  - *Extraction of information from the data*
  - *Discovery of new knowledge*

Data types


quantitative data
Univariate
bivariate
trivariate
multivariate
Relationship
Graphs
Trees
mapping, figures
Document Visualization

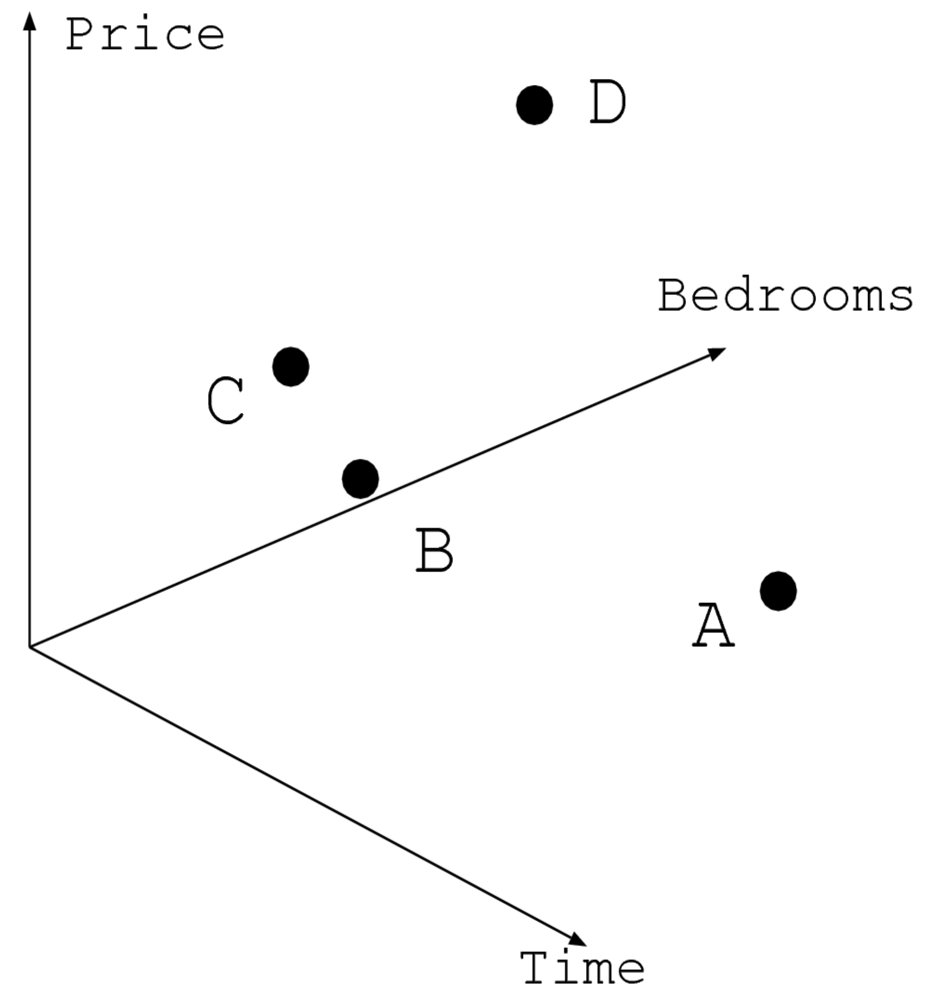
Univariate Data, Bivariate Data

Can use scatter plots, histograms



Trivariate Data

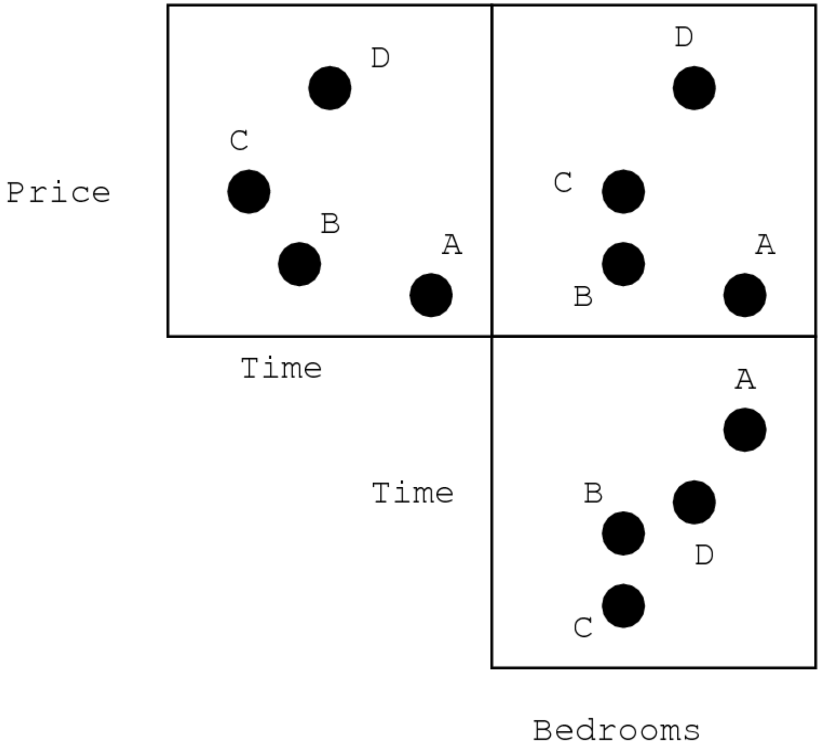
Scatterplots

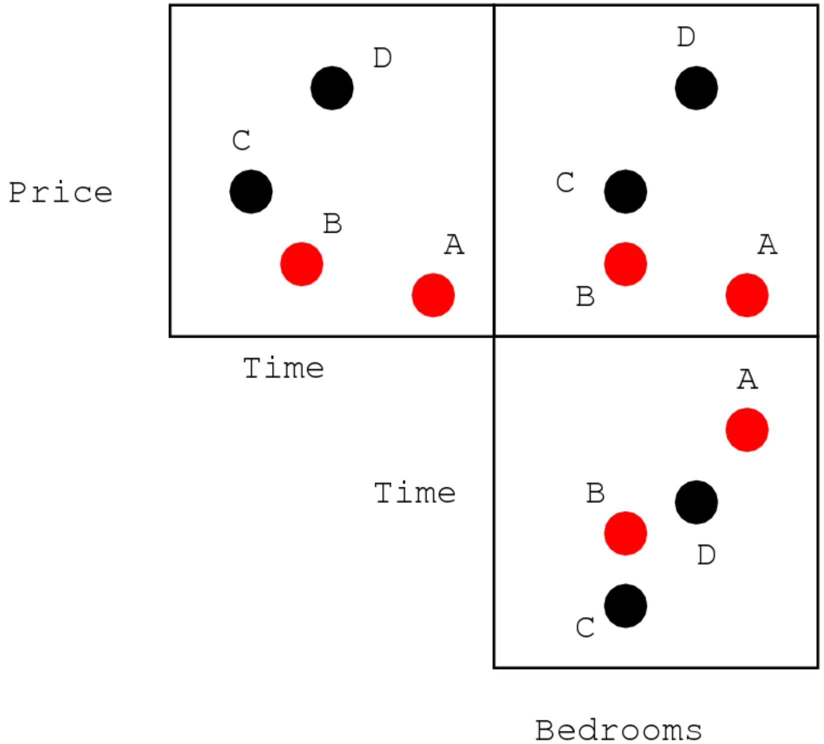


Not clear if D is more expensive
Than B and C

Trivariate Data

Scatterplot Matrix : Visualizing the relations of every two variables



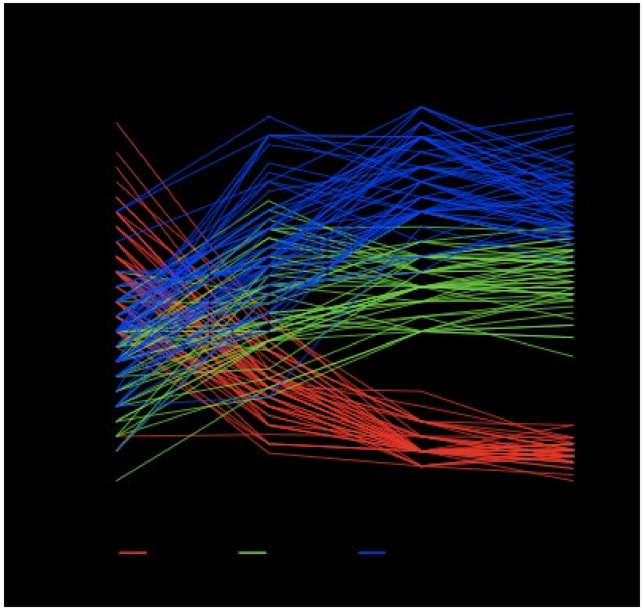


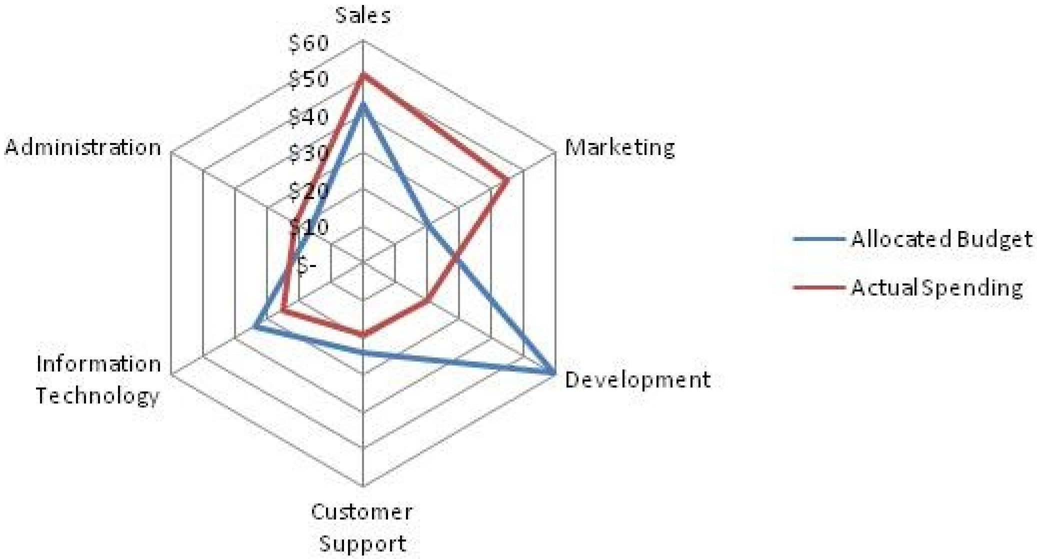
Multivariate Data

Parallel Coordinates

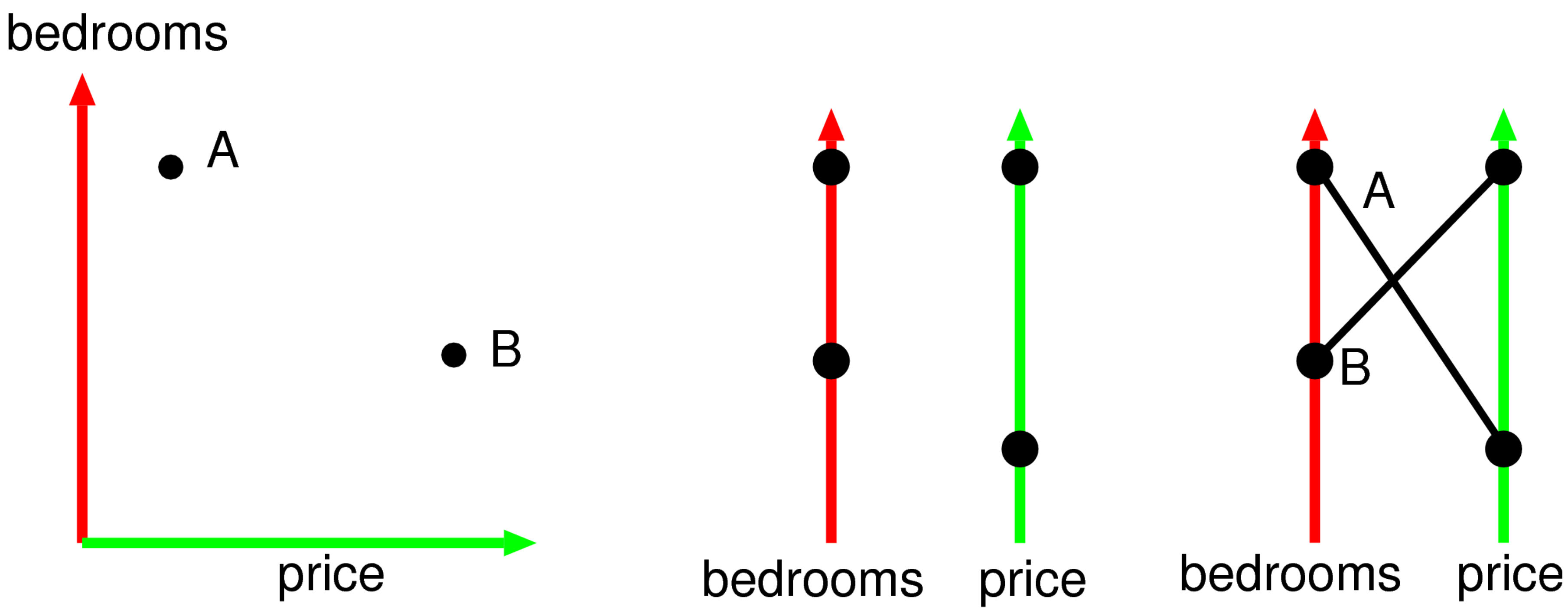
Star plots

Scattered plot matrix

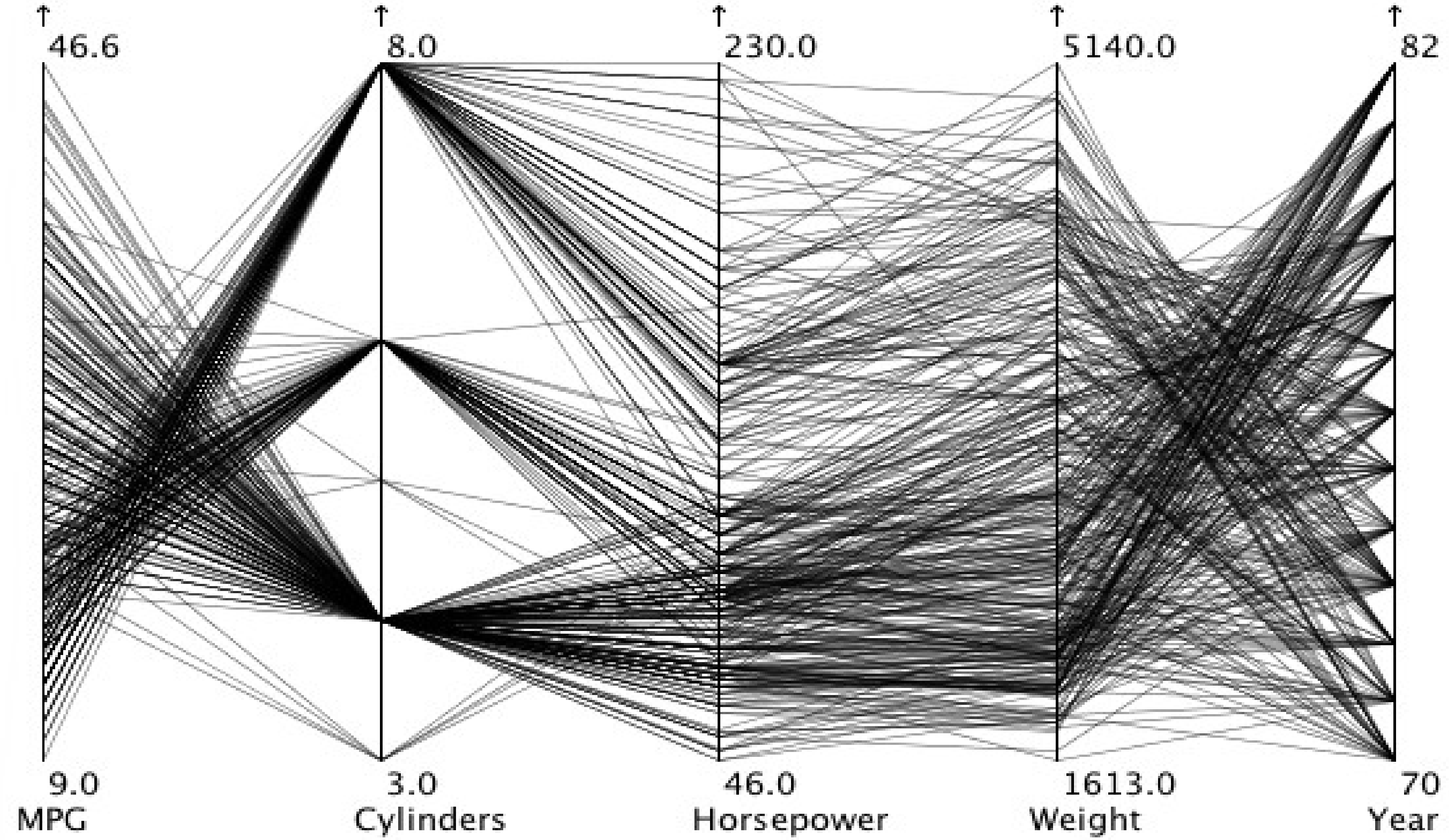




Parallel Coordinates



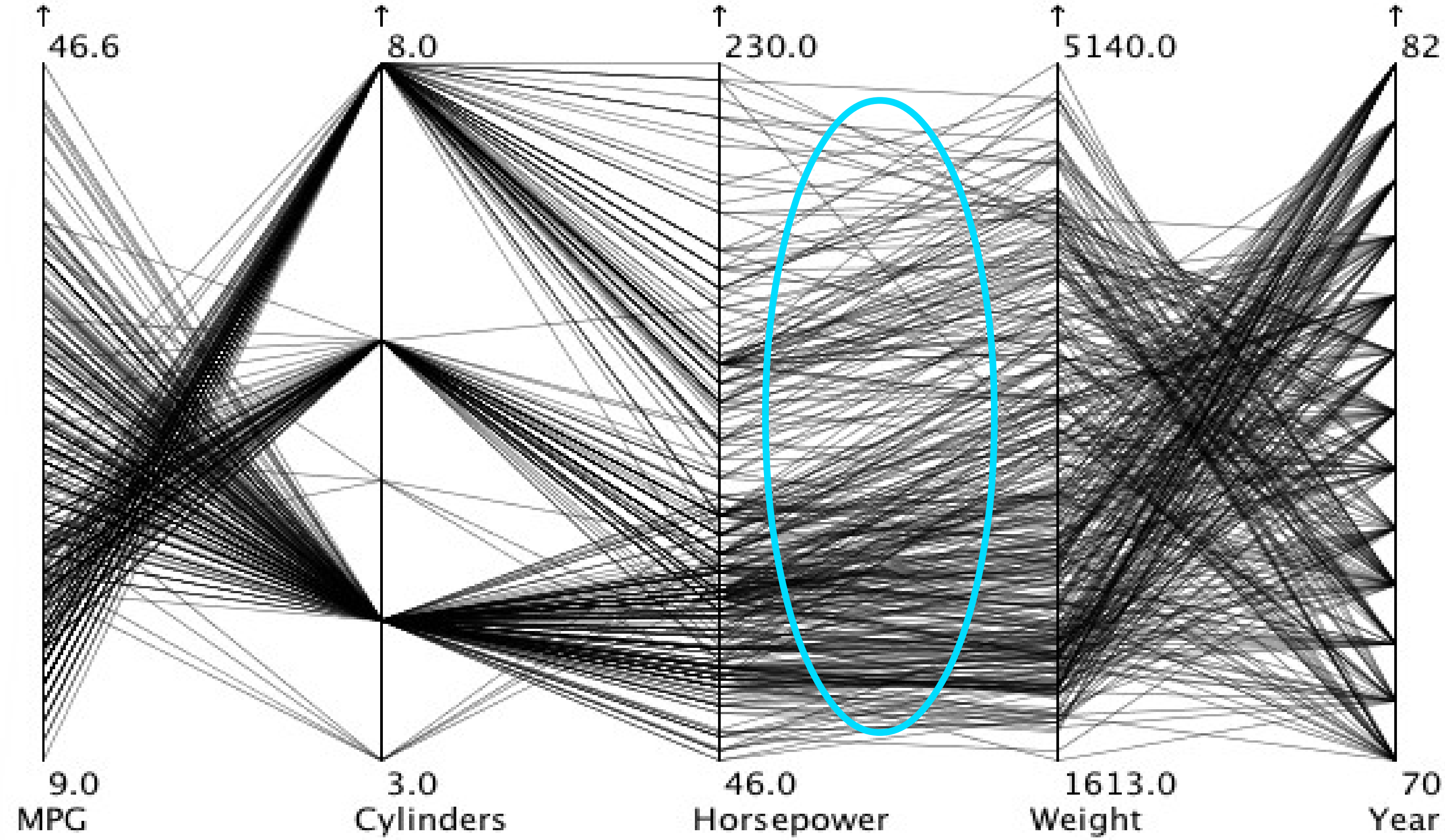
Parallel Coordinates



Car data :

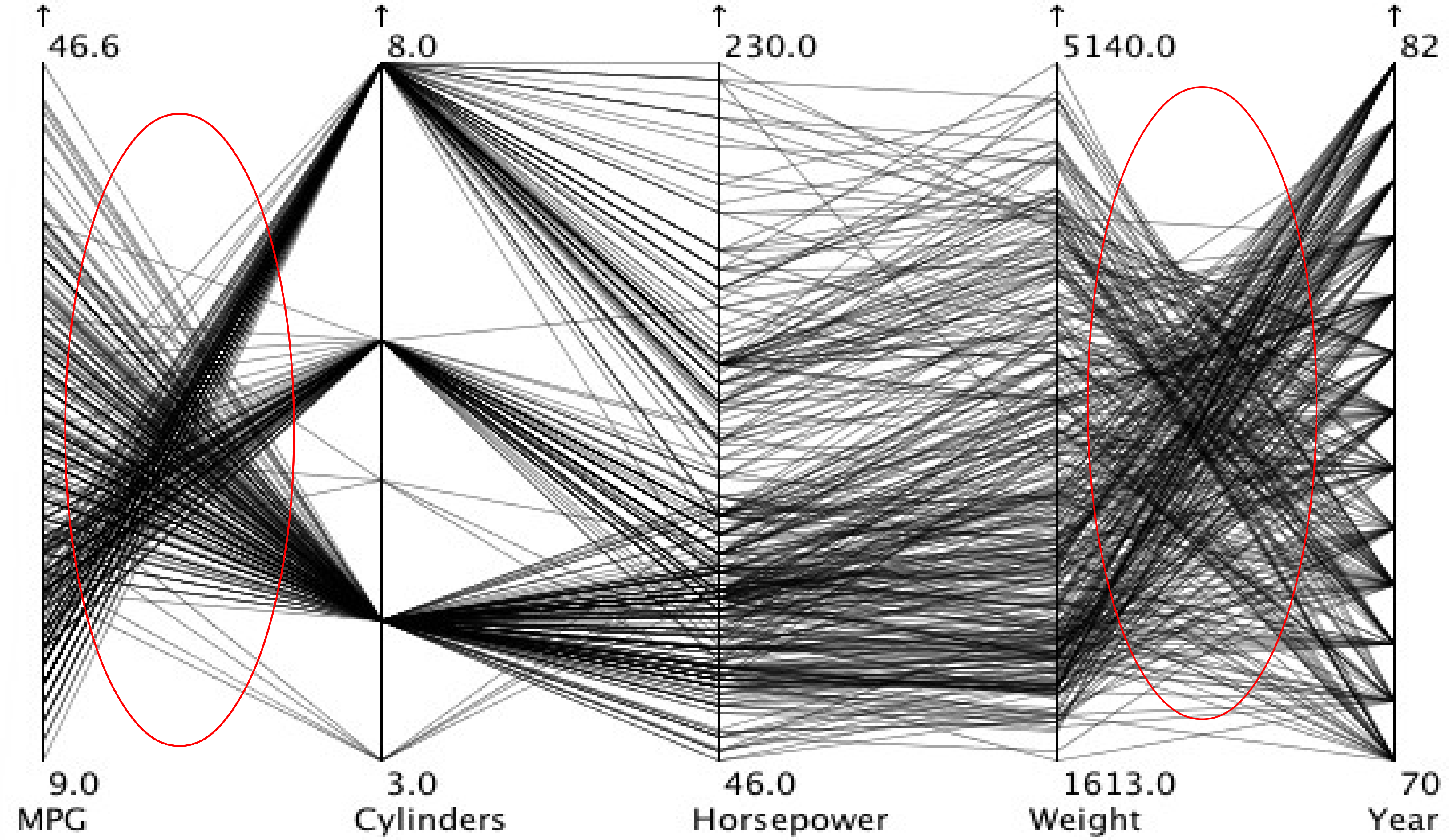
http://eagereyes.org/techniques/parallel-coordinates

Parallel Coordinates



Direct correlation

Parallel Coordinates

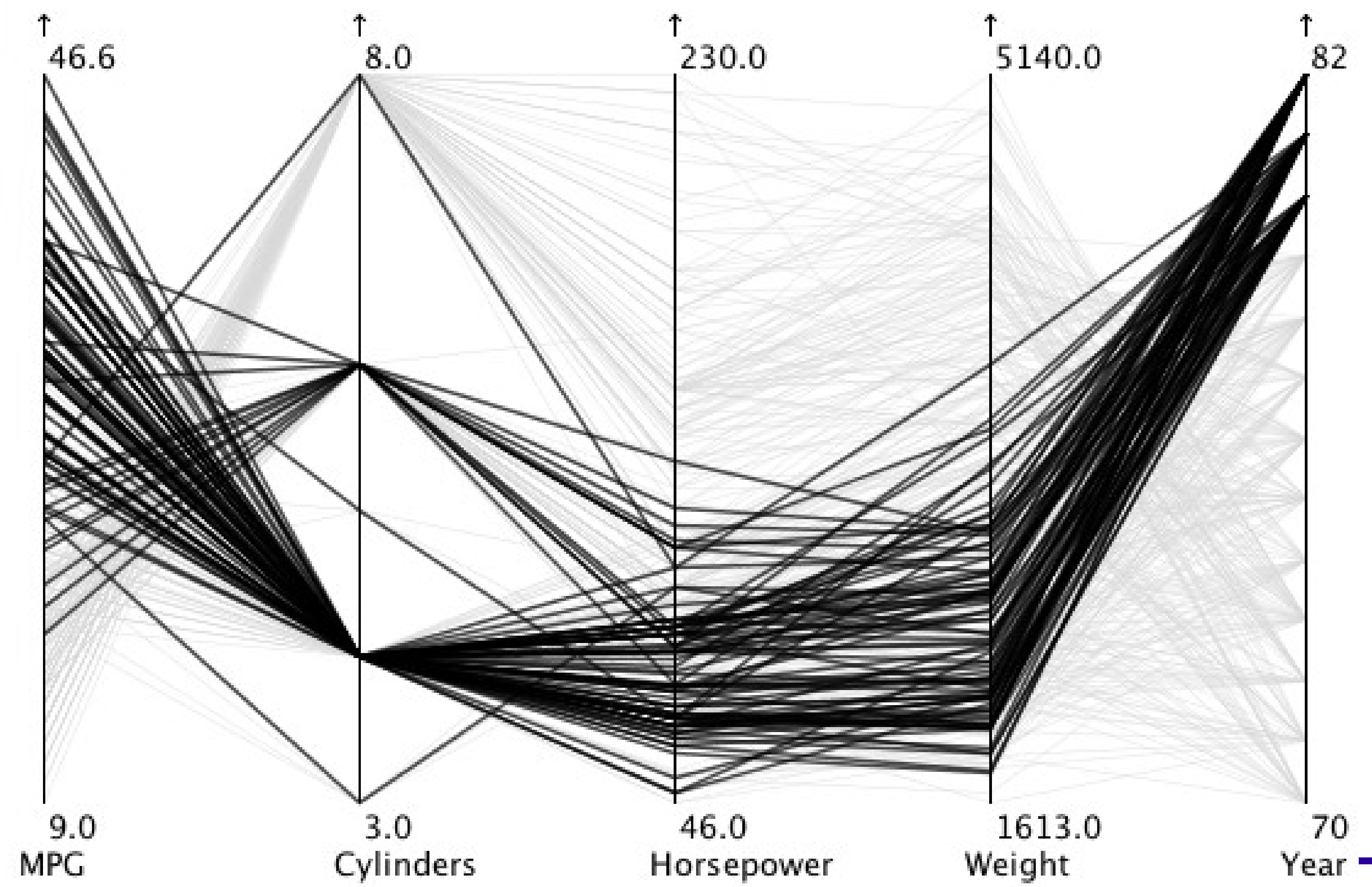


Inverse Relations

Brushing

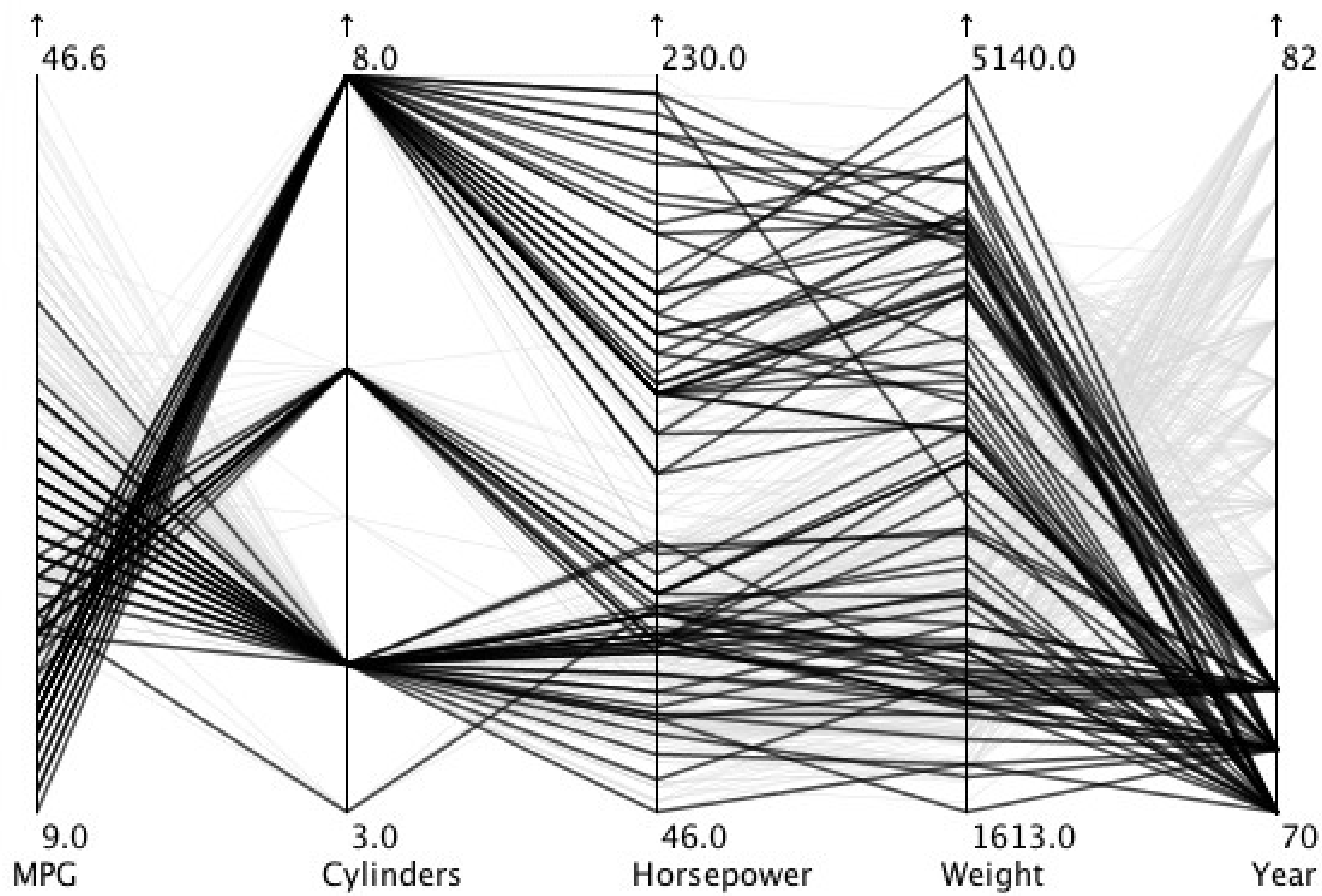
Select some data using one of the coordinates

Brushing years 1980 to 1982



Brushing

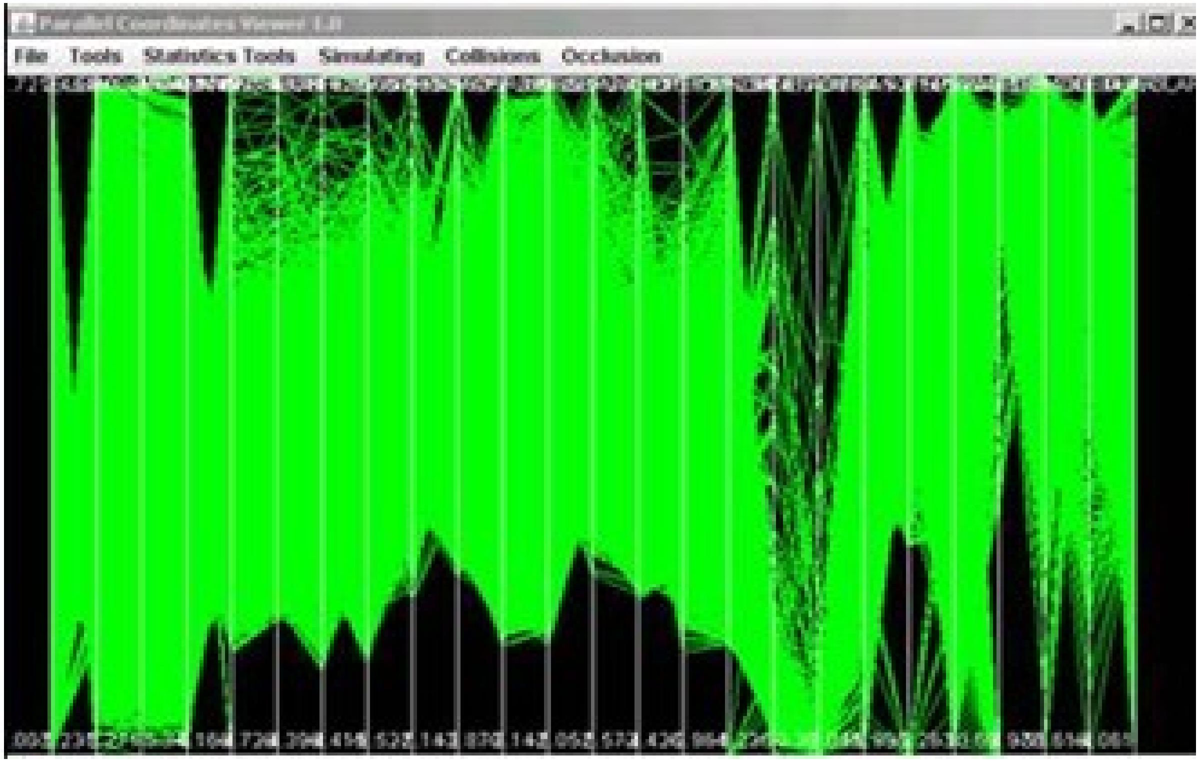
Brushing the years 1970 to 1972



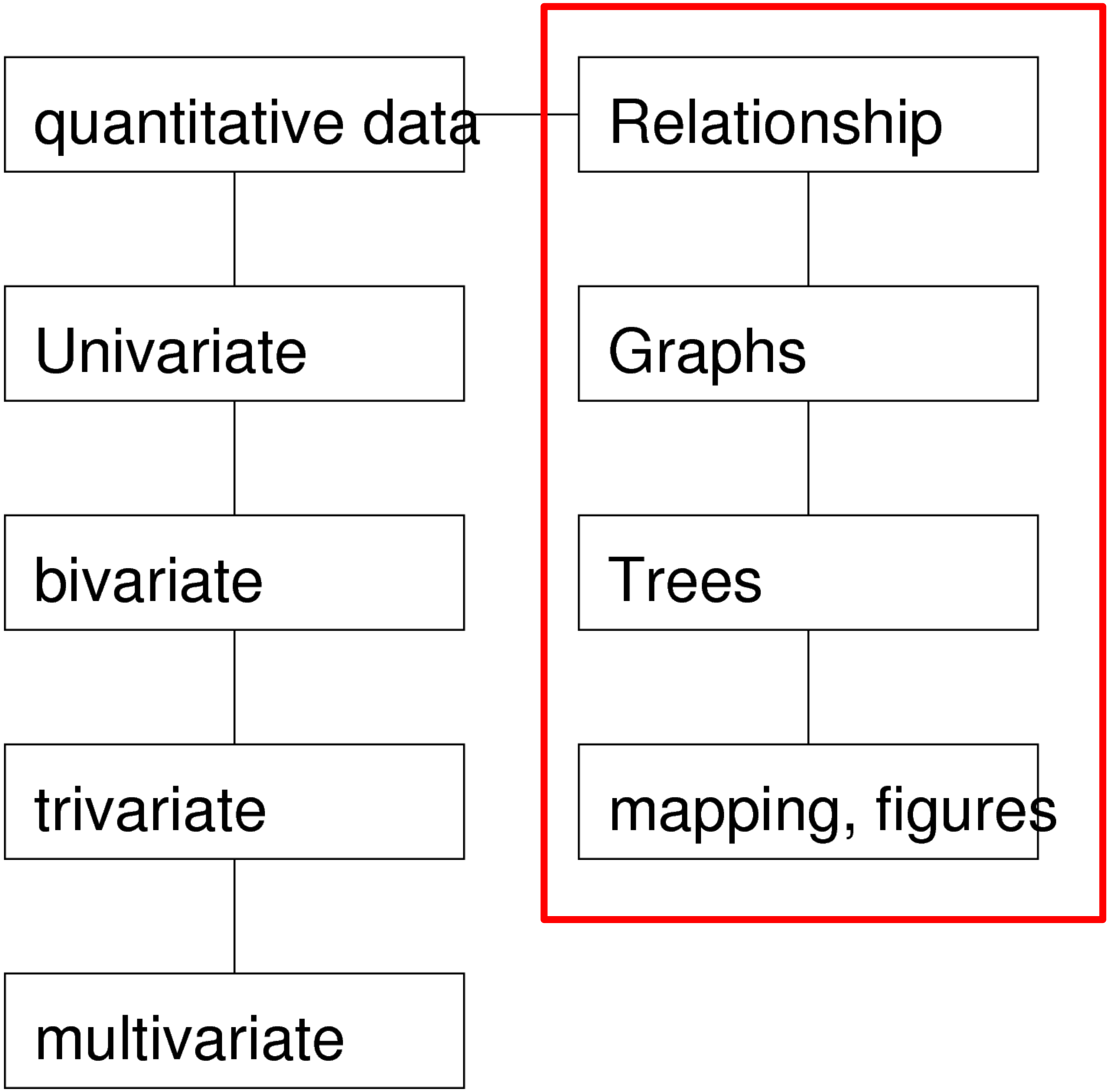
Limitations

Visual clutter

- Many lines cluttered together making it impossible to see anything
- Too many dimensions make things difficult to see



Data types

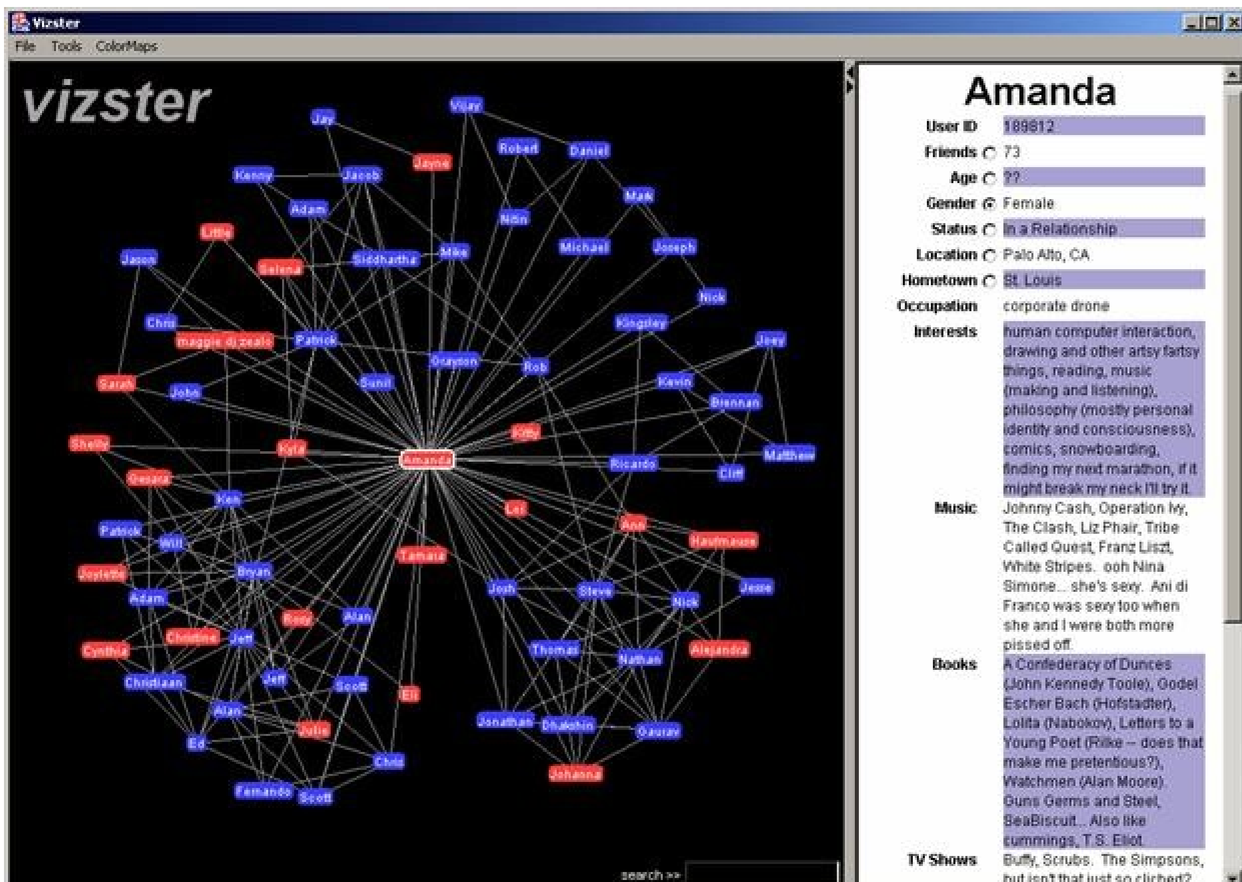


Document Visualization

Visualizing Relations

Relation: A logical or natural association between two or more things; relevance of one to another; connection

Usually use lines to represent the relations



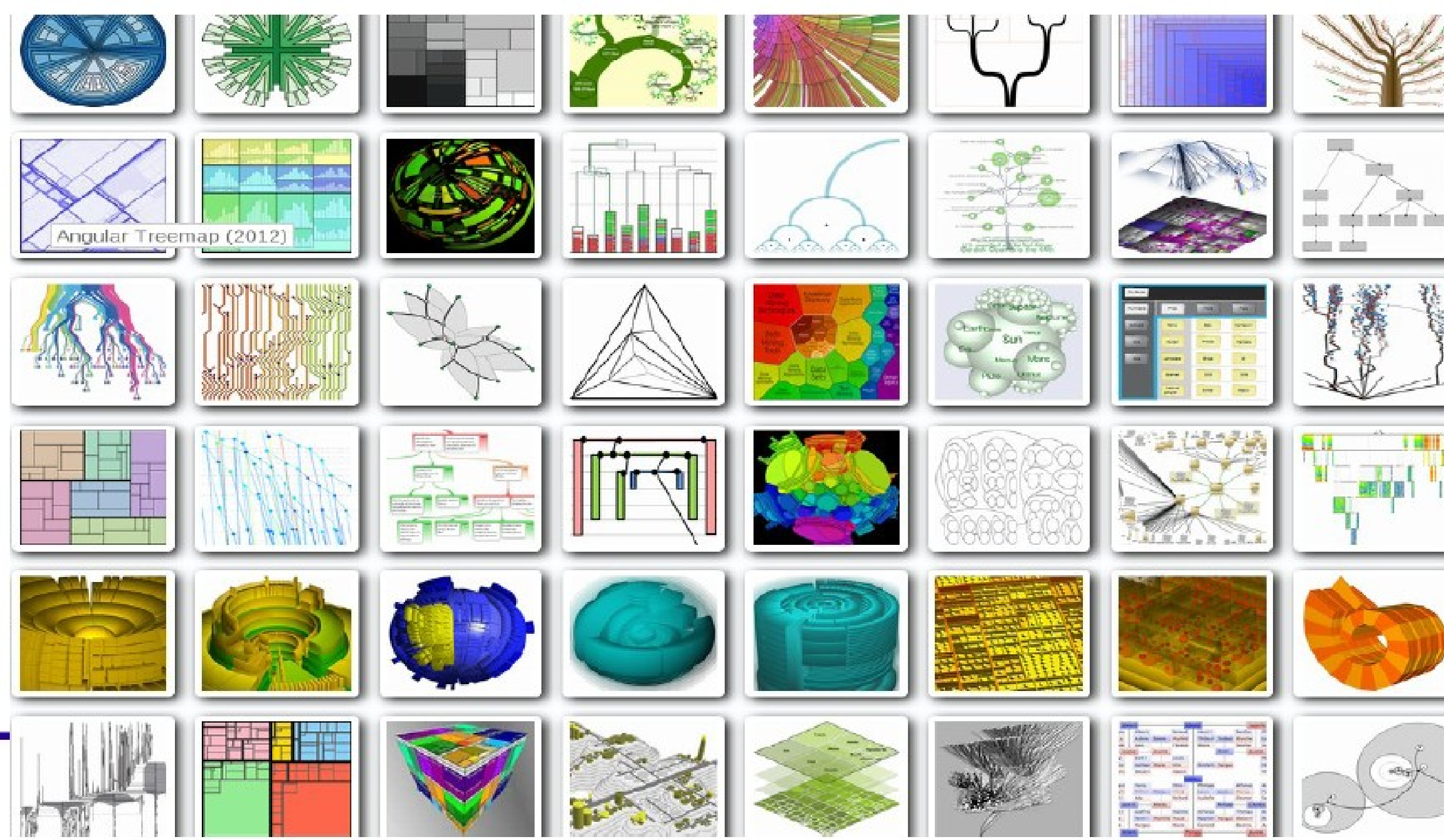
Tree visualization

Trees have hierarchical structures

No close loops

So many methods : see

http://vcg.informatik.uni-rostock.de/~hs162/treeposter/poster.html

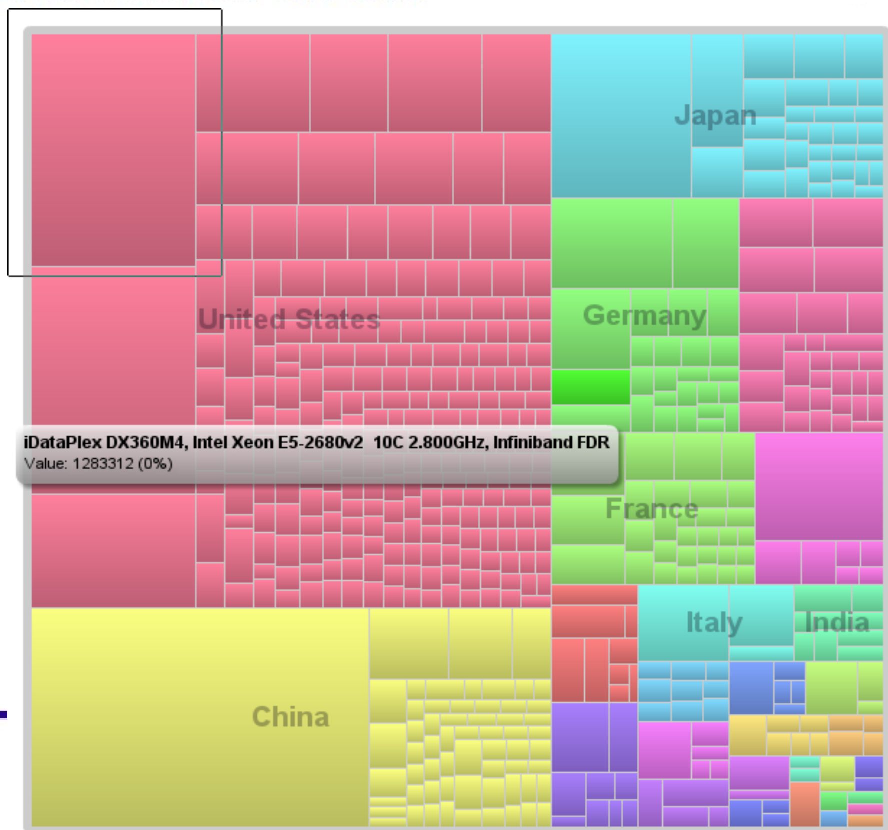


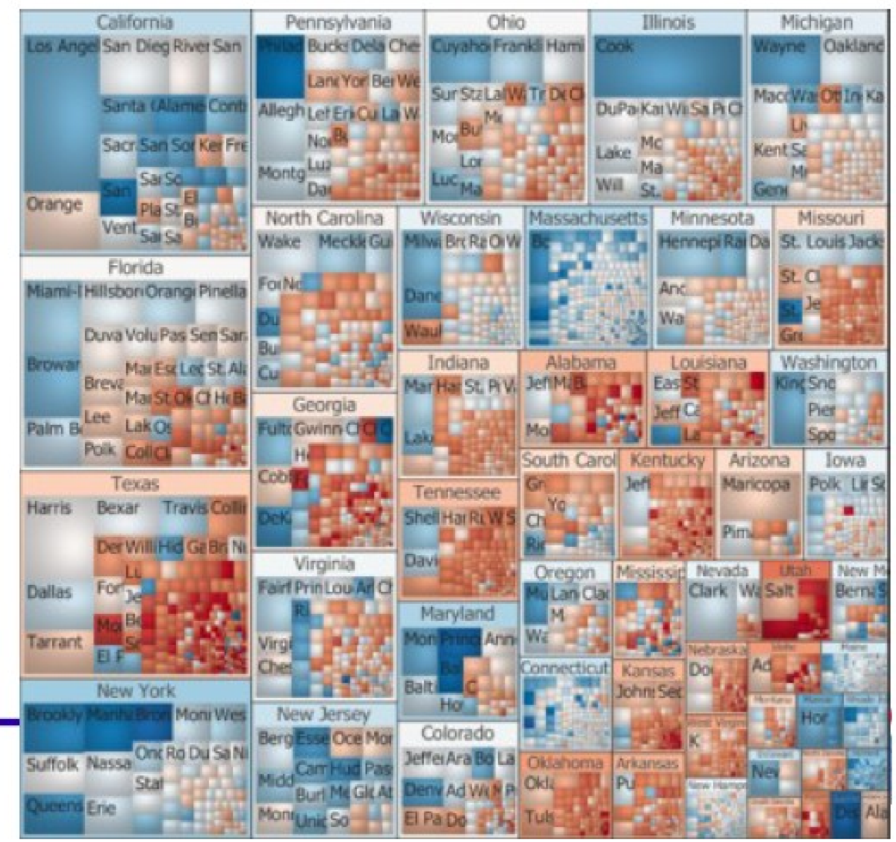
Treemaps

Display hierarchical (tree-structured) data as a set of nested rectangles

The area of the rectangles representing a scalar attribute

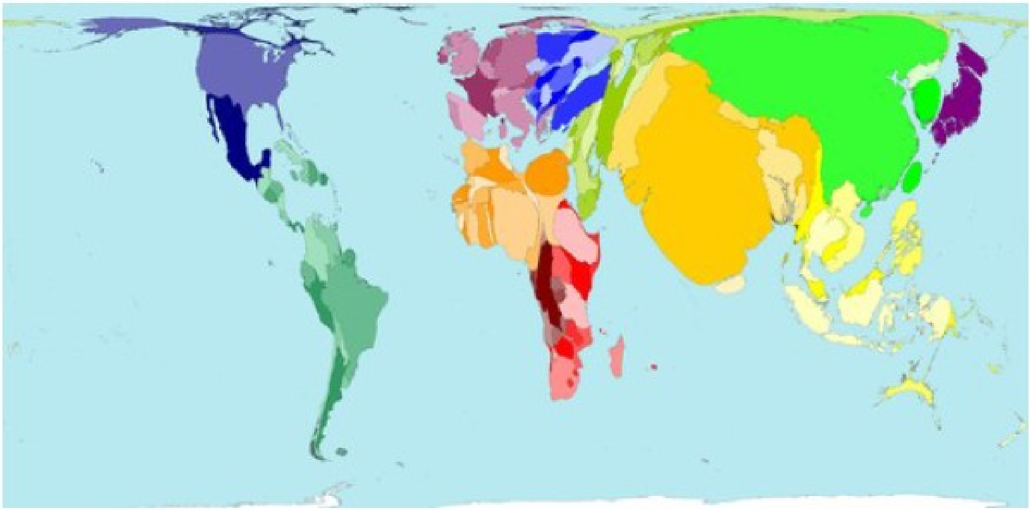
The leaf nodes are often colored to visualize another attribute data

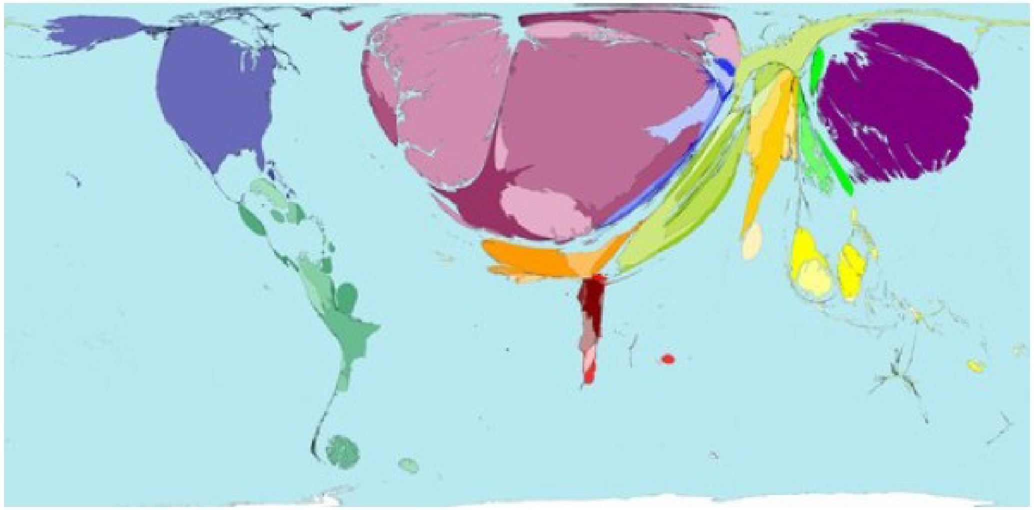




Worldmapper

http://sasi.group.shef.ac.uk/worldmapper/





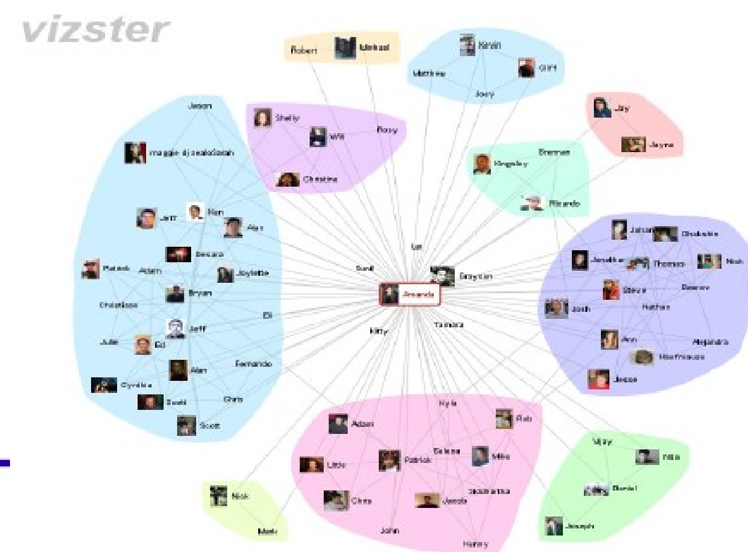
Distorted maps according to numbers: **Cartograms**

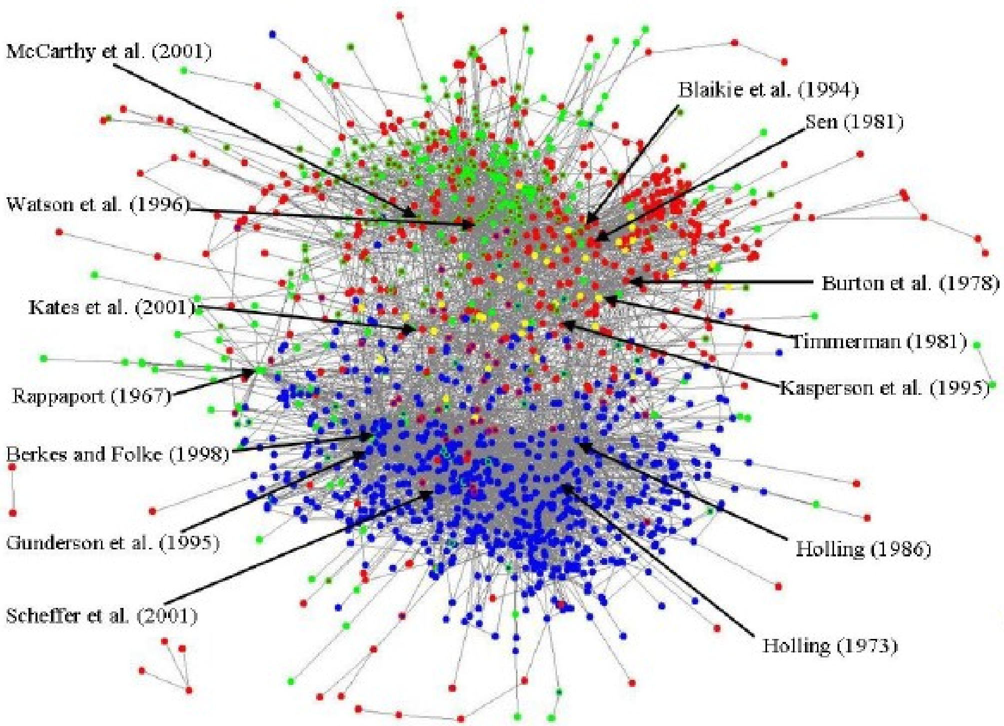
Graph Visualization

Visualizing correlation of different nodes

E.g.

- social networks
- citation networks





Facebook relations

Facebook relations

"I defined weights for each pair of cities as a function of the Euclidean distance between them and the number of friends between them. Then I plotted lines between the pairs by weight, so that pairs of cities with the most friendships between them were drawn on top of the others. I used a color ramp from black to blue to white, with each line's color depending on its weight. I also transformed some of the lines to wrap around the image, rather than spanning more than halfway around the world."

Formal Aesthetics Metrics

Minimize node-node / node-edge occlusion

Minimize edge crossings

Minimize edge bends

Maximize symmetry

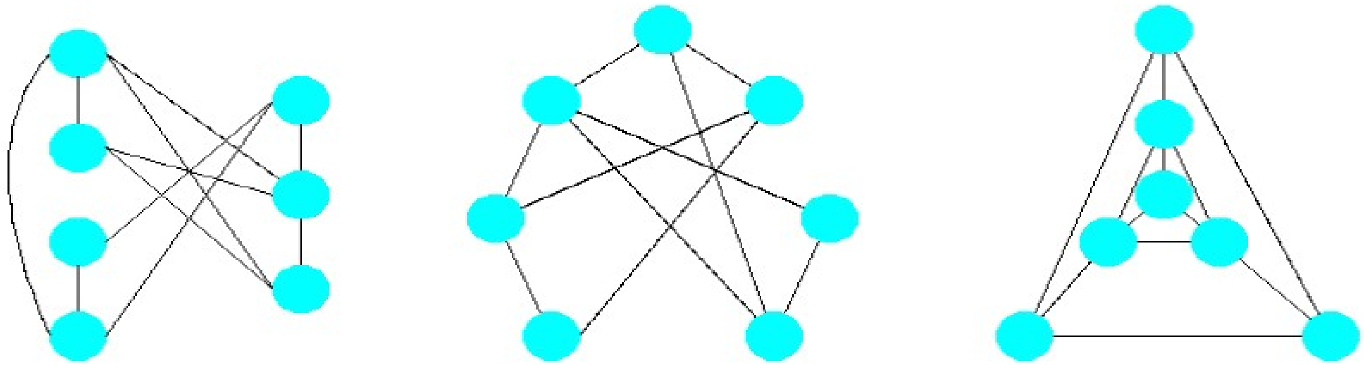
Maximize the minimum angle between neighbor edges

Maximize edge orthogonality

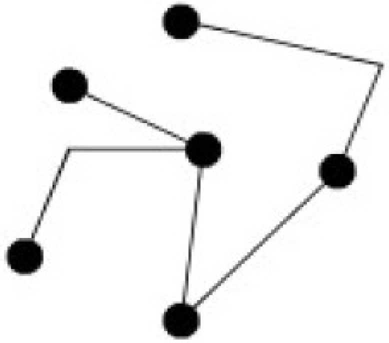
Maximize node orthogolnality

Formal Aesthetics Metrics

Minimize edge crossings

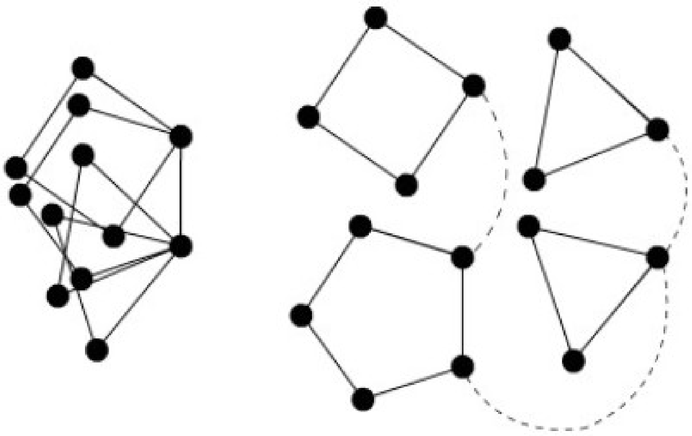


Minimize edge bends

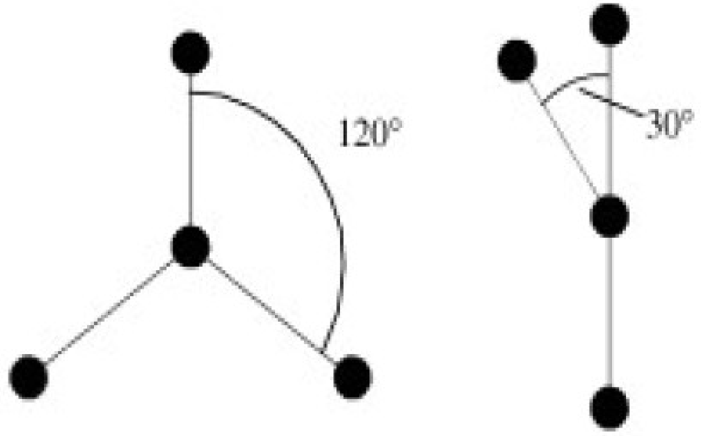


Formal Aesthetics Metrics

Maximizing symmetry

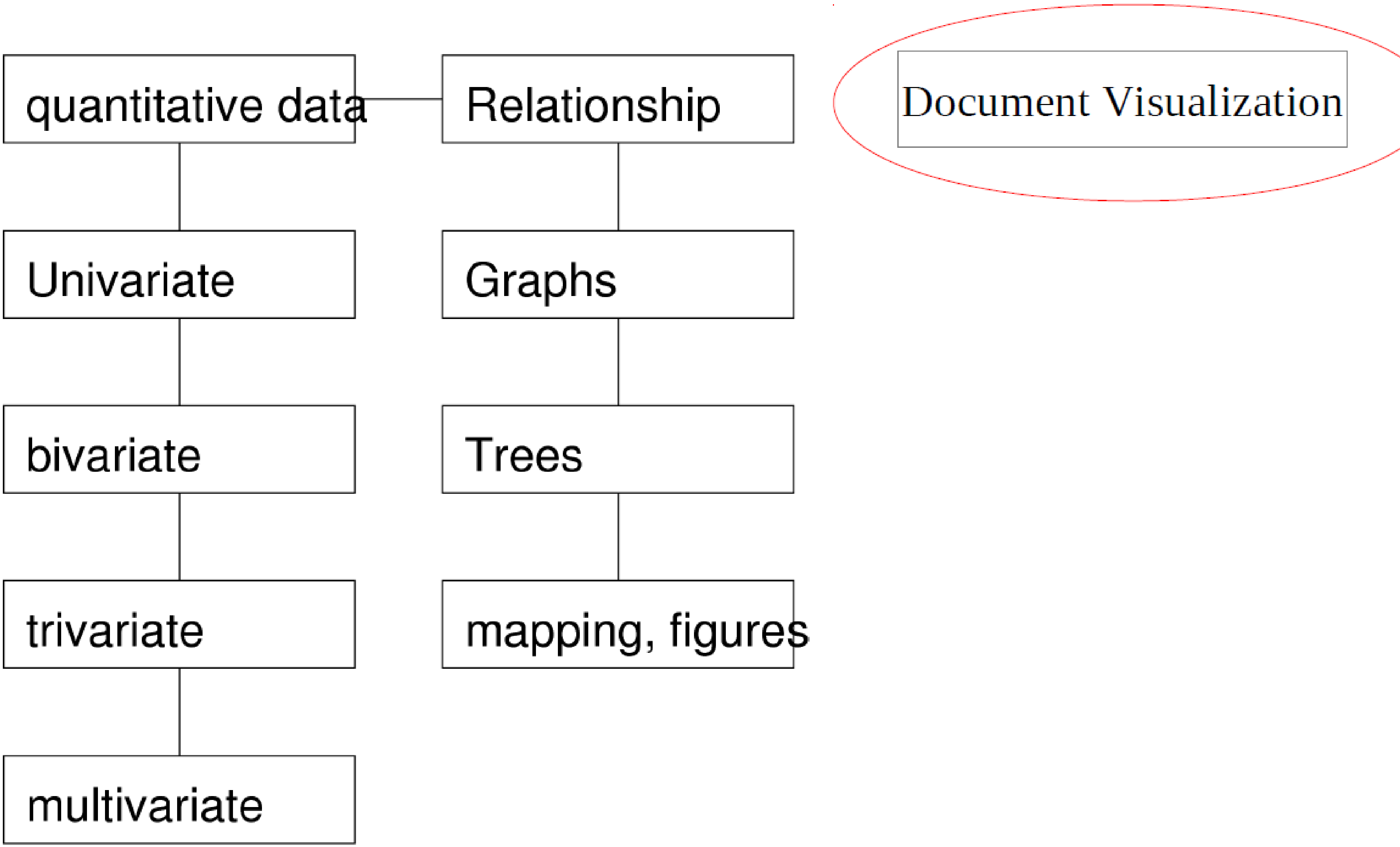


Maximizing the minimum angle between edges leaving a node



CAV : Lecture 18

Data types



Document Visualisation

- Motivation:

Action	Units of Information transfer
Typing at 10 bytes per second	1
Mouse Operations.	2
Reading	3-40
Hearing	60
Visualisation and Pattern Recognition	12,500

Source :
Silicon Graphics Inc.

  - visualisation is considerably faster than hearing / reading!

Visualisation of Documents

- **Motivation : large bandwidth of human visual system**
  - 100s millions of documents available on-line
  - information only in textual form
- **‘Visualising the non-visual’**
  - searching for scientific papers
  - analysing witness statements
  - awareness of events in news bulletins

Document Visualisation - Stages

- **Representation of results**
  - form high-dimensional vector (one for each word, ~10000+)
  - cluster documents based on vector similarity (e.g. Nearest-Neighbour)
- **Visualisation of clustered results**
  - projection to lower dimensional space
  - 3D “galaxy” / 2D “theme-scape” / 1D “theme-river”

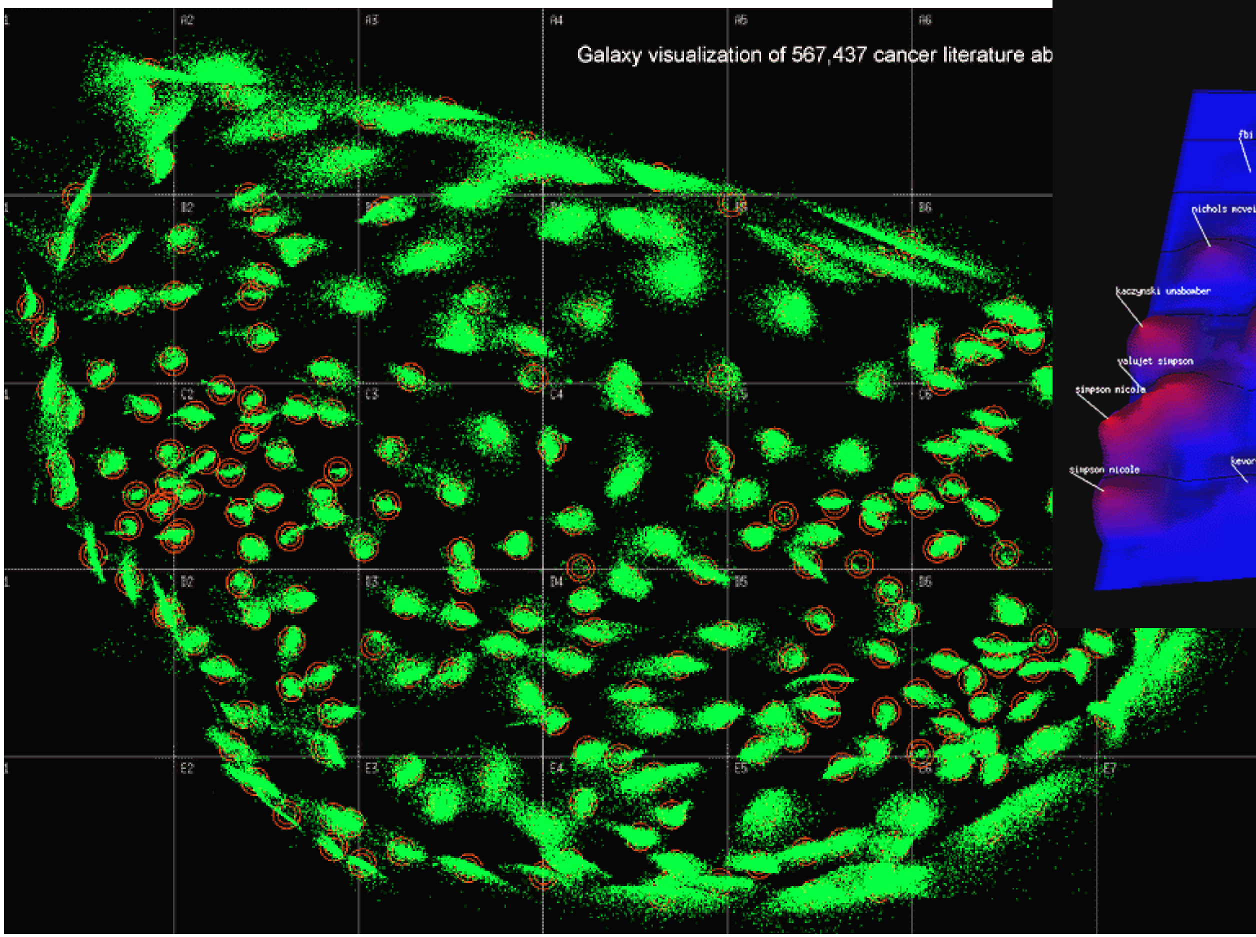
Query

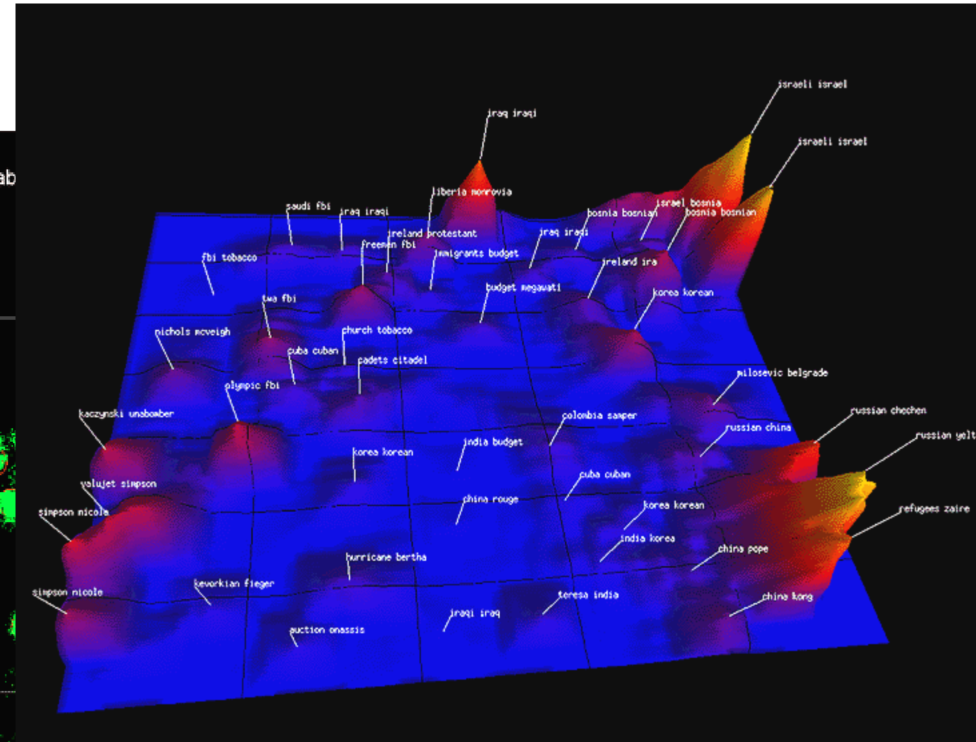
“keywords” from user specification

comparison to sample “reference” document

2D and 3D projections of documents

3D Visualisation of 567,000 cancer literature abstracts.



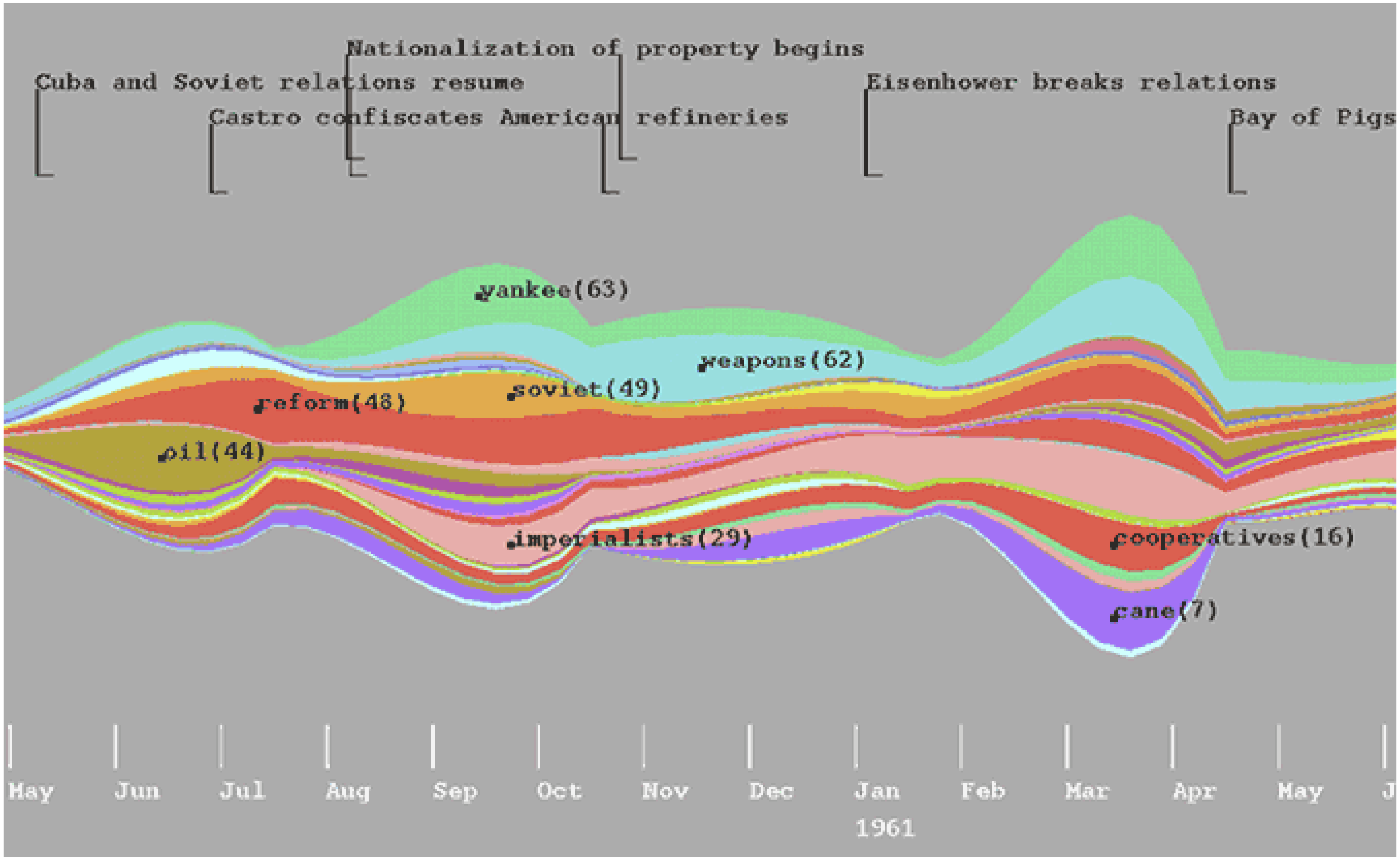


Articles in a collection of news items (2D).

Pacific Northwest National Laboratory.

1D visualisation of news articles

A 'Theme River' shows the relative importance of themes over the course of a year from press articles.

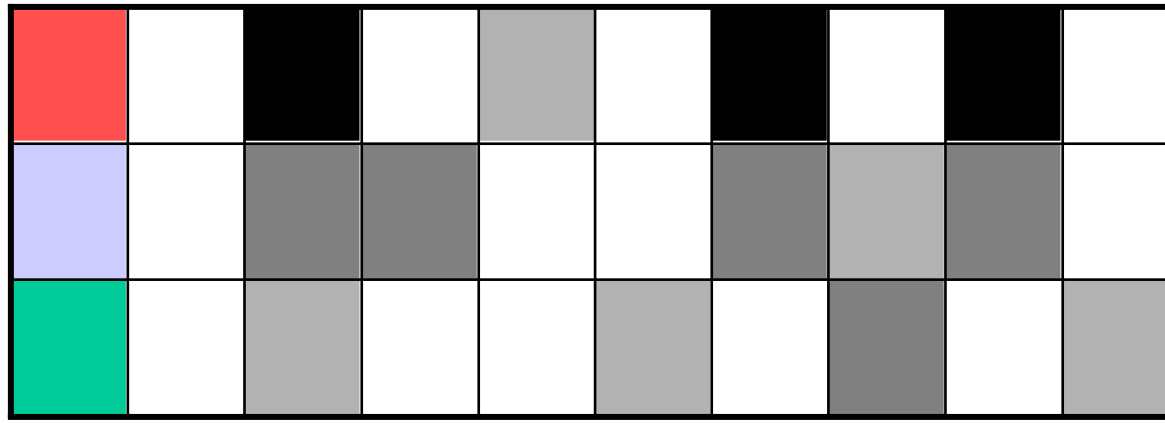


Pacific Northwest National Laboratory.

Document Querying

- **Keyword search is problematic**
  - **ambiguity**
  - ~7-18% of people describe same concept with same word (Barnard '91)
- Interested in
  - **distribution of keywords in the document**
  - **related articles** to the keyword entered
- **Tile bar scheme** (Hearst 1995)
  - display a list of documents with a **tile bar**
  - tile bar **shows the occurrence of keywords** in document

Title Bar Method



Columns **represent** paragraphs or pages in a document.
Shade indicates relevance shown by word occurrence.
Shows length and likely relevance.
System allows **interactivity** by clicking on box.

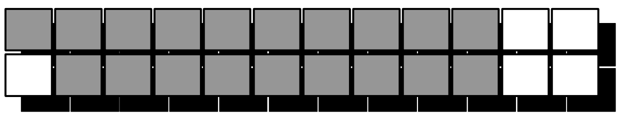
- Visualisation - Use of document topology / colour-mapping / interaction

Example : Title Bar Query / Result

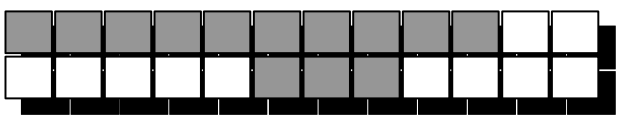
Query terms: DBMS (Database Systems)

Reliability

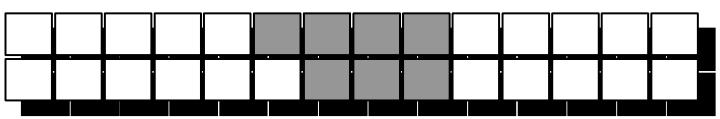
What roles do they play in retrieved documents?



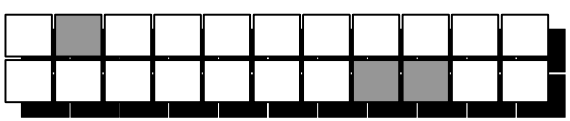
Mainly about both DBMS
& reliability



Mainly about DBMS, discusses
reliability



Mainly about, say, banking, with
a subtopic discussion on
DBMS/Reliability



Mainly about
something different

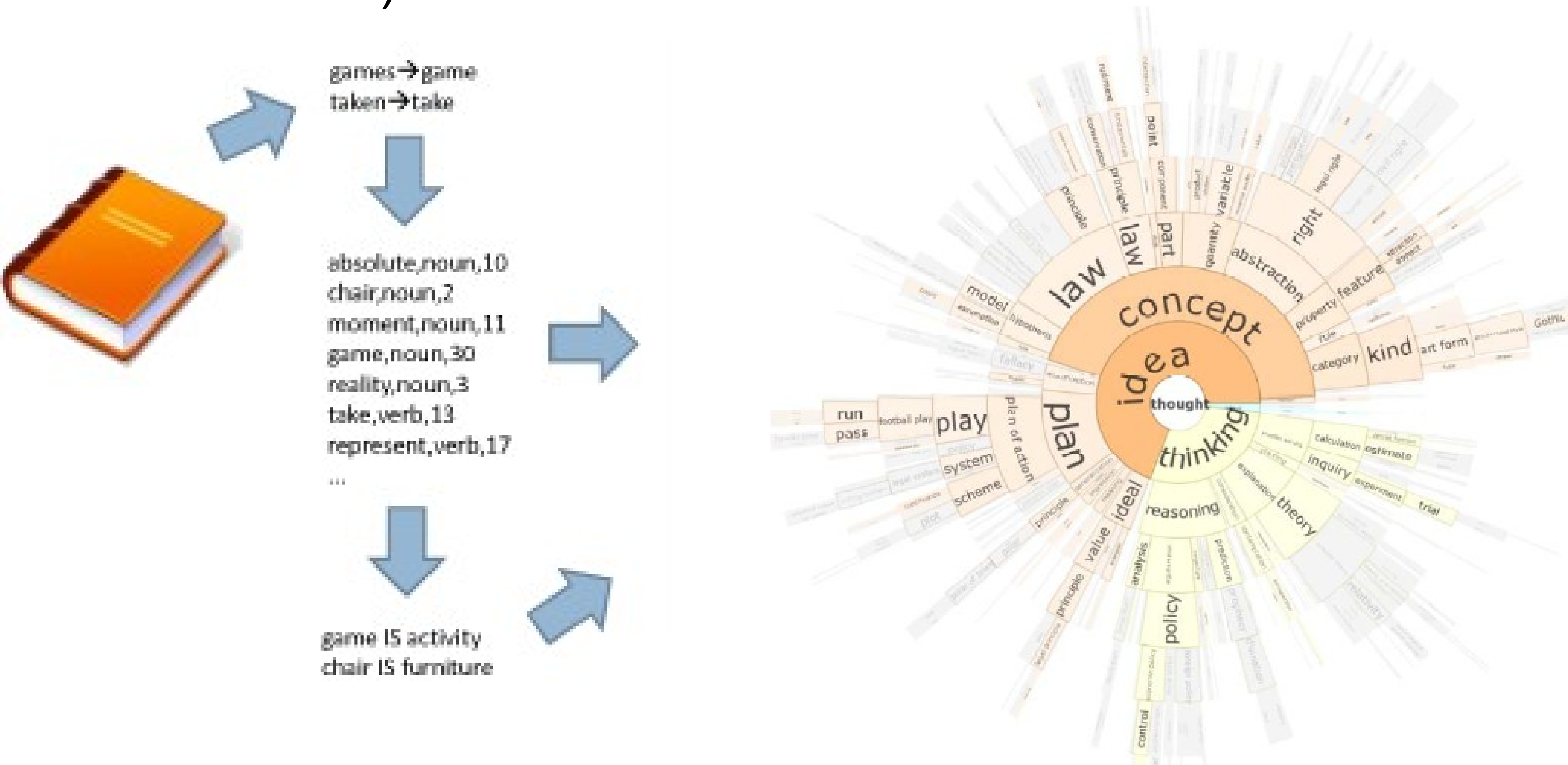
Wordle

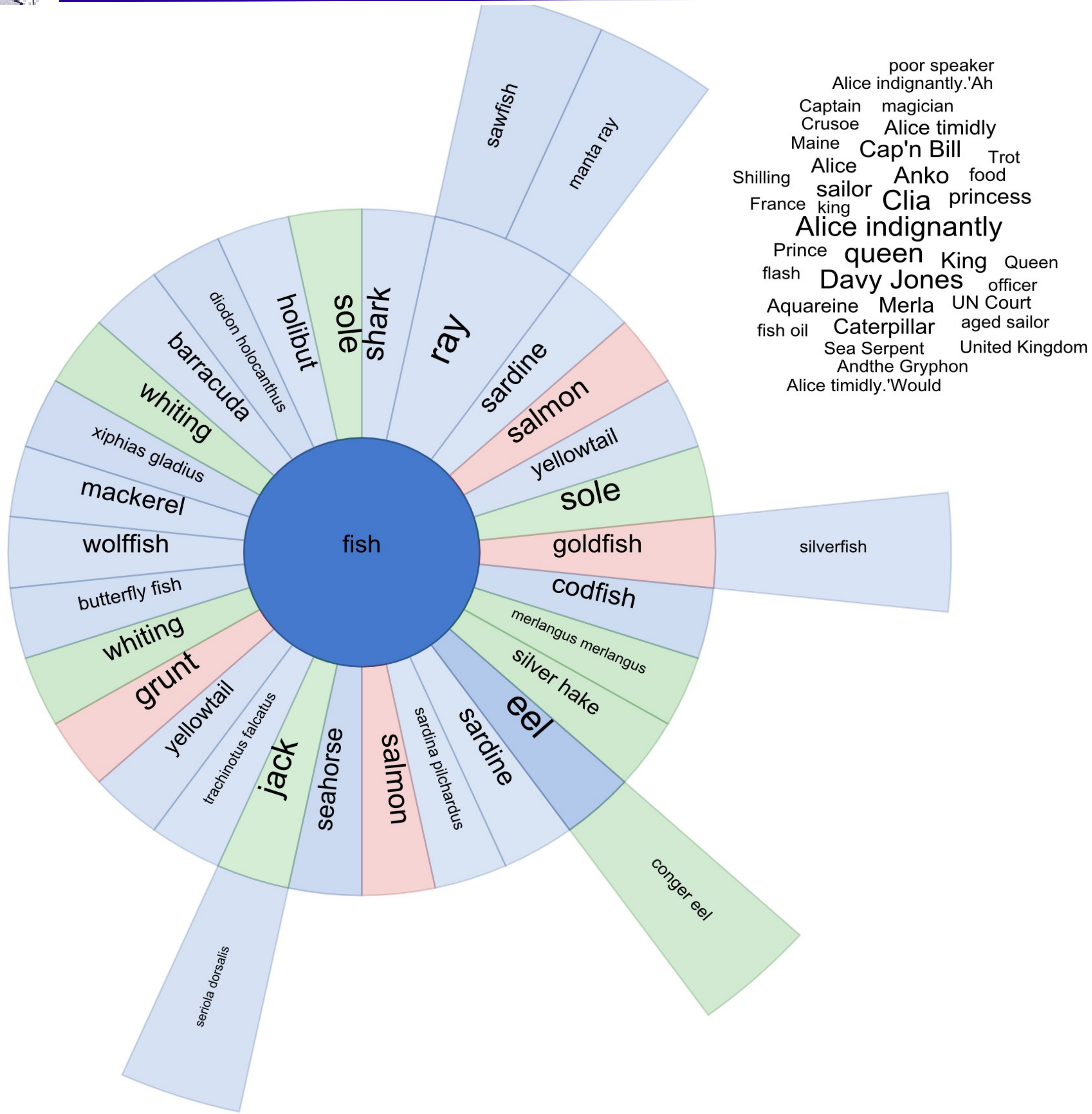
http://www.wordle.net/create

Produces a word cloud from a document

DocuBurst

- A radial, space-filling layout of hyponymy (IS-A relation)





http://vialab.science.uoit.ca/docuburst/

Summary

- **Tensor Visualisation**
  - **challenging**
  - for common rank 2 tensors in $\mathbb{R}^3$
    - common sources **stress / strain / MRI data**
  - a number of methods exist via **eigenanalysis decomposition of tensors**
    - **3D glyphs** – specifically **ellipsoids**
    - **vector and scalar field** methods
    - **hyper-streamlines**
    - **LIC** in 3D volumes
- Information Visualisation
  - Univariate, bivariate, trivariate, multi-variate data
  - Relations visualized by lines, tree visualization
  - Document visualization

Reading

- Processing and Visualization of Diffusion Tensor MRI [Westin et al. '02]
- Tensor field visualisation using adaptive filtering of noise fields combined with glyph rendering [Sigfridsson et al. '02]
- *Marti A. Hearst* **TileBars: Visualization of Term Distribution Information in Full Text Information Access**
- **Collins, Christopher;** Carpendale, Sheelagh; and Penn, Gerald. DocuBurst: Visualizing Document Content using Language Structure. Computer Graphics Forum (Proceedings of Eurographics/IEEE-VGTC Symposium on Visualization (EuroVis '09)), 28(3): pp. 1039-1046, June, 2009
- Westin et al. '02, "Processing and visualization for diffusion tensor MRI"
- http://faculty.uoit.ca/collins/research/docuburst/index.html
- **http://searchuserinterfaces.com/book/sui_ch11_text_analysis_visualization.html**