Tensor Visualisation

Computer Animation and Visualisation
Lecture 16

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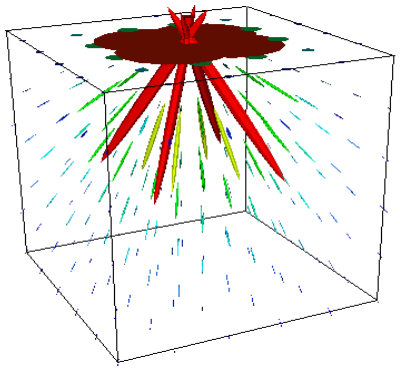
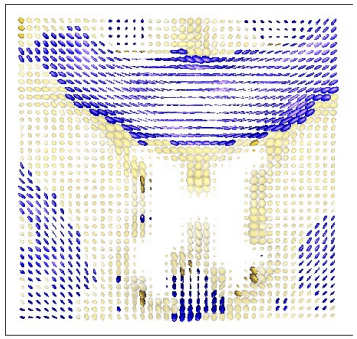
Overview

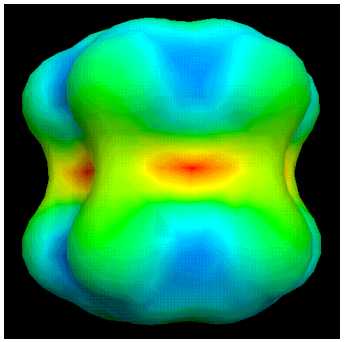
- **Tensor Visualisation**
  - **What is tensor**
  - **Methods of visualization**
    - **3D glyphs**
    - **vector and scalar field**
    - **hyper-streamlines**
    - **LIC** in 3D volumes

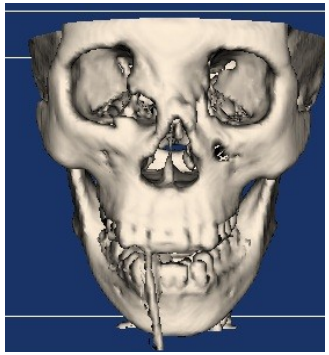




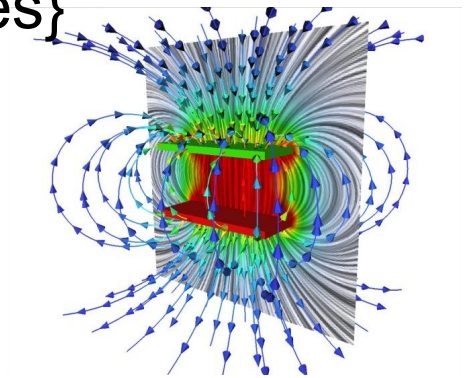
Reminder : Attribute Data Types

- **Scalar**
  - colour mapping, contouring





- **Vector**
  - lines, glyphs, stream {lines | ribbons | surfaces}



- **Tensor**
  - complex problem
  - today : **simple techniques for tensor visualisation**

What is a tensor ?

- A tensor is a table of rank *k* defined in *n*-dimensional space ($\mathbb{R}^n$)
  - generalisation of vectors and matrices in $\mathbb{R}^n$
    - Rank 0 is a scalar
    - Rank 1 is a vector
    - Rank 2 is a matrix
    - Rank 3 is a regular 3D array
  - *k* : rank defines the **topological dimension** of the attribute
    - i.e. it can be indexed with *k* separate indices
  - *n* : defines the ***geometrical dimension*** of the attribute
    - i.e. k indices each in range 0→(n-1)

Tensors in $\mathbb{R}^3$

- Here we limit discussion to tensors in $\mathbb{R}^3$
  - In $\mathbb{R}^3$ a tensor of rank *k* requires 3^k numbers
    - A tensor of rank 0 is a scalar *($3^0 = 1$)*
    - A tensor of rank 1 is a vector *($3^1 = 3$)*
    - A tensor of rank 2 is a 3x3 matrix (9 numbers)
    - A tensor of rank 3 is a 3x3x3 cube (27 numbers)

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad T = \begin{bmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{bmatrix}$$

- We will only treat rank 2 tensors – i.e. matrices

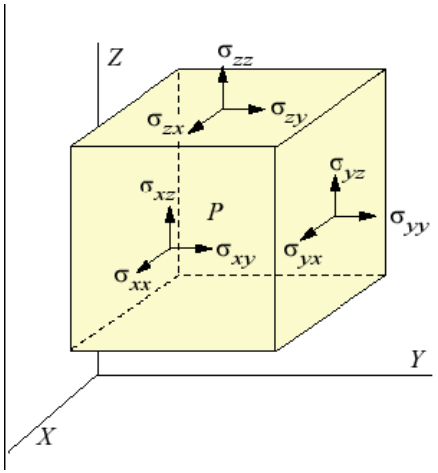
Where do tensors come from?

- **Stress/strain tensors**
  - analysis in engineering
- **DT-MRI**
  - molecular diffusion measurements
- *These are represented by 3x3 matrices*
  - *Or three normalized eigenvectors and three corresponding eigenvalues*

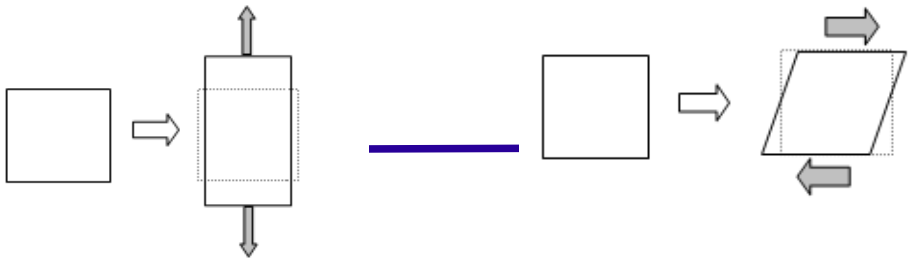
Stresses and Strain 1

- The **stress tensor:**

	In the direction of x:	y:	z:
stress on the face normal to x:	σ_{xx}	σ_{xy}	σ_{xz}
stress on the face normal to y:	σ_{yx}	σ_{yy}	σ_{yz}
stress on the face normal to z:	σ_{zx}	σ_{zy}	σ_{zz}

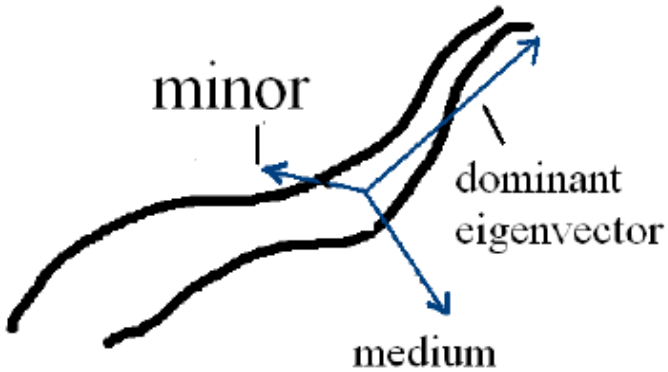


  - A **'normal' stress is a stress perpendicular (i.e. normal)** to a specified surface
  - A **shear stress acts tangentially** to the surface orientation
  - Stress tensor : characterised by **principle axes of tensor**
    - **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)
    - Form 3D co-ordinate system (locally) with mutually perpendicular axes



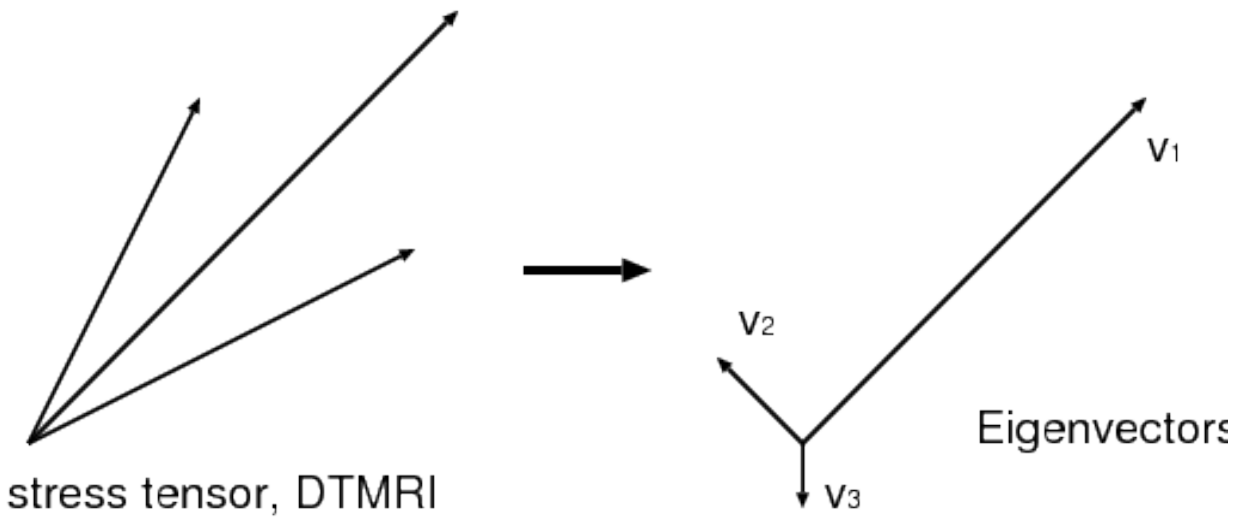
MRI : diffusion tensor

- Water molecules have **anisotropic diffusion in the body due to the cell shape and membrane properties**
  - Neural fibers : long cylindrical cells filled with fluid
  - Water diffusion rate is fastest along the axis
  - Slowest in the two transverse directions
  - brain functional imaging by detecting the anisotropy



Computing Eigenvectors

- 3x3 matrix results in **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)
- form 3D coordinate system (locally) with mutually perpendicular axes
- ordering by eigenvector referred to as **major, medium and minor eigenvectors**



Tensors : Visualisation Methods

- 2 main techniques : glyphs & vector methods

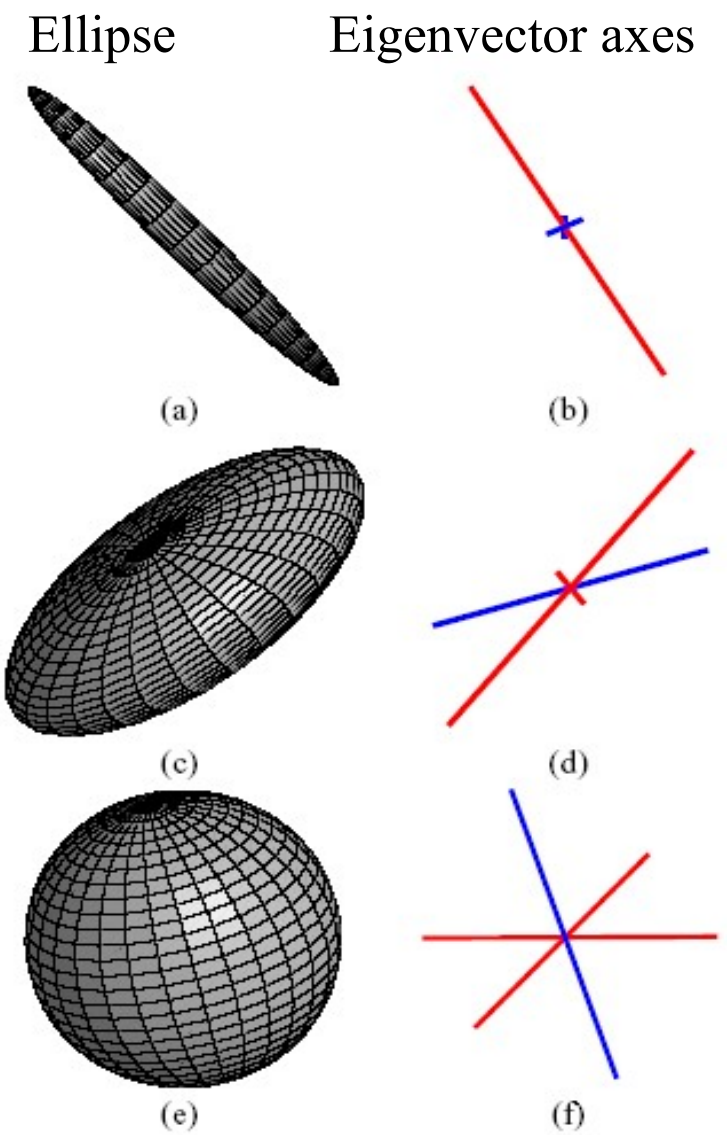
- **Glyphs**
  - 3D ellipsoids particularly appropriate (3 modes of variation)

- **Vector methods**
  - **a symmetric rank 2 tensor can be visualised as 3 orthogonal vector fields** (i.e. using eigenvectors)
  - **hyper-streamline**
  - Noise filtering algorithms – **LIC variant**

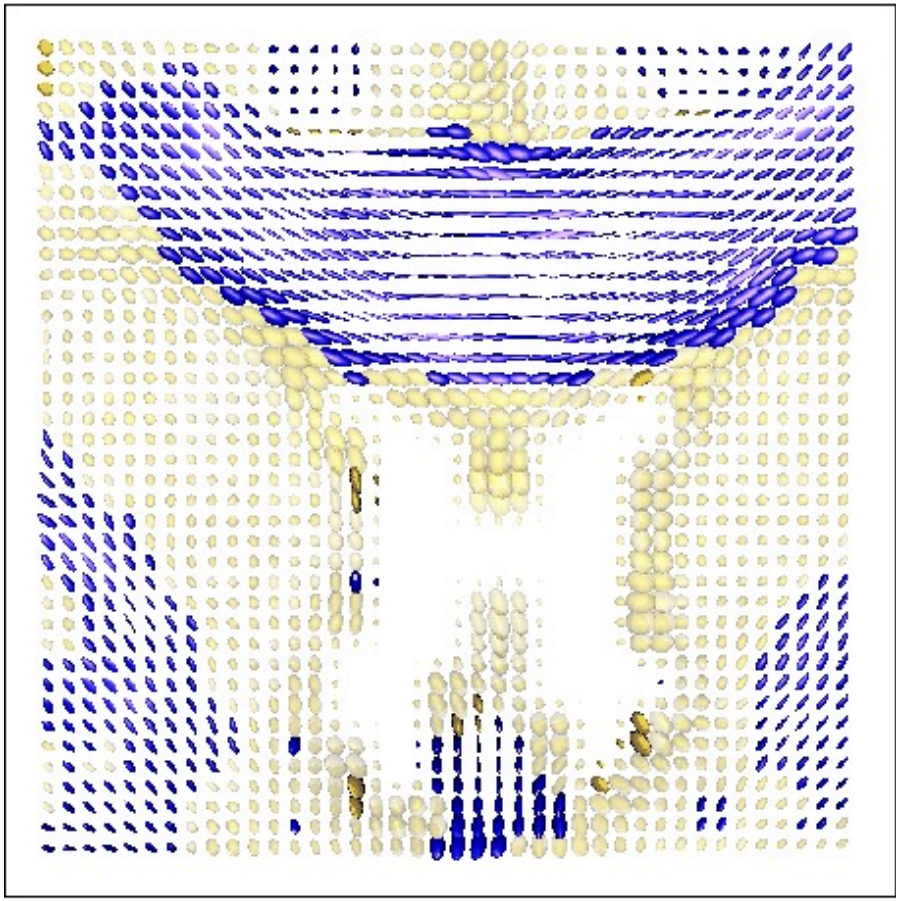
Tensor Glyphs

- **Ellipsoids**
  - **rotated into coordinate**

  system defined by eigenvectors of tensor
  - axes are **scaled by the eigenvalues**
  - **very suitable as 3 modes of variation**
- **Classes of tensor:**
  - (a,b) - **large major eigenvalue**
    - ellipse approximates a **line**
  - (c,d) - **large major and medium eigenvalue**
    - ellipse approximates a **plane**
  - (e,f) - **all similar** - ellipse approximates a **sphere**



Diffusion Tensor Visualisation



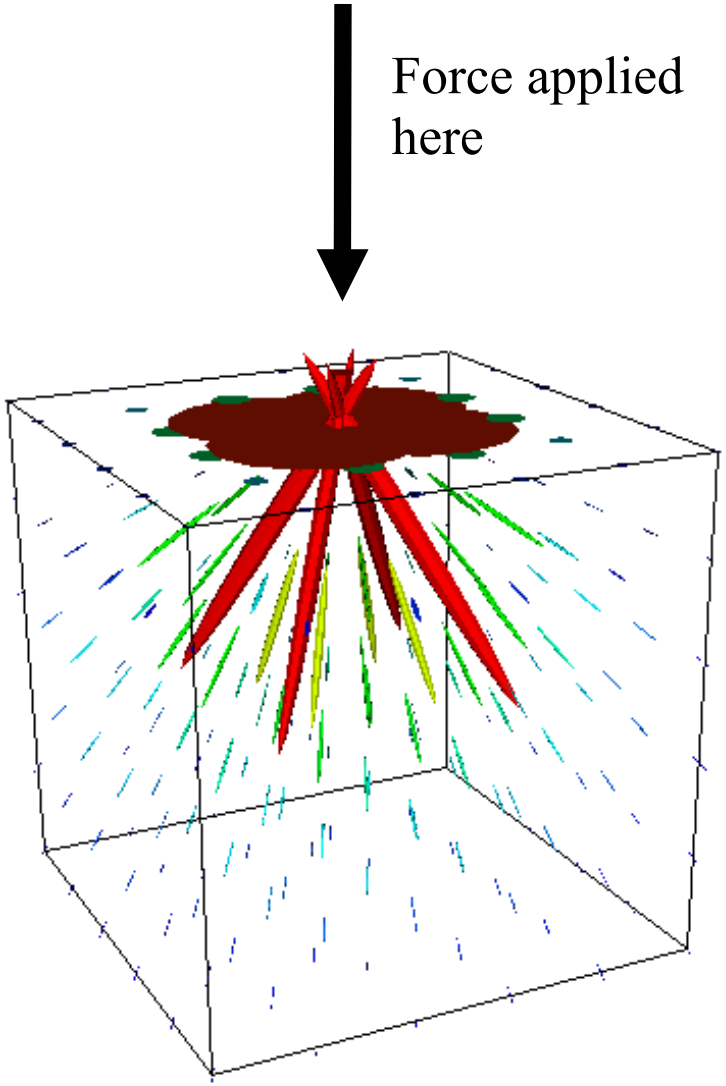
Baby's brain image (source: R.Sierra)

Anisotropic tensors indicate nerve pathway in brain:

- **Blue shape** – tensor approximates a line.
- **Yellow shape** – tensor approximates a plane.
- **Yellow transparent shape** – ellipsoids approximates a sphere

Colours needed due to **ambiguity in 3D shape** – a line tensor viewed 'end-on' looks like a sphere.

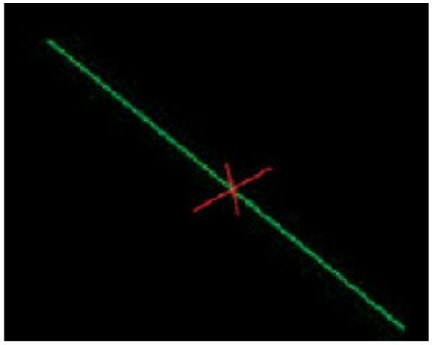
Stress Ellipses

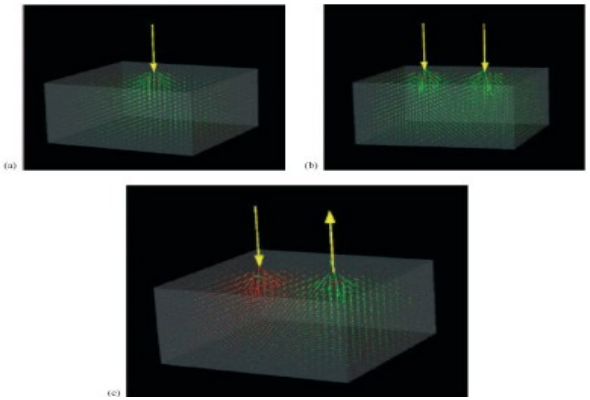


- Force applied to dense 3D solid – resulting **stress at 3D position in structure**
- Ellipsoids visualise the stress tensor
- Tensor Eigenvalues:
  - Large **major eigenvalue indicates principle direction of stress**
  - ‘Temperature’ **colourmap indicates size of major eigenvalue (magnitude of stress)**

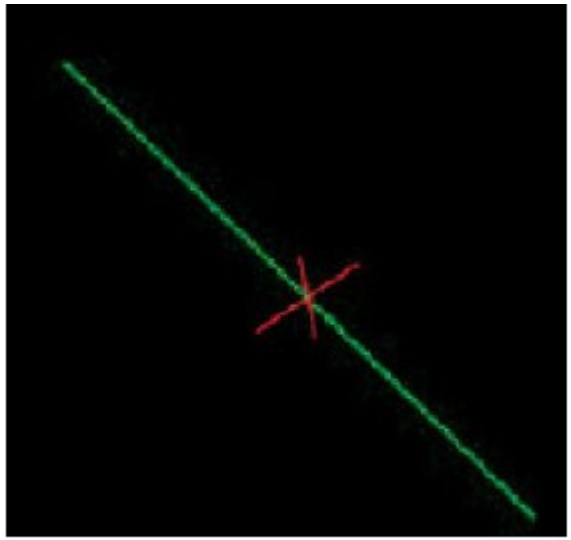
Tensor Visualisation as Vectors

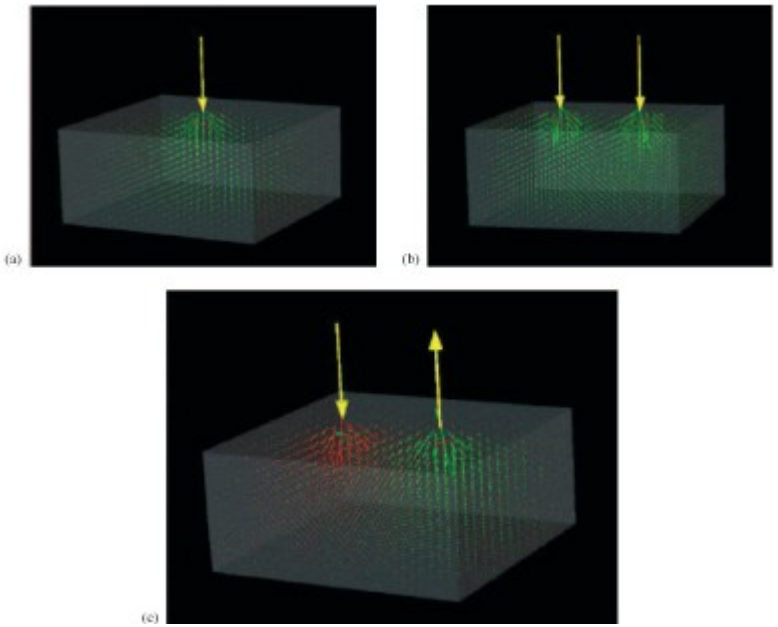
- Visualise just the major **eigenvectors as a vector field**
  - alternatively medium or minor eigenvector
  - use any of vector visualisation techniques from lecture 15





Lines, Hedgehogs





- Using hedgehogs to draw the three eigenvectors

  The length is the stress value
- Good for simple cases as above
  - Applying forces to the box
  - Green represents positive, red negative

Streamlines for tensor visualisation

- Often major eigenvector is used, with medium and minor shown by other properties
  - **Major vector is relevant in the case of anisotropy** - indicates nerve pathways or stress directions.

http://www.cmiv.liu.se/

Streamlines for tensor visualisation

- **Each eigenvector defines a vector field**
- Using the eigenvector to create the streamline
  - **We can use the Major vector, the medium and the minor vector to generate 3 streamlines**

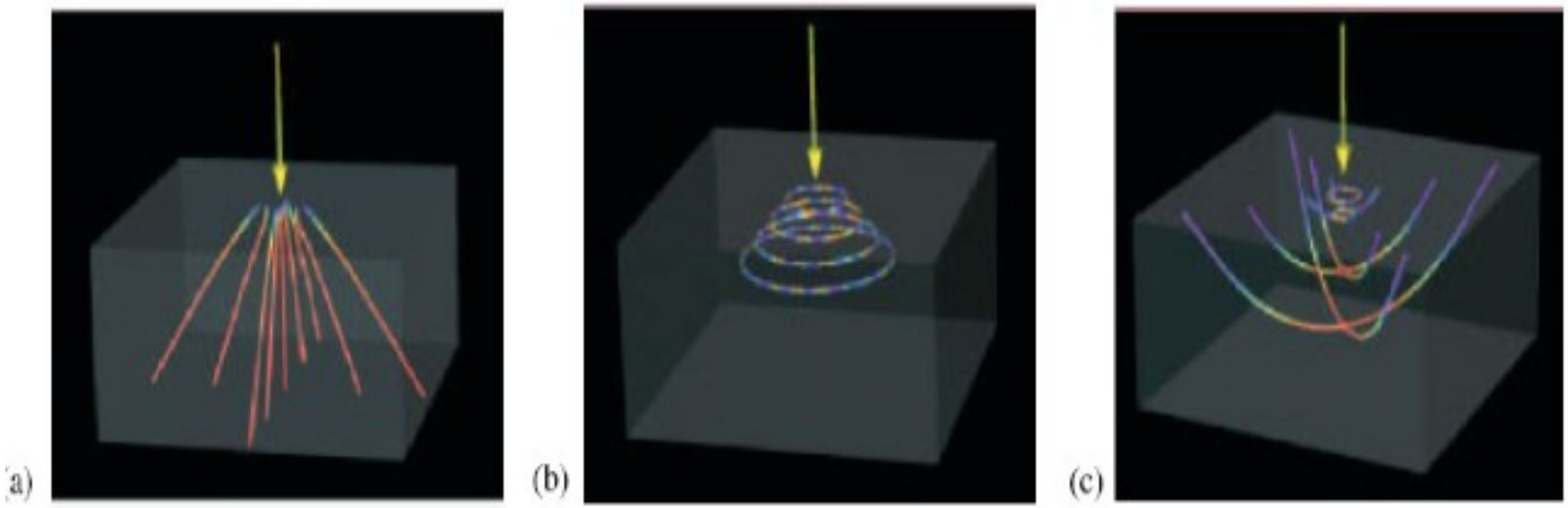
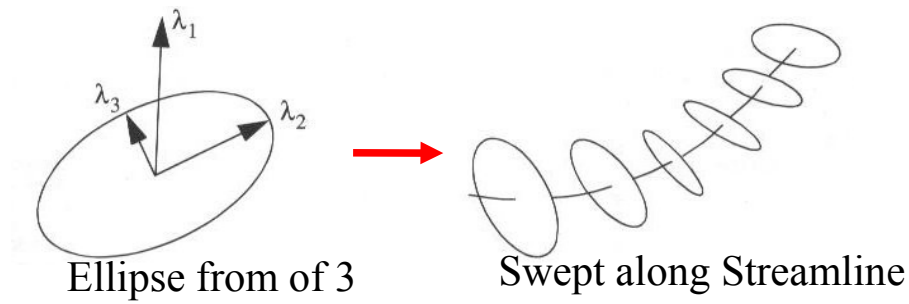


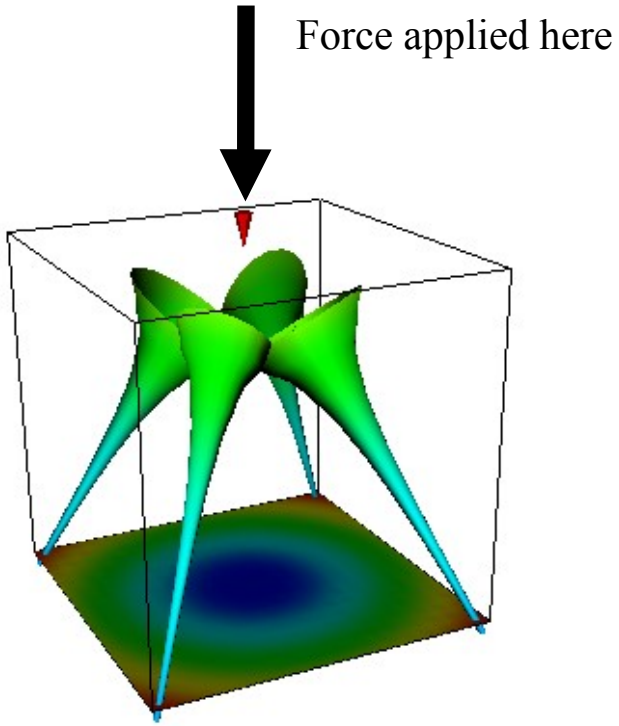
Figure 8. Hyperstreamlines for minor, intermediate and major principal stress for a point-load.

Hyper-streamlines [Delmarcelle et al. '93]

- Construct a **streamline from vector field of major eigenvector**

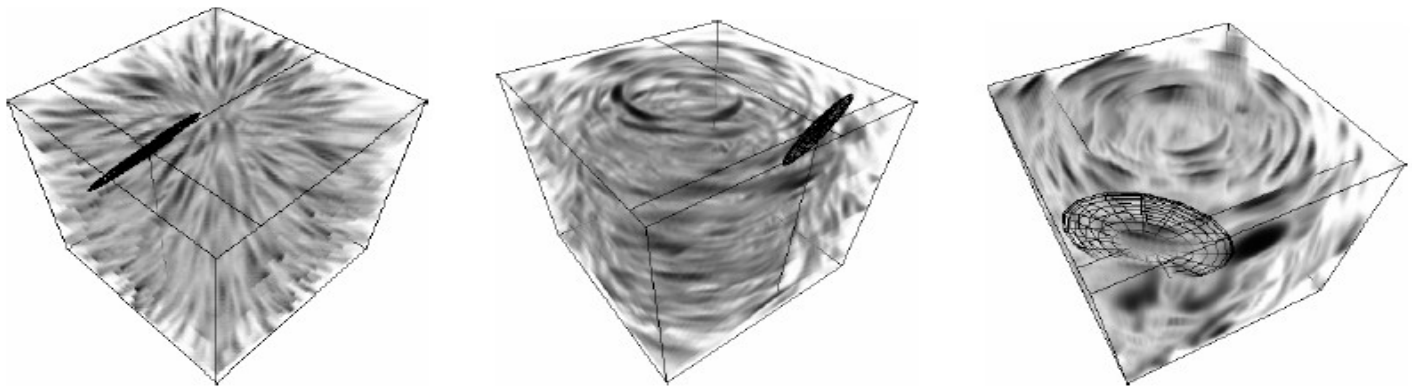


- **Form ellipse together with medium and minor eigenvector**
  - both are orthogonal to streamline direction
  - use major eigenvector as surface normal (i.e. orientation)
- **Sweep ellipse** along streamline
  - ***Hyper-Streamline*** ***(type of stream polygon)***



LIC algorithm for tensors

- **Linear Integral Convolution – LIC**
  - **‘blurs’ a noise pattern with a vector field**
  - For **tensors**
    - can **apply ‘blur’ consecutively for 3 vector field directions** (of eigenvectors)
    - using result from previous blur as input to next stage
    - use **volume rendering** with opacity = image intensity value **for display**

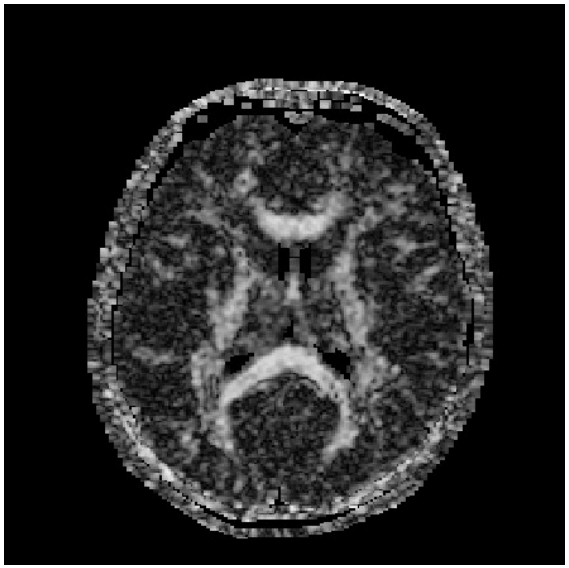


(a) (b) (c)

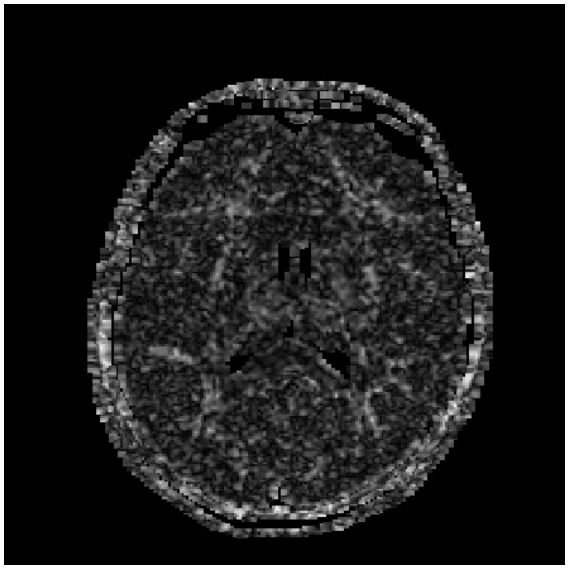
[*Sigfridsson et al. '02*]

Scalar field Method for Tensors

- **Scalarfield** : Produce grayscale image intensity in relation to tensor class (or closeness too). *(scalar from tensors)*



Greyscale image shows **how closely the tensor ellipsoids approximate a line.**



Greyscale image shows **how closely the tensor ellipsoids approximate a plane.**

Greyscale image shows **how closely the tensor ellipsoids approximate a sphere.**

Summary

- **Tensor Visualisation**
  - **challenging**
  - for common rank 2 tensors in $\mathbb{R}^3$
    - common sources **stress / strain / MRI data**
  - a number of methods exist via **eigenanalysis decomposition of tensors**
    - **3D glyphs** – specifically **ellipsoids**
    - **vector and scalar field** methods
    - **hyper-streamlines**
    - **LIC** in 3D volumes

Reading

- Processing and Visualization of Diffusion Tensor MRI [Westin et al. '02]
- Tensor field visualisation using adaptive filtering of noise fields combined with glyph rendering [Sigfridsson et al. '02]