Tensor Visualisation

Computer Animation and Visualisation
Lecture 16

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Overview

• Tensor Visualisation
  - What is tensor
  - Methods of visualization
    - 3D glyphs
    - vector and scalar field
    - hyper-streamlines
    - LIC in 3D volumes
Reminder : Attribute Data Types

- **Scalar**
  - colour mapping, contouring

- **Vector**
  - lines, glyphs, stream {lines | ribbons | surfaces}

- **Tensor**
  - complex problem
  - today : **simple techniques for tensor visualisation**
What is a tensor?

• A tensor is a table of rank $k$ defined in $n$-dimensional space ($\mathbb{R}^n$)
  
  - generalisation of vectors and matrices in $\mathbb{R}^n$
    
    - Rank 0 is a scalar
    - Rank 1 is a vector
    - Rank 2 is a matrix
    - Rank 3 is a regular 3D array

  - $k$ : rank defines the **topological dimension** of the attribute
    - i.e. it can be indexed with $k$ separate indices
  
  - $n$ : defines the **geometrical dimension** of the attribute
    - i.e. $k$ indices each in range $0 \rightarrow (n-1)$
Tensors in $\mathbb{R}^3$

- Here we limit discussion to tensors in $\mathbb{R}^3$
  - In $\mathbb{R}^3$ a tensor of rank $k$ requires $3^k$ numbers
    - A tensor of rank 0 is a scalar $(3^0 = 1)$
    - A tensor of rank 1 is a vector $(3^1 = 3)$
    - A tensor of rank 2 is a 3x3 matrix (9 numbers)
    - A tensor of rank 3 is a 3x3x3 cube (27 numbers)

\[ V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad T = \begin{bmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{bmatrix} \]

- We will only treat rank 2 tensors – i.e. matrices
Where do tensors come from?

• Stress/strain tensors
  – analysis in engineering

• DT-MRI
  – molecular diffusion measurements

• These are represented by 3x3 matrices
  – Or three normalized eigenvectors and three corresponding eigenvalues
Stresses and Strain 1

- The stress tensor:
  - A ‘normal’ stress is a stress perpendicular (i.e. normal) to a specified surface
  - A shear stress acts tangentially to the surface orientation
  - Stress tensor: characterised by principle axes of tensor
    - **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)
    - Form 3D co-ordinate system (locally) with mutually perpendicular axes
MRI: diffusion tensor

- Water molecules have **anisotropic diffusion in the body due to the cell shape and membrane properties**
  - Neural fibers: long cylindrical cells filled with fluid
  - Water diffusion rate is fastest along the axis
  - Slowest in the two transverse directions
  - Brain functional imaging by detecting the anisotropy
Computing Eigenvectors

• 3x3 matrix results in **Eigenvalues** (scale) of normal stress along **eigenvectors** (direction)

• form 3D coordinate system (locally) with mutually perpendicular axes

• ordering by eigenvector referred to as **major, medium and minor eigenvectors**
Tensors : Visualisation Methods

• 2 main techniques : glyphs & vector methods

• Glyphs
  – 3D ellipsoids particularly appropriate (3 modes of variation)

• Vector methods
  – a symmetric rank 2 tensor can be visualised as 3 orthogonal vector fields (i.e. using eigenvectors)
  – hyper-streamline
  – Noise filtering algorithms – LIC variant
Tensor Glyphs

- **Ellipsoids**
  - rotated into coordinate system defined by eigenvectors of tensor
  - axes are *scaled by the eigenvalues*
  - very suitable as 3 modes of variation

- **Classes of tensor:**
  - (a,b) - *large major eigenvalue*
    - ellipse approximates a *line*
  - (c,d) - *large major and medium eigenvalue*
    - ellipse approximates a *plane*
  - (e,f) - *all similar* - ellipse approximates a *sphere*
Anisotropic tensors indicate nerve pathway in brain:

- **Blue shape** – tensor approximates a line.
- **Yellow shape** – tensor approximates a plane.
- **Yellow transparent shape** – ellipsoids approximate a sphere

Colours needed due to ambiguity in 3D shape – a line tensor viewed ‘end-on’ looks like a sphere.

Baby's brain image (source: R.Sierra)
Stress Ellipses

- Force applied to dense 3D solid – resulting **stress at 3D position in structure**
- Ellipsoids visualise the stress tensor
- Tensor Eigenvalues:
  - Large major eigenvalue indicates principle direction of stress
  - ‘Temperature’ **colourmap indicates size of major eigenvalue** (magnitude of stress)
Tensor Visualisation as Vectors

- Visualise just the major eigenvectors as a vector field
  - alternatively medium or minor eigenvector
  - use any of vector visualisation techniques from lecture 15

Source: R. Sierra
• Using hedgehogs to draw the three eigenvectors
  The length is the stress value
• Good for simple cases as above
  – Applying forces to the box
  – Green represents positive, red negative
Streamlines for tensor visualisation

- Often major eigenvector is used, with medium and minor shown by other properties
  - **Major vector is relevant in the case of anisotropy** - indicates nerve pathways or stress directions.

http://www.cmiv.liu.se/
Streamlines for tensor visualisation

- Each eigenvector defines a vector field
- Using the eigenvector to create the streamline
  - We can use the Major vector, the medium and the minor vector to generate 3 streamlines

Figure 8. Hyperstreamlines for minor, intermediate and major principal stress for a point-load.
Hyper-streamlines [Delmarcelle et al. '93]

- Construct a streamline from vector field of major eigenvector

- Form ellipse together with medium and minor eigenvector
  - both are orthogonal to streamline direction
  - use major eigenvector as surface normal (i.e. orientation)

- Sweep ellipse along streamline
  - **Hyper-Streamline** (type of stream polygon)
LIC algorithm for tensors

- **Linear Integral Convolution – LIC**
  - ‘blurs’ a noise pattern with a vector field
  - For tensors
    - can apply ‘blur’ consecutively for 3 vector field directions (of eigenvectors)
    - using result from previous blur as input to next stage
    - use **volume rendering** with opacity = image intensity value for display

[Sigfridsson et al. '02]
Scalar field Method for Tensors

- **Scalarfield**: Produce grayscale image intensity in relation to tensor class (or closeness too). *(scalar from tensors)*

Greyscale image shows **how closely the tensor ellipsoids approximate a line.**

Greyscale image shows **how closely the tensor ellipsoids approximate a plane.**

Greyscale image shows **how closely the tensor ellipsoids approximate a sphere.**
Summary

• Tensor Visualisation
  - challenging
  - for common rank 2 tensors in $\mathbb{R}^3$
    - common sources stress / strain / MRI data
  - a number of methods exist via eigenanalysis decomposition of tensors
    - 3D glyphs – specifically ellipsoids
    - vector and scalar field methods
    - hyper-streamlines
    - LIC in 3D volumes
Reading

• Processing and Visualization of Diffusion Tensor MRI [Westin et al. '02]

• Tensor field visualisation using adaptive filtering of noise fields combined with glyph rendering [Sigfridsson et al. '02]