## Computer Animation

## Lecture 2.

## Basics of Character Animation

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## Overview

$\square$ Character Animation
■ Posture representation
■ Hierarchical structure of the body
■ Joint types
$\square$ Translational, hinge, universal, gimbal, free

- Euler angles
-Gimbal lock
- Quaternions
- Coordinate Transformations

Creating the animation (Keyframe animation)

- Interpolation

■ Inverse Kinematics

- Analytical
- CCD

■ Pseudo Inverse

## Characters include

$\square$ Human models
$\square$ Virtual characters
$\square$ Animal models


## Controlling the Skeleton

- We control the skeleton of the character
$\rightarrow$ skeletal motion
- The skin follows the movement of the skeleton
$\rightarrow$ skinning deformation



## Representation of Postures

## -The body has a hierarchical structure $\bullet$ Many \& different types of joints



## Hierarchical structure of the body

-The first joint is typically called the "root" of the character

- The position of the joints lower in the hierarchy are affected by those above it
- Each joints can have 1 to 6 degrees of freedom (DOF)
-For rotational joints, usually it is 1,2 , or 3
- The root of the body has 6 DOF in total (3 DOF for translation, 3DOF for rotation)



## Joints

$\square$ The Degrees of Freedom (DOF) is defined for various joints
$\square$ There are several kinds of joints

- Translational joint (1,2,3 DOF)
- Hinge joints (1 DOF)
- Universal joint (2 DOF)
- Gimbal joint (3 DOF)
- Free joint (3 DOF)


## Translational joint

$\square$ A sliding joint
$\square$ Can be 1,2 or 3 DOF


## Hinge Joint

$\square$ A 1 DOF rotational joint
$\square$ Can be defined by the axis of rotation
$\square$ Knee, elbow


## Universal Joint

- 2DOF
- Rotation around 2 axes perpendicular to each other
$\square$ Wrist joint



## 3DOF rotational joints

$\square$ Shoulder, hip, neck
$\square$ Two ways to represent the rotations

- Gimbal joint (Euler Angles)

■ Free joint (Quaternions)

## Gimbal joint: Euler Angles

$\square$ 3DOF joints
$\square$ Comes from Robotics
$\square$ 3DOF joints in robots were designed by connecting three motors pointing different axes


## Gimbal joint: Euler Angles

$\square$ Rotation defined by the three axes and the angle of rotation around them
$\square$ the rotation order has to be specified such as $X-Y-Z, Z-X-Y, Y-Z-X$, etc
The one below is $Z-X$


## Problem: Gimbal Lock

$\square$ Two rotational axis of an object pointing in the same direction - 1DOF is lost

- For example for rotation defined in the order of $X-Y$ Z
- Gimbal lock occurs when rotating Y for 90 degrees.
- $X$ and $Z$ axis get pointed down the same axis



## Problem: Gimbal Lock


https://www.youtube.com/watch?v=zc8b2Jo7mn

## Free joint

$\square$ A ball joint
$\square 3$ DOF
-Do not have to worry about gimbal lock


## Free joint: Quaternion

$\square$ Do not have to worry about gimbal lock
$\square$ The rotation is represented by a vector of four components $\left(q_{x}, q_{v}, q_{z}, q_{w}\right)$


## Free joint: Quaternion

$\square$ Do not have to worry about gimbal lock
$\square$ The rotation is represented by a vector of four components ( $q_{x}, q_{v}, q_{z}, q_{w}$ )
A rotation about the unit vector $\boldsymbol{q}$ by an angle alpha around an axis ( $a_{x}, a_{y}, a_{z}$ ) makes a quaternion:

$$
q_{a}(\alpha)=\left(q_{x}, q_{y}, q_{z}, q_{w}\right)=\left(a_{x} S \frac{\alpha}{2}, a_{y} S \frac{\alpha}{2}, a_{z} S \frac{\alpha}{2}, C \frac{\alpha}{2}\right)
$$


(S, C are sin, cos)

## Generalized Coordinates

A vector to specify the posture of the body

$$
q=\left(q_{1}, q_{2}, q_{3}, q_{4}, q_{5}, q_{6}, q_{7}, \ldots, q_{n}\right)
$$

Usually, the first three numbers: location of the root The next three numbers: orientation of root
The rest: the joint angles of the body


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## Keyframe Animation

$\square$ The keyframe postures are designed by the animator
$\square$ The inbetween motion is created by interpolation


## Keyframe Design

$\square$ Each postures are created by the user interface
■ Forward Kinematics (FK, Virtual Track Ball)
$\square$ The user clicks the segment and rotates it around the joint origin
$\square$ The movement of the mouse is mapped to the rotation of the joint
$\square$ Example:

- Inverse Kinematics (IK)
$\square$ The user clicks the segment and drags it in the 3D coordinate
$\square$ The motion of the mouse is mapped to the translation in 3D coordinate
$\square$ The movement of the segment is achieved by moving each joint of the body
$\square$ Example:
$\square$ Interactive Demo:


## Keyframe animation by Poser

$\square$ Poser is a commercial software to generate human animation
$\square$ There is another free software called MikuMikuDance

- Blender plugin called "Manuel Bastioni Labs"
$\square$ Unity3D In-Built IK \& Keyframe-Animation

https://www.youtube.com/watch?v=QAXOdj8mr7s


## Interpolation

The generalized coordinates can be interpolated by - Linear interpolation

- High-order polynomials (e.g. Bsplines, Bezier) Bezier: 2 end points, two points to control the tangent vector



## Linear interpolation

## Bsplines

## Interpolation of Quaternions

$\square$ Interpolation of two rotations (SLERP)
$\square$ Changing the orientation from q 1 to q 2 by rotating around a single axis $\boldsymbol{u}$
$\square \quad$ angle of rotation around $\boldsymbol{u}$ to change from q1 to q2


## Problems with interpolating the generalized coordinates

■ Important constraints might not be satisfied
$\rightarrow$ The feet on the ground can slide
$\rightarrow$ Mainly reasoned by interpolating the root wrong $\rightarrow$ "Error-propagation" along the hierarchy

- Solution
$\rightarrow$ Specify the position of the joints and use IK to calcualte the joint angles


## Forward and Inverse Kinematics

■ Forward kinematics solution is straightforward to compute and unique. However, hard to control. $\rightarrow$ "We don't know the joint angles to reach a location."

■ Inverse kinematics can have zero to infinite solutions. Solutions tell us the joint angles.


## Inverse Kinematics Problem

$\square$ Calculate the joint angles from the position of the end effector


## Inverse Kinematics

$\square$ Forward Kinematics: calculating the joint positions from the joint angles
$\square$ Inverse Kinematics: Calculate the joint angles based on the joint positions

■ Many different approaches
$\square$ Analytical approaches (analytical solution exists)
$\square$ CCD (Cyclic Coordinate Descent)
$\square$ Jacobian-based methods (compute by optimization)
$\square$
■ Originated from robotics, but also essential in animation. Main difference: speed vs. accuracy

## Analytical Approaches

$\square$ Using an analytical solver for calculating the joint angles
$\square$ e.g. suppose the positions of the wrist and shoulder are given, calculate the elbow angle

$$
\theta=\arccos \frac{\mathrm{r}_{1}^{2}+\mathrm{r}_{2}^{2}-\mathrm{r}^{2}}{2 \mathrm{r}_{1} \mathrm{r}_{2}}
$$



## Cyclic-Coordinate Descent

Moving the joints closest to the end effectors first and minimize the distance between the end effector and the target
Move up the hierarchy and move the next joint to minimize the distance between the end effector and the target
Repeat the process until the base is reached
Move to the first joint again and repeat the same process until the end effector reaches the target or max iterations reached

- This is needed to handle cases that the target is not reachable
- 2D Tutorial: http://www.ryanjuckett.com/programming/cyclic-coordinate-descent-in-


figure 38.


FIGure 3 c.

figure 30.

## Pseudo-inverse method

- The relationship between the end effector position and the joint angles are non-linear

$$
\mathbf{e}=\mathrm{f}(\mathbf{q})
$$

- But for small movements, their relation can be considered linear
$\boldsymbol{\Delta} \mathbf{e}=\mathbf{J} \boldsymbol{\Delta} \mathbf{q}$



## Jacobian matrix

- Correlates the movement of the end effector $\Delta \mathrm{e}$ with movements of the joints $\Delta \mathbf{q}_{\mathrm{i}}$ Each column describing how much $\Delta \mathbf{e}$ changes when the $\boldsymbol{\Delta q i}$ is changed

$$
\Delta e=\mathbf{J} \boldsymbol{\Delta} q
$$

Jacobian $J=\frac{\partial e}{\partial q} \approx\left|\begin{array}{lll}\frac{\Delta e_{x}}{\Delta q_{1}} & \frac{\Delta e_{x}}{\Delta q_{n}} \\ \frac{\Delta e_{y}}{\Delta q_{1}} & \cdots & \frac{\Delta e_{y}}{\Delta q_{n}} \\ \frac{\Delta e_{z}}{\Delta q_{1}} & \frac{\Delta e_{z}}{\Delta q_{n}}\end{array}\right|$

## Pseudo-inverse method

- The relationship between the end effector position and the joint angles are non-linear

$$
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$$

- But for small movements, their relation can be considered linear

$$
\Delta \mathrm{e}=\mathbf{J} \boldsymbol{\Delta} \mathbf{q}
$$

This is Forward mapping Inverse mapping?
$\boldsymbol{\Delta} \mathbf{e} \longrightarrow \boldsymbol{\Delta} \mathbf{q}$


## Pseudo Inverse Matrix

$\square$ The minimal joint angle movements that achieves the end effector movement e can be computed by the pseudo inverse matrix

## Pseudo-inverse Method

Iteratively updating the generalized coordinates so that the position constraints are satisfied
e: end effector position
g : target location
while ( $\Delta \mathrm{e}$ is too far from $\mathbf{g}$ ) \{

compute the Jacobian matrix $\mathbf{J}$

$$
\Delta e=J \Delta q
$$

compute the pseudoinverse of the Jacobian matrix $\mathbf{J}^{+}$
compute change in joint DOFs: $\Delta \mathbf{q}=\mathbf{J}^{\boldsymbol{+}} \boldsymbol{\Delta} \mathbf{e}$
apply the change to DOFs, move a small step of

$$
\mathbf{q}=\mathbf{q}+\Delta \mathbf{q}
$$

## Singularity Problem

$\square$ There can be postures that the end effector cannot be moved to some directions

- i.e. when all the joints are fully extended
$\square$ The system becomes unstable
$\square$ Test:
$\square$ Solution: Damped Least Squares
(imposing soft constraints): $J^{\star}=J^{T}\left(J J^{T}+k^{2} I\right)^{-1}$


## Which method will be good for controlling each of the below?



## Cons and Pros of IK

$\square$ Analytical : Fast, but for most cases have no analytical solution
$\square$ CCD : simple, fast, easy to implement, can take into account the limit of the joint angles, may have oscillation problems
$\square$ Pseudo Inverse:

- Used in robotics often, can handle any topological structure, multiple constraints
- Can incorporate physics
- Singularity problems (unstable when the limb is fully extended) -> damped least squares


## Summary

$\square$ Representation of the Posture
$\square$ Euler angles, generalized coordinates, quaternions
$\square$ Character Animation by Interpolation
$\square$ Inverse Kinematics
$\square$ Analytical
$\square$ CCD
$\square$ Jacobian Pseudoinverse
$\square$ Damped-Least-Squares

## Links, Readings

$\square$ Poser http://poser.smithmicro.com/poser.html
$\square$ MikuMikuDance
$\square$ https://sites.google.com/view/vpvp/
$\square$ About CCD http://graphics.cs.cmu.edu/nsp/course/15464/Fall09/assignments/asst2/jlander gamedev nov98.pdf
$\square$ IK introduction

- http://www.math.ucsd.edu/~sbuss/ResearchWeb/ikmethods/iksu rvey.pdf
$\square$ A robotics textbook (can be downloaded from within the university, see chapter 3)
- http://link.springer.com/content/pdf/10.1007\%2F978-1-84628-642-1.pdf

