Simulating Pathological Gait using the Enhanced Linear Inverted Pendulum Model

Taku Komura*, *Member, IEEE,* Akinori Nagano, Howard Leung, *Member, IEEE,* Yoshihisa Shinagawa, *Member, IEEE,*

***Abstract*— In this study, we propose a new method to simulate human gait motion when muscles are weakened. The method is based on the enhanced version of 3D linear inverted pendulum model that is used for generation of gait in robotics. After the normal gait motion is generated by setting the initial posture and the parameters that decide the trajectories of the center of mass and angular momentum, the muscle to be weakened is specified. By minimizing an objective function based on the force exerted by the specified muscle during the motion, the set of parameters that represent the pathological gait was calculated. Since the number of parameters to describe the motion is small in our method, the optimization process converges much more quickly than in previous methods. The effects of weakening the gluteus medialis, the gluteus maximus, and vastus were analyzed. Important similarities were noted when comparing the predicted pendulum motion with data obtained from an actual patient.**

***Index Terms*— bipedal gait, musculoskeletal model, static optimization, inverse dynamics**

I. Introduction

Computer simulations based on musculoskeletal models are often used to analyze the role of specific muscles during motions. Many researchers have analyzed the contribution of some specific muscles to the performance of certain motions by using optimization methods based on forward dynamics [1]. For example, Pandy *et al.*[2] analyzed the role of biarticular muscles in maximal jumping, and they concluded that biarticular muscles contribute to jumping performance by redistributing segmental energy within the musculoskeletal system but do not contribute to the energy of jumping (a conclusion that is consistent with that made by Jacobs *et al.* [3]). Neptune *et al.* [4] analyzed the role of the plantarflexor muscles during gait, and they calculated the degree to which these muscles contribute to propelling the trunk in the forward direction (induced acceleration [5]). Piazza *et al.*[6] examined the contribution of muscle forces to knee flexion during the swing phase of normal gait. Human gait motion have also been simulated by combining forward dynamics and optimization [7],[8],[4],[9],[10].

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Researchers in biomechanics often mention about using musculoskeletal models tuned to individual bodies for diagnosis and scheduling for rehabilitation [11]. This can be done by first generating the musculoskeletal model of the patient by estimating parameters such as the origin and insertion points, physiological cross sectional area, and tendon and muscle fiber length using CT scan and MRI images of the patient. Then, by capturing the motion of the patient using a motion capture device and estimating the bone-on-bone contact force and neural activities of the muscles, it will be possible to estimate what are the problems with the gait motion. Finally, the scheduling of rehabilitation can be done by searching for the most efficient way to convert the patholigcal gait to a normal gait. This scenario is attractive, as the patients will be able to know one of the best strategies for recovery based on the current status of their body. Providing such a system for diagnosis and scheduling of rehabilitation is the ultimate objective of our study.

When trying to achieve this goal, one of the tools that can be considered for use is the dynamic optimization approach [12],[7],[8]. However, there are several shortcomings with dynamic optimization approaches, that make it difficult to apply them for practical use.

1) First of all, when doing dynamic optimization, researchers must provide a good initial guess of the activation data to prevent the optimization process from getting caught into some local minima. In most of the cases, this is done through trial-and-error by researchers. This is quite time consuming, and the researcher needs special experience to determine the good set of muscle activations to generate realistic gait motion. In addition, since this process is done completely manually, the motion can become subjective to the researcher.
2) Next, since all the time sequence data for the muscles must be determined through the optimization, lots of computation are required until it converges to the optimal result. Neptune *et al.* [13] reported that more than thousands of iterations of the motion simulation were needed before the optimized values were obtained. In [14], sixty thousand simulations were conducted until the final optimal motion was obtained.
3) Third, although maintaining balance is one of the most important factor for gait motion, there was no explicit way to describe the balanced motion in these researches. As a result, the search space for the parameter is much larger since it also includes those parameter values that

lead to unbalanced motion.

In robotics, biped locomotion of humanoid robots is one of the most exciting topics these days. Many researchers have developed humanoid robots that are capable of performing biped gait motion [15], [16], [17], [18]. A method commonly used to generate gait motion is the 3D Linear Inverted Pendulum Mode (3DLIPM) [15],[18]. The great advantage of the 3DLIPM approach is that the trajectory of the center of mass (COM) can be written in an explicit form. The 3DLIPM approach is a hierarchical approach, in which the abstract motion such as the trajectory of COM is determined first, and the details of the motion such as the kinematic data are calculated later. The advantage of the hierarchical approach is that the motion planner only needs to specify the high level abstract motion; then the low level motion by the generalized coordinates are then decided, accordingly. In addition, as the motion of the COM is explicitly planned in the begining in a way the Zero Moment Point (ZMP) comes under the supporting region, the balance of the humanoid robot is assured in the feedforward stage. A balanced walking motion is defined here as follows:

1) the body does not fall down onto the ground during the motion,
2) the only contact with the external world is at the sole of the feet, and
3) the posture of the body and the trajectories of the COM, ZMP are cyclic.

No angular momentum has been generated by the 3DLIPM since it assumes that the COM is a mass point and that the ground force vector always passes through the COM of the system (Figure 1(a)). It is known that angular momentum is consistantly generated around the COM when humans walk [19]. The amount of angular momentum gets even larger as the motion involves larger bendings of the thorax, which can be often observed in pathological gait. It is therefore difficult to model human gait motion by using 3DLIPM. To represent trajectory of COM and angular momentum of human gait motion in an explicit form, we have extended the 3DLIPM model so that angular momentum can be induced by the ground reaction force [20]. This model is called *Angular Momentum inducing inverted Pendulum Model* (AMPM).

In this study, we propose a new approach to simulate the effect of impairment and recovery of muscles in human gait motion by combining AMPM with the musculoskeletal model. The effects of weakening the gluteus medialis, gluteus maximus muscles and vastus were simulated. The results were evaluated by comparing the motion and muscle neural activities with human motion data. The objective is to provide a computationally efficient framework for conversion of gait motion which threapists and medical doctors can utilize for diagnosing underlying pathologies, figuring out potential compensations for neuromuscular impairments of human gait, or scheduling the rehabilitation plan.

The trajectory of the COM and angular momentum is first calculated by using AMPM. Next, the trajectories of the generalized coordinates including the position of the pelvis and the joint angles of the whole body are determined using inverse kinematics. After determining the muscles of the body to be weakened, an objective function based on the time series data of the force exerted by this muscle is formed. By searching for an optimal set of AMPM parameters that minimizes the objective function defining the gait motion, the pathological gait motion is calculated.

The method proposed in this study has the following advantages compared with previous methods:

- Since the gait motion is described by the AMPM parameters, there is no need to specify all the input parameters of the muscles. As a result, the computational cost for the optimization is much less than previous methods.
- As the balance of the human body model is explicitly kept by using the AMPM model, the optimizer only needs to search for the optimal set of parameters in terms of muscle force, and hence does not need to go through a large number of trials to generate the balanced motion.

By using our method, it is possible to find out the general idea how an impairment of a muscle can affect biped gait. By doing further concise modeling of the musculoskeletal and development of techniques to import parameters from individual humans, it will be possible to use the proposed method for analysis, diagnosis, and rehabilitation training of patients with impairments at the muscles.

II. METHODS

A. Angular Momentum inducing inverted Pendulum Model

In this section, we review the Angular Momentum inducing inverted Pendulum Model (AMPM) [20]. The AMPM enhances the 3DLIPM in the following ways; (1) the zero moment point (ZMP), which represents the center of pressure for the force applied from the ground to the body [21], is allowed to move over the ground; ZMP is a very important concept used in robotics to generate stable gait motion. It represents the center of pressure between the ground and the feet on a plane, around which no rotational moment is generated; (2) the ground force vector is calculated to be not only parallel to the vector connecting the ZMP and the COM; its horizontal element can be linearly correlated to the ZMP-COM vector (Figure 1(b)). As a result, rotational moment will be generated by the ground force. The other advantage of AMPM is that the motion in the sagittal plane and the frontal plane can be planned independently; regarding the trajectory of the COM and angular momentum around it, the motion in one plane does not actually affect the motion in the other plane. Therefore, in the following explanation about AMPM, the trajectories of the COM are explained in a 2D plane. In this study, we assume the position of the ZMP is linearly correlated to the position of the COM. Therefore, the position of the COM and ZMP can be defined by (x, H) and $(zmp, 0) = (cx + d, 0)$, where (c, d) are constant parameters. According to the enhancement (2) of the AMPM model, the ground force vector is modelled to be parallel to the vector $(a(x - zmp) + b, g)$, where (a, b) are constant parameters that correlate the ZMP-COM vector and the ground force vector.

As the height of the COM is assumed to have a constant value H, the relationship between the acceleration of the COM

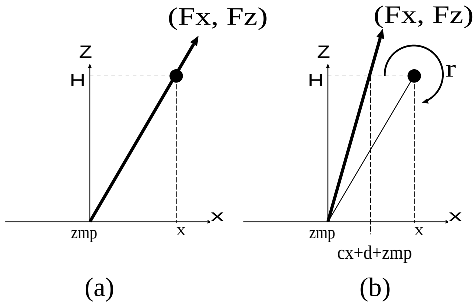


Fig. 1. The standard 3DLIPM (a) and AMPM (b). The 3DLIPM restricted the ground force to pass through COM while the AMPM allows the horizontal component of the ground force to have linear relationship with the position of the COM.

and its position can be written by:

$$F_x : F_z = \ddot{x} : (\ddot{z} + g) = \frac{H}{g}(a(x - (cx + d)) + b) : H.$$

The differential equation of the COM can then be written by the following form:

$$\ddot{x} = a(1 - c)x + b - ad. \tag{1}$$

The explicit solution for this differential equation can be written as

$$x = -\frac{b - ad}{a(1 - c)} + C_1 e^{-(\sqrt{a-ac})t} + C_2 e^{(\sqrt{a-ac})t} \tag{2}$$

where C_1, C_2 are constant values. As initial parameter values are set as $x = x_0$ and $\dot{x} = v_0$ at $t = 0$, the constant values C_1, C_2 will be as follows:

$$C_1 = \frac{1}{2}(x_0 - \frac{v_0}{\sqrt{a - ac}} + \frac{b - ad}{a(1 - c)}),$$
$$C_2 = \frac{1}{2}(x_0 + \frac{v_0}{\sqrt{a - ac}} + \frac{b - ad}{a(1 - c)}).$$

Then, the ground force vector can be written as

$$\begin{aligned} F_x &= m\ddot{x} = m(a - ac)\left(C_1 e^{-(\sqrt{a-ac})t} + C_2 e^{(\sqrt{a-ac})t}\right) \\ F_z &= mg \end{aligned}$$

where m is the mass of the system. The rotational moment r around the y-axis can be calculated by

$$\begin{aligned} r &= m(1 - c)(aH - g)\left(C_1 e^{-(\sqrt{a-ac})t} + C_2 e^{(\sqrt{a-ac})t}\right) \\ &+ mg(\frac{b}{a}) \end{aligned}$$

and the angular momentum ω_{t_1,t_2} generated by the rotational momentum between times $t = t_1, t_2$ can be obtained as

$$\omega_{t_1,t_2} = [\frac{m(1 - c)(aH - g)}{\sqrt{a - ac}}(-C_1 e^{-(\sqrt{a-ac})t} + C_2 e^{(\sqrt{a-ac})t}) + mgt(\frac{b}{a})]_{t_1}^{t_2} + \omega_1 \tag{3}$$

where ω_1 is the angular momentum at $t = t_1$.

B. Modeling Human Gait by AMPM

Since AMPM only allows linear relationship between the COM and the ground force vector, the gait motion must be divided into several phases to be represented by the AMPM (See Appendices I and II for further explanation in detail). As the motion of the gait is assumed symmetric, the half cycle of the gait motion in the sagittal plane can be divided into four stages, by the postures known as other toe off (OTO), heel rise (HR), opposite initial contact (OIC), middle stance (MS), and toe off (TO), as shown in Figure 2. The half cycle of the motion in the frontal plane can be represented by two consecutive AMPM models, one representing the single support phase, and the other representing the double support phase. A reference walking motion of a human obtained using a motion capture device is used as a reference. The parameters to explain these AMPMs can be calculated from the boundary conditions and kinematic parameters of the motion. The following parameters are used to define the gait motion in this research:

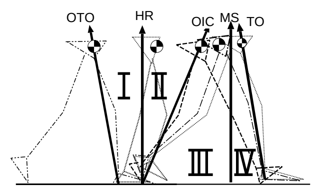


Fig. 2. The half cycle of the gait is divided into four phases, each motion represented by AMPM.

- the position of COM at OTO, HR, OIC, and its velocity at OTO in the sagittal plane,
- the difference of the body's moment of inertia in the initial posture from that in the reference motion,
- and the lateral distance between the left and right foot and the parameter that determines the angular momentum in the frontal plane.

These variables are represented by a vector $\boldsymbol{p}$ here. The initial posture in the new motion is calculated from the initial posture in the reference motion. The difference of the moment of inertia and the position of the COM is used to calculate the initial posture from that in the reference motion using inverse kinematics. The process of this methodology is explained later in this section.

At this point of the discussion, we have assumed that the height of the COM is constant. The adjustment can be easily handled, since it is known that the trajectory can be expressed in a sinusoidal form [22]. Therefore, to simulate the motion of the body more precisely, the trajectory of the height of the COM must be adjusted. The trajectory of the COM is therefore calculated by the following function:

$$z_g = H + z_h sin(\frac{t}{T}),$$

where T is a half cycle of gait motion, z_h is the amplitude of the motion, and z_g is the height of the COM. It is known that the COM reaches the bottom in the middle of the double support phase and the top in the middle of the single support phase. Although motion of the COM along the vertical axis changes from a constant value to a sinusoidal curve, we assume that its motion along the lateral and anterior axes remain the same as stated in the previous sections. This actually does not affect the motion of the COM and ZMP. However, the angular momentum function must be adjusted because change in the height results in a different z component of the ground force, and extra rotational momentum must therefore be added to the values that had been calculated before. This is not a serious problem because (1) this angular momentum difference is quite small compared to the original angular momentum value and (2) the angular momentum can be compensated anyway because our algorithm is based on the assumption of compensation of the angular momentum either in the sagittal or lateral plane.

After the trajectory of the COM and the angular momentum are determined by the AMPM, the motions of the body segments that satisfy those trajectories are calculated using inverse kinematics. It is known that methodologies of inverse kinematics combined with concepts of COM [23] and dynamics [24] give fairly realistic motions. Natural behavior such as swinging of arms and thorax can be obtained using this methodology. This process is also explained in [20], [25] and Appendix A3. By using inverse kinematics, the generalized coordinates, velocities and accelerations of the joints can be calculated. Then, the torque developed at each joint was calculated from these kinematic data using inverse dynamics [26]. During double support phase, to solve the redundancy problem caused by the closed-loop formed by the ground and the feet, the ground force vector was divided into two parts and applied to each foot in a manner such that the total amount of torque exerted by the joints is minimized [27]. In summary, by specifying the parameters $\boldsymbol{p}$ for the AMPM, the motion of the human body model can be calculated. As the kinematic data and external force data from the ground are available, the torques exerted by the joints can be calculated.

C. Musculoskeletal Model

To simulate the motion by pathological patients, it is necessary to prepare a physiological model of the human body. The musculoskeletal model developed by Delp [28] was used in this study. This data includes the attachment sites of 43 muscles on each leg and physiological parameters such as the tendon slack length, optimal fiber length, pennation angles, and maximum exertable force. Muscles are attached only to the legs, and no muscles are put on the upper half of the body. The human body model used in this study is shown in Figure 3. The upper half of the body is composed of the thorax, head, upper arms, lower arms, and hands. However, only the thorax (3 degrees of freedom (DOF)) and the upper arms (3 DOF each) are allowed to move among these segments. The lower half of the body is composed of the pelvis, and the femur, tibia, patella, talus, calcaneous, and toes in each leg. The joints of each leg are composed of a 3-DOF gimbal joint (hip joint) and 1-DOF joint (knee, ankle, calcaneous, and metatarsal joint). Therefore, the total degrees of freedom of the body, including the 6DOF of the pelvis in the global coordinate system, is twenty nine.

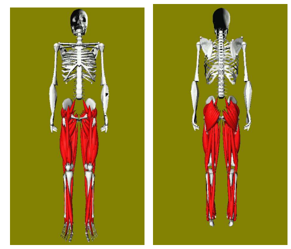


Fig. 3. The frontal (left) and backward (right) views of the human body model

Hill's muscle model [29] is used to represent each musculotendon. It is composed of three elements: a contractile element (CE, representing all the muscle fibers), a parallel elastic element (PEE, representing all connective tissues around the muscles fibers), and a series elastic element (SEE, representing all series elasticity, including tendons). At each time step, the musculotendon length was determined from the posture (i.e., as a function of joint angles). By using the musculotendon dynamics, the maximum and minimum force can be calculated using the musculotendon length and the muscle length in the previous time step [30]:

$$F_i^{min} \leq F_i(t) \leq F_i^{max} \tag{4}$$

where $F_i(t)$ is the musculotendon force of the ith muscle, F_i^{min} and F_i^{max} are the minimum and maximum force developed by this muscle, which are determined by their force-length-velocity properties.

D. Calculating pathological gait

Torque $\tau_j(t)$ developed at joint j is theoretically generated as follows by the muscles crossing the joint:

$$\tau_j(t) \quad = \quad \sum_i F_i(t) r_{i,j}, j = 0, ..., n_{dof} \tag{5}$$

where $r_{i,j}$ is the moment arm of muscle i about the jth joint axis, and n_{dof} is the number of degrees of freedom whose torque are assumed here to be generated by the muscles. They include the flexion/extension, adduction/abduction, and rotation at the hip, flexion/extension at the knee, and plantarflexion/dorsiflexion at the ankle, and therefore, by taking both legs into account, $n_{dof} = 10$.

The muscle forces at each time step can be calculated by minimizing the sum of the squared muscle stresses [31]:

$$J = \sum_i^{n_m} (\frac{F_i(t_j)}{F_i^0})^2 \tag{6}$$

where n_m is the total number of muscles ($n_m = 43 \times 2 = 86$). J was minimized using quadratic programming [32], which is a method to minimize a quadratic form while satisfying linear equality and inequality constraints. In summary, the muscle forces can be calculated by minimizing (6) while using (5) and (4) as constraints. This method is known as the *static optimization method* to estimate muscle force in biomechanics [8],[33].

To describe the process to calculate the pathological gait, a new function is defined here that summarize all the processes explained previously, including the AMPM, inverse kinematics, inverse dynamics and static optimization:

$$F_i = f_i(\boldsymbol{p}, t)(i = 0, ..., n_m - 1) \tag{7}$$

where $\boldsymbol{p}$ is the vector defined in Section II-B that includes the parameters defining the gait motion by the AMPM, and F_i is the force by muscle i calculated by minimizing (6). To simulate the effect of weakening muscle i, the following problem is solved:

$$\begin{gathered}\min_{\boldsymbol{p}} J_i \qquad \text{until } J_i <= sT \\ \text{where } J_i = \int_0^T [f_i(\boldsymbol{p}, t) + \alpha(\boldsymbol{p}, t)]dt\end{gathered} \tag{8}$$

where T is the half cycle of the motion, and s is a threshold value which can be considered as a reference of the maximum average output of the specified muscle during the gait cycle; for example, if the user wants to limit the average output of the muscle to less than 50% of the maximum output constant, it can be set to $0.5TF_i^0$. $\alpha(\boldsymbol{p}, t)$ is a penalty function based on the squared sum of the minimum external assisting torque at the joints that has to be applied to the body to assist the musculoskeletal model accomplish the motion when there is no solution found for the static optimization problem at time t [30],[34]:

$$\alpha(\boldsymbol{p}, t) = \min_{\boldsymbol{F}, \boldsymbol{\tau}_{ext}} ||\boldsymbol{\tau}_{ext}||^2$$

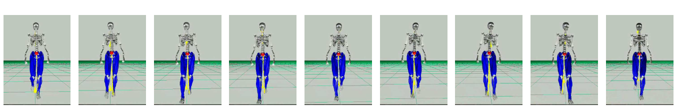
$$\text{subject to} \begin{cases} \boldsymbol{\tau} = \boldsymbol{A}\boldsymbol{F} - \boldsymbol{\tau}_{ext} \\ \boldsymbol{F}^{min} \leq \boldsymbol{F} \leq \boldsymbol{F}^{max} \end{cases}$$

where $\boldsymbol{A}$ is the matrix form of $r_{i,j}$ in (5). The penalty function helps to avoid the motion to converge to one that is not feasible by the musculoskeletal model. This optimization is done by sequential quadratic programming [32].

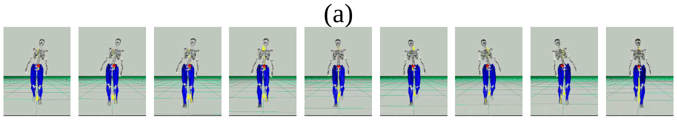
III. Experimental Data Analysis

To examine the validity of the method proposed in this paper, we have generated normal and pathological human gait motion using our method, and compared the kinematical and dynamical data with those by humans.

A set of AMPM parameters $\boldsymbol{p}$ that reproduce a gait motion with a step length of 0.6 m and velocity of 1 m/s were used. The gait motion is shown in Figure 4(a). By minimizing an objective function based on muscle dynamics that has the form of (8), the effects of weakening the gluteus medialis, gluteus maximus and vastus during gait were simulated. First, we conducted an experiment by minimizing the force exerted by the gluteus medialis. The threshold value for the objective function was set to 30% of the current exertion-rate. As the optimization proceeds, features known as lateral trunk bending appears in the motion. The trunk swings from one side to the other, producing a gait pattern known as waddling. During the double support phase, the trunk is generally upright, but as soon as the single support phase begins, the trunk leans over the support leg, returning to the upright attitude again at the beginning of the next double support phase. The trajectory of the gait motion after the optimization is shown in Figure 4 (b).



(a)

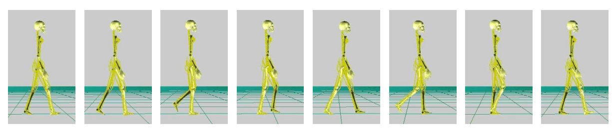


(b)

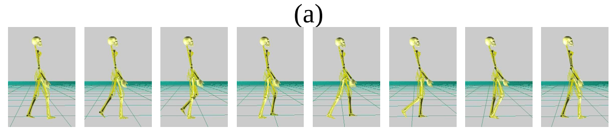
Fig. 4. The trajectory of the AMPM-generated motion before (a) and after (b) the optimizing an objective function based on gluteus medialis.

Next, starting from another walking motion with shorter strides shown in Figure 5(a), we conducted an experiment by minimizing the force exerted by the gluteus maximus. This time, features known as backward bending appears in the motion. The initial posture of the motion was converted to a leaning back posture, and this posture was kept during the whole cycle. The trajectory of the gait motion after the optimization is shown in Figure 5 (b).

Finally, we conducted an experiment by minimizing the force exerted by the vastus. Again, the motion shown in Figure 5 (a) was used as the initial motion. Features known as forward bending appears in the motion. As same as when weakening the gluteus maximus, the initial posture of the motion was converted to a leaning back posture, and this posture was kept during the whole cycle. The trajectory of the gait motion after the optimization is shown in Figure 5 (c).



(a)



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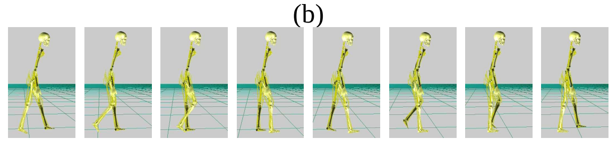


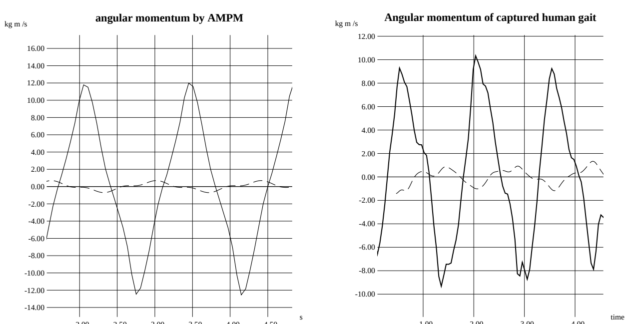
(c)

Fig. 5. The trajectory of the AMPM-generated motion before (a) and after (b) the optimizing an objective function based on gluteus maximus.

The threshold value was set to 50% of the maximum output in these examples. In both of the above examples, the objective function becomes smaller than the threshold value within one hundred iterations. The time-series data of the muscle force calculated by applying the static optimization method to normal gait motion and the pathological gait motion obtained using our method are plotted in Figure 8 and 9. The human muscle force data estimated by the electromyography (EMG) pattern of muscles during normal gait from [35] are plotted by dotted line together. This was estimated by multiplying the maximum force parameter F_i^0 estimated using scaling, by the activation data listed in [35], which were calculated by manual muscle tests.

A 27-year-old female subject with body weight of 56kg and height of 160cm with congenial dislocations of both hip joints, performed a waddling gait motion during which kinematic data were recorded using a VICON 512 apparatus (Vicon Motion Systems, Oxford, United Kingdom) for comparison. A normal gait motion by a healthy 50-year-old male subject with body weight of 51 kg and height of 164cm was also recorded. This data collection procedure has been reviewed and approved by the institutional review board (IRB) of RIKEN (Wako-Shi, Saitama, Japan). The adduction/abduction torque exertable at the hip by a patient who has congenital dislocation is much less than that by the the healthy subject. Therefore, it is known that congenial dislocations and low output by the gluteus medialis give similar effects to gait motion. The musculoskeletal model of the human body was scaled to the subject based on the information of the mass and the size of the segments, and the physiological average cross sectional area of the muscles were calculated by scaling the data in [28].

The trajectory of the angular momentum of the body around the sagittal axis before and after the optimization is shown in Figure 6 (a). Because of the waddling, a large amount of angular momentum is generated by the gait motion by the weakened gluteus medialis. The results are compared with the data by humans, one by a healthy subject and the other by a patient who has congenital dislocation in Figure 6 (b).



(a) (b)

Fig. 6. The comparison of the angular momentum around the sagittal axis during the gait motion: (a) the data calculated by the motion created by the AMPM, before (dashed line) and after (bold line) the optimation process. The amplitude of the angular momentum increases as the force exerted by the gluteus medialis decreases. These results match with the data calculated using human motion, which is shown in (b). In (b), the same data by a healthy subject (dashed line) and a patient with waddling gait (bold line) caused by congenital dislocation are shown.

IV. Discussions

A method to calculate pathological gait by combining the musculoskeletal system and the enhanced version of the 3D linear inverted pendulum model was proposed in this research. The motion is planned by the AMPM, and it is evaluated by using the musculoskeletal model. As the number of the parameters to define the motion is less than previous appraoches including dynamic optimization-based methods [7],[8], [4],[9],[10], or static optimization-based methods [30], [36], the optimization process converges more quickly than previous methods.

As we compare the muscle force calculated from the motion by the AMPM with the muscle force calculated based on the EMG data, there are some differences on the timing of the on-set and off-set of the muscle force. However, the overall patterns of the time-series data match very well. From these results we can conclude that the motion based on the AMPM is not only dynamically realistic, but also physiologically realistic.

In our experiment, by minimizing the amount of force exerted by the gluteus medialis during the gait motion, the waddling gait motion was automatically induced, and by minimizing the amount of force exerted by the gluteus maximus and vastus, motion such as backward trunk leaning and anterior trunk leaning were generated, respectively. These are natural phenomenon, as it is known that weak abductor muscles cause waddling, weak hip extender muscles cause backward trunk leaning, and weak knee extender muscles cause backward trunk leaning [35].

Waddling is a well-known abnormality of gait motion which is caused not only by weak abductor muscles, but also by congenital dislocation of hip joints and pain in the joints. Waddling reduces the torque and the bone-on-bone contact force at the hip of the support leg during the single support phase. As shown in Figure 8(e), the muscle force by the gluteus medialis is greatly reduced after the optimization process. The muscle force history of the musculoskeletal model performing normal and waddling gait calculated using static optimization are shown in Figure 7.

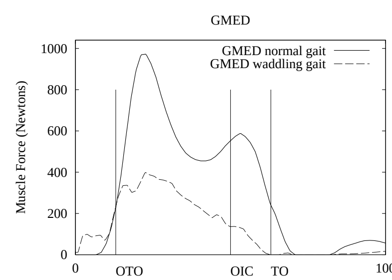


Fig. 7. The gluteus medialis (GMED) force calculated by static optimization method using human motion by a healthy subject (bold line) and by a patient with congenital dislocation (dashed line).

Anterior and backward trunk bending are also typical well known abnormality that are caused by weak vastus and hip extensors, respectively. As shown in Figure 9(d) and (e), the output of the vastus and gluteus maximus are decreased after the optimization.

Compared with dynamic optimization methods that have recently been used in biomechanics to simulate human gait and estimate muscle force, the method proposed in this study has the following advantages.

1) Our method to calculate human gait motion requires only a small number of parameters that define the motion of the AMPM. For example, if the musculoskeletal system is composed of 54 muscles, and during a half gait cycle each muscle is allowed to change its input value

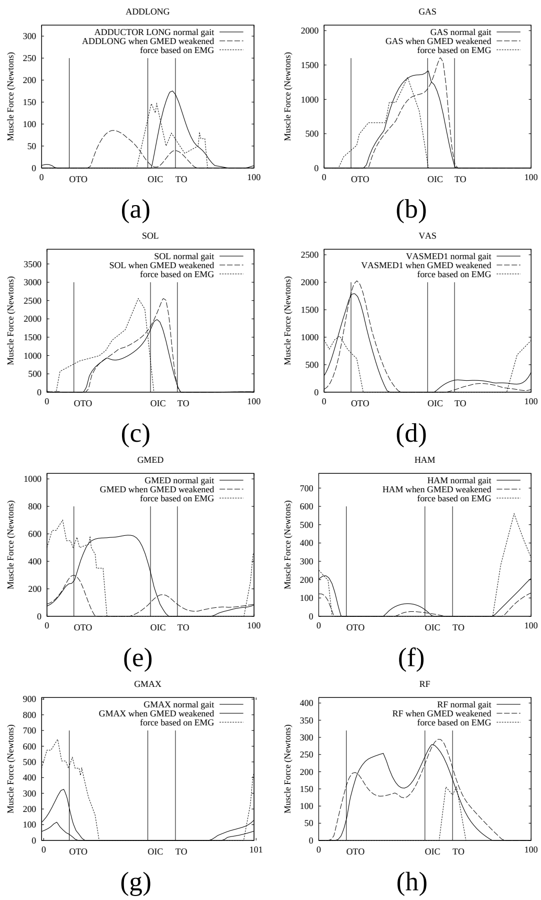


Fig. 8. Muscle force characteristics of a normal and waddling gait generated by AMPM. The muscle force are calculated by the static optimization method. The plotted data include the force during normal gait (bold line), pathological gait by weakening the gluteus medialis (dashed line) and the corresponding EMG data of a normal gait (dotted line) from [35]. The figures show the characteristics of (a) adductor longus (ADDLONG) (b) gastrocnemius (GAS), (c) soleus (SOL), (d) vastus (VAS), (e) gluteus medialis (GMED), (f) hamstrings (HAM), (g) gluteus maximus (GMAX), and (h) rectus femoris (RF).

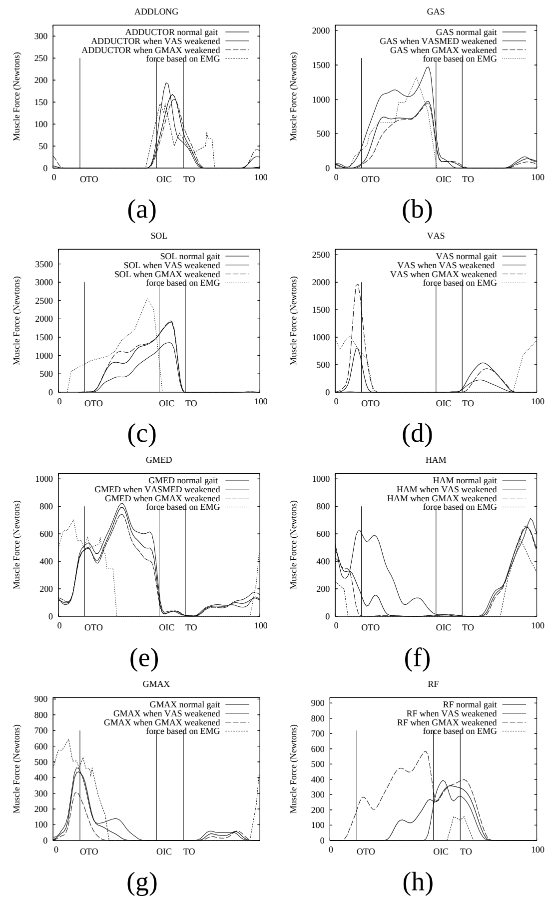


Fig. 9. Muscle force characteristics of normal, leaning forward, and leaning backward gait motion by the AMPM. The muscle forces were calculated by the static optimization method. The plotted data include the force during normal gait (bold line), pathological gait by weakening the gluteus maximus (dashed line) and vastus (plain line), and the corresponding EMG data of a normal gait (dotted line) from [35]. The figures show the characteristics of (a) adductor longus (ADDLONG) (b) gastrocnemius (GAS), (c) soleus (SOL), (d) vastus (VAS), (e) gluteus medialis (GMED), (f) hamstrings (HAM), (g) gluteus maximus (GMAX), and (h) rectus femoris (RF).

15 times [8], the total number of parameters that define the whole motion becomes 810. This means at least 810 iterations are needed for calculating the derivative of the criteria of optmization. Although techiniques such as using the same input signals for muscles with similar roles [12], or controlling muscles by "Bang-Bang" methods to reduce the search space [7], are used, the number of parameters still remains very large. On the other hand, when using the proposed method, the number of parameters is less than ten. Since the number of the parameters is small, the optimization converges much more quickly than dynamic optimization methods. Comparing the amount of computation needed, our method requires less than one tenth of the forward dynamics approach.

2) Since the balance of the motion is assured by the algorithm of the motion generation, the optimizer does not have to spend further effort in keeping the balance through the gait cycle. This is one of the most important contribution of the paper, because when using optimization methods based on forward dynamics to simulate bipedal gait, most of the iterations are done for unbalanced motions tumbling down onto the ground. If the system can avoid evaluating such motions, the search for the optimal motion can be done much more efficiently.
3) It is not necessary to search manually the initial guess of the muscle-activation parameters that decide the motion, as the parameters that define the gait based on the AMPM are intuitive kinematic parameters such as the position and velocity of the COM. As explained in the Introduction, when using optimization methods based on forward dynamics, a good initial guess of the activation data must be provided in order to prevent the optimization process from getting caught into some local minima. This is quite time consuming, and the researcher needs special experience to determine the good set of muscle activations to generate realistic gait motion. In addition, as such process is done completely manually, the motion can become subjective to the researcher.

On the other hand, our method has the following disadvantages.

1) Some pathological gait cannot be generated by the same combination of the differential equations as for normal gait. For example, sudden fall of the pelvis during the single support phase can result from weak knee extender muscles, and it cannot be expressed by the same combination of differential equations as normal gait.
2) In this study, only the AMPM parameters were used as variables of the optimization. The general coordinates of the initial posture of the body, the keyframe postures of the foot, and the trajectory of the swing leg were not used as parameters during the optimization stage. Therefore, pathological effects could not be produced for the swing leg.
3) The motion is assumed to be symmetric, although pathological gaits due to injuries or paralysis cause the gait to be asymmetric, and such effect cannot be simulated.
4) As static optimization is used to calculate the muscle force, our method assumes the muscles exert the minimal amount of force to generate the motion. This is not true for some cases, for example, when the antagonist muscles also exert force to increase the stiffness of the joints, and to take such effect into account, it is necessary to add extra parameters that express the stiffness of the joints.

One way to simulate a motion with high frequency movements is to adaptively increase the number of AMPMs to represent the trajectory of the COM/ZMP/angular momentum during the optimization process. In that case, it is necessary to define the condition of splitting a single AMPM into multiple pieces. This can be another issue for research in the future. Although high frequency motion cannot be represented by the same number of sets of AMPMs as normal gait, it is still possible to model various kinds of pathological gait with the same number of AMPMs as far as the motion is a dynamically stable biped gait. In daily life, when humans get adjusted to the pain of injuries or the disfunction of their body, they find out a new dynamically stable and feasible motion resulting in another pattern of bipedal gait. Such kind of motions mostly do not include any high frequency data and therefore there is a high chance that the gait motion can be simulated by the same number of AMPMs.

In order to simulate the motion of the swing leg, it is necessary to increase the number of parameters that define the motion. The trajectory of the leg must be represented not only by the information of the four keyframes, but also by those of additional intermediate keyframes. As the number of parameters increase, the optimization process can suffer from falling into local minima. In order to avoid such case, global optimization methods such as simulated annealing [37] must be used. Although the current parameter set is not enough to simulate the pathlogical motion of the swing leg, it is enough to simulate the effect of the support leg. Because the support leg needs to exert much more force than the swing leg, in many cases the the pathological motion is prominent during the support phase, just as the example shown in this paper.

One possibility to simulate the motion of the swing leg without increasing a number of parameters is to use forward simulations to produce its trajectory, or conduct another optimization to generate an efficient swing motion. This idea comes from the natural and realistic swing leg motion that can be observed in passive gait [38]. The motion of the swing leg highly depends on the laws of dynamics. It is necessary to reduce the output of the torques of the leg while reaching to the goal position after the duration decided by the AMPM parameters. This is one key problem to be solved.

Regarding the discussion that the motion must be symmetric, the application of the proposed method will greatly increase if it can also handle asymmetric gait. The approach used in this study can be enhanced to simulate asymmetric gait by connecting two sets of AMPM parameters each representing a half-cycle gait motion. To keep the motion cyclic, the position and velocity of the COM and the angular momentum at the initial posture of the first half-cycle must be same as those values in the last frame of the second half-cycle. However, this will double the number of parameters, moreover the computational cost and the chance to fall into local minima will both increase. Handling asymmetric gait is the future work of this study, that is currently in progress. Although our method only supports symmetric gait, still there are various pathological gait that can be simulated by our method, such as paralysis causing symmetric effects to the body or degeneration of gait due to aging. The impairment does not necessarily have to be symmetric as far as we analyze only half a cycle. In such case, an objective function based on half cycle will be prepared, and the user will set the impaired leg to be either the support leg or the swing leg. This will help the user roughly estimating how the impairment affects the motion. Therefore, even at this stage, our method is useful from medical and biomechanical point of view.

V. Summary and Future Work

In this study, we have proposed a new method to simulate the gait motion when muscles are weakened. The method is based on the AMPM which is an enhanced model of the 3DLIPM that is used in robotics to generate gait motion. Compared with previous methods, the proposed method is simple, and the computational cost is much smaller.

The ultimate objective of this research is to generate a system that medical doctors and therapists can use to diagnose and find out what are the impairments of the body causing the abnormalities of the gait; once the impairment is known, the system should be able to provide plans of rehabilitation or other ideas of compensations.

In order to achieve this objective the following issues have to be addresed:

1) a technique to concisely model the musculoskeletal system of individual subject must be developed, and
2) a method to estimate the AMPM parameters from the gait motion data ; by proposing such techniques, a motion similar to individual gait can be represented by the AMPM model.

3) a method to convert the pathological gait to a normal gait by gradually changing the AMPM parameters according to the effect of rehabilitation and other compensations.

Regarding issue 1, this is a very difficult problem that involves not only the measurement of physical and kinematical parameters such as the position of the joints, mass distribution, and moment of inertia of the body segments but also the measurement of physiological parameters including attachment sites, direction of muscle fibers and average cross sectional area. Although researches such as [39] appear to deal with such problems, further tools and interfaces are required for practical use.

Regarding issue 2, the research is now in progress. The problem is that all the externally measured values such as the trajectory of the ZMP and ground force vectors must be matched with to those trajectories calculated by the trajectories of COM and angular moementum of the body. Usually those values do not perfectly match due to noise, filtering, and wrong estimation of the inertial parameters of the body. Therefore, a good error correction method to minimize the gap between the externally measured values and the trajectories calculated through kinematical data must be introduced.

Regarding issue 3, if a model of rehabilitation can be generated, it is possible to search for the "optimal path" to convert the current abnormal gait to a normal gait. This can be considered as a path-planning problem. As the number of parameters of AMPM is little, it will be possible to search a solution using methods such as A* algorithms.

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APPENDIX I
APPLICATION OF THE ENHANCED 3DLIPM FOR CALCULATION OF MOTION IN THE SAGITTAL PLANE

Since AMPM only allows linear relationship between the COM and the ground force vector, the gait motion must be divided into several phases to be represented by the AMPM. By substituting the position and acceleration data at the boundary postures into (1), the coefficients of the AMPM can be calculated for each stage. For example, as the motion of the COM for the ith stage is defined by

$$\ddot{x} = A_i x + B_i, \tag{9}$$

where A_i and B_i are the AMPM coefficients that define the motion of the COM at this stage, they can be calculated by substituting the position and acceleration data of the COM at the beginning and end of this stage:

$$A_i = \frac{\ddot{x}_{i+1} - \ddot{x}_i}{x_{i+1} - x_i} \tag{10}$$

$$B_i = \frac{\ddot{x}_i x_{i+1} + \ddot{x}_{i+1} x_i}{x_{i+1} - x_i} \tag{11}$$

where x_i and $\ddot{x}_i$ are the position and acceleration of the COM at the beginning of stage i. The position of the ZMP has a linear relationship with the position of the COM:

$$zmp_x = c_i x + d_i \tag{12}$$

where c_i and d_i are constant coefficients that determine the motion of the ZMP in each stage, and zmp_x is the position of the ZMP along the x-axis. In the sagittal plane, the position of the ZMP at OTO, HR, OIC, and TO are set under the heel, metatarsals, and tiptoe, respectively. The position of the ZMP at MS is determined in a manner that the angular momentum around the COM at the beginning and end of half cycle are identical. The coefficients c_i, d_i can be calculated using these boundary conditions:

$$c_i = \frac{zmp_{x,i+1} - zmp_{x,i}}{x_{i+1} - x_i}$$

$$d_i = \frac{x_{i+1} zmp_{x,i} - x_i zmp_{x,i+1}}{x_{i+1} - x_i}$$

where $zmp_{x,i}$ is the position of the ZMP at the beginning of stage i. Among the variables described above, the following parameters are used for the optimization:

- the position of COM at OTO, HR, OIC, and
- the velocity at OTO.

The position of the COM at MS is determined so as to compensate for the angular momentum generated during the single support phase. Therefore, the number of parameters for the AMPM in the sagittal plane is four.

APPENDIX II
APPLICATION OF THE AMPM FOR CALCULATION OF MOTION IN THE FRONTAL PLANE

The coordinate systems used here are shown in Figure 10. The distance between the feet when they are both on the ground is $2s + 2\beta$. The COM moves 2β during the double support phase. After switching to the single support phase, it moves along until it stops and returns back the same path. The relationships between the ZMP, COM and ground force during the single support phase are

$$\ddot{y} : g = c_y y : H, \tag{13}$$

where y is the position of the COM along the horizontal axis, c_y is a constant value that is correlated with ground force direction and position of the COM (Figure 10(a)). Using the terminal condition $y = s$ and $\dot{y} = v_{ex}$ at time $t = 0$, the motion of the COM can be finally written as:

$$y = s \cosh \frac{t}{T_{ls}} - v_{ex} \sinh \frac{t}{T_{ls}}, \tag{14}$$

where v_{ex} is the velocity when the single support starts, and $T_{ls} = \sqrt{H/(c_y g)}$. Since the duration of the single support phase T is determined by the motion in the sagittal plane, v_{ex} can be calculated by setting $t = T, y = s$ in (14). As a result, v_{ex} can be calculated as

$$v_{ex} = \frac{-s + s \cosh \frac{T}{T_{ls}}}{\sinh \frac{T}{T_{ls}}}.$$

The y and z-component of the ground force can be written as

$$F_y = \frac{m}{T_{ls}^2}\left(y_0 \cosh \frac{t}{T_{ls}} - v_{ex} \sinh \frac{t}{T_{ls}}\right),$$
$$F_z = mg.$$

Since the trajectories of the ground force, COM, and ZMP are known, rotational moment around the sagittal axis can be calculated as

$$r_x = yF_z - HF_y.$$

The double support phase can be modeled as follows. Since

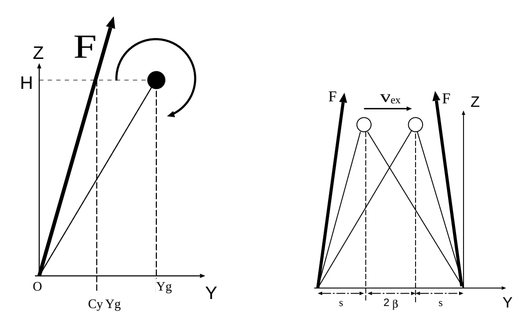

Fig. 10. Coordinate systems used in the frontal plane for (a) the single support phase and (b) the double support phase. The origin is set to the position of the ZMP in the single support phase and to the center of the feet in the double support phase.

the motion in the frontal plane is symmetric with respect to time, it can be considered that the ZMP and COM satisfy the following relationship:

$$zmp_y = \frac{\beta}{\beta + s} y.$$

where zmp_y is the position of the ZMP along the y-axis. Since rotational momentum is generated in the frontal plane and since rotational momentum decreases as the COM approaches the origin of the coordinate system, motion of the COM can be approximated by the following function:

$$\ddot{y} : g = c_3(y - zmp_y) : H, \tag{15}$$

where c_3 is a constant. Using the boundary conditions of the single support phase, c_3 can be calculated as

$$c_3 = \frac{\beta + (1 - c_y)s}{\beta} < 0.$$

The trajectory of y then becomes

$$y = C_1 \cos \frac{t - t_2}{T_c} + C_2 \sin \frac{t - t_2}{T_c} - \frac{b}{T_c^2},$$

where C_1 and C_2 are arbitrary constant values, $T_c = \sqrt{\frac{H(\beta + s)}{g c_3 s}}$, and t_2 is the time the double support started. Using the terminal conditions, the final form becomes

$$y = (\beta + s) \cos \frac{t - t_2}{T_c} + v_e x \sin \frac{t - t_2}{T_c}.$$

The y- and z- components of the ground force can be written as

$$\begin{aligned} F_y &= -\frac{m}{T_c^2} \left((\beta + s) \cos \frac{t - t_2}{T_c} + v_{ex} \sin \frac{t - t_2}{T_c} \right), \\ F_z &= mg. \end{aligned}$$

Rotational moment around the sagittal axis can be calculated by (15). Same as in the sagittal plane, because of the symmetry of the motion with respect to time, we do not have to tune any parameters to compensate for the angular momentum. The only variables used for optimization in the frontal plane are c_y and s.

APPENDIX III
CALCULATING THE JOINT ANGLES USING INVERSE KINEMATICS

As we have already defined trajectories of the COM and angular momentum, the next step is to calculate kinematic parameters that satisfy these trajectories. Inverse kinematics is used for this purpose. At first, positions and rotational trajectories of the feet, which are defined here as (p_l, θ_l) and (p_r, θ_r), are calculated using footstep data specified in advance. As shown in Figure 11, four keyframes of the support foot are specified. The data include postures of the foot at initial contact, initial full contact, heel rise, and toe-off. The x- component of the velocity of the motion of the foot of the swing leg when it is lifted from the ground is calculated by

$$v^0_{swing} = \frac{l_s}{T_{swing}}.$$

where l_s is the step length. The final velocity of the motion of the foot when it comes into contact with the ground is set to zero. The trajectory of the swung foot is calculated by interpolating the keyframes with a cubic spline curve. Trajectories of generalized coordinates of the human body

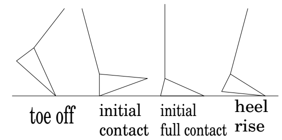

Fig. 11. Keyframes of the foot rotation

model are defined here as $\boldsymbol{q(t)} = (q_1(t), q_2(t), ..., q_{dof}(t))$, where dof is the number of degrees of freedom of the human body model, which is actually twenty-nine as explained in Section II-C.

The relationship between velocity of the COM and velocity of the generalized coordinates can be written as:

$$\dot{\boldsymbol{x_g}} = \boldsymbol{J_{cog}} \dot{\boldsymbol{q}},$$

where $\boldsymbol{J_{cog}}$ is the Jacobian matrix that consists of the partial differentials of the COM by the generalized coordinates:

$$\boldsymbol{J_{cog}} = \frac{\partial \boldsymbol{x_g}}{\partial \boldsymbol{q}}.$$

Acceleration of the COM can then be obtained as follows:

$$\ddot{\boldsymbol{x_g}} = \boldsymbol{J_{cog}} \ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_{\boldsymbol{cog}} \dot{\boldsymbol{q}}. \tag{16}$$

Angular momentum ω and the first derivative of the generalized coordinates have a linear correlation:

$$\boldsymbol{\omega} = \boldsymbol{R} \dot{\boldsymbol{q}}$$

where $\boldsymbol{R}$ is a $3 \times dof$ matrix that correlates the angular momentum and the velocity of the generalized coordinates. The derivative of the angular momentum can then be calculated as follows:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{R} \ddot{\boldsymbol{q}} + \dot{\boldsymbol{R}} \dot{\boldsymbol{q}}. \tag{17}$$

Acceleration of the motions of the feet can be expressed as functions of $\ddot{\boldsymbol{q}}$

$$\begin{pmatrix} \ddot{p_l} \\ \ddot{p_r} \\ \ddot{\theta_l} \\ \ddot{\theta_r} \end{pmatrix} = \boldsymbol{J_f} \ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_{\boldsymbol{f}} \dot{\boldsymbol{q}}, \tag{18}$$

where $\boldsymbol{J_f}$ is the Jacobian matrix of the feet data.

The trajectories of the feet are calculated by scaling the trajectories of the original feet using the subsequent positions of the foot steps;

$$(p_x, p_y) = \\ (\frac{s_x^{i+1} - s_x^i}{s_{x,0}^{i+1} - s_{x,0}^i}(p_x^0 - s_x^i) + s_x^i, \frac{s_y^{i+1} - s_y^i}{s_{y,0}^{i+1} - s_{y,0}^i}(p_y^0 - s_y^i) + s_y^i)$$

where (s_x^i, s_y^i) and (s_x^{i+1}, s_y^{i+1}) are the position of the ith and $(i+1)$th footsteps on the floor in the newly generated motion, $(s_{x,0}^i, s_{y,0}^i)$ and $(s_{x,0}^{i+1}, s_{y,0}^{i+1})$ are the corresponding positions of the footsteps in the original motion, and (p_x, p_y) is the position of the foot in the horizontal plane in the newly generated motion, and (p_x^0, p_y^0) is the corresponding position in the original motion. The rotation of the feet in the new motion will be calculated by using the step length as a scaling factor: $\theta_y = (s_l/s_l^0)\theta_y^0$. This is due to the fact that the orientation of the feet enlarges as the step length gets larger.

By combining (16), (17) and (18), linear constraints that must be satisfied can be written in the following form:

$$\dot{\boldsymbol{\lambda}} = \boldsymbol{J_{all}}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_{\boldsymbol{all}}\dot{\boldsymbol{q}}, \tag{19}$$

where $\boldsymbol{\lambda} = (\dot{\boldsymbol{x}}_{\boldsymbol{g}}, \boldsymbol{\omega}, \dot{\boldsymbol{p}}_{\boldsymbol{l}}, \dot{\boldsymbol{\theta}}_{\boldsymbol{l}}, \dot{\boldsymbol{p}}_{\boldsymbol{r}}, \dot{\boldsymbol{\theta}}_{\boldsymbol{r}})^{\boldsymbol{T}}$, and $\boldsymbol{J_{all}} = (\boldsymbol{J_{cog}}, \boldsymbol{R}, \boldsymbol{J_f})^{\boldsymbol{T}}$. Calculating $\ddot{\boldsymbol{q}}$ that satisfies (19) can be considered as an inverse kinematics problem.

The $\ddot{q}$ that minimizes the quadratic form

$$\begin{aligned}&(\ddot{\boldsymbol{q}} - \boldsymbol{k}(\boldsymbol{q} - \boldsymbol{q_0}) + \boldsymbol{d}(\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\boldsymbol{0}}))\\ &\qquad\qquad(\ddot{\boldsymbol{q}} - \boldsymbol{k}(\boldsymbol{q} - \boldsymbol{q_0}) + \boldsymbol{d}(\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\boldsymbol{0}}))^{\boldsymbol{T}},\end{aligned} \tag{20}$$

was used to update the generalized coordinates $\boldsymbol{q}$, where $\boldsymbol{q_0}$ and $\dot{\boldsymbol{q}}_{\boldsymbol{0}}$ are the target generalized coordinates and its derivative at each moment based on the reference captured gait motion, $\boldsymbol{k}$ is the vector of elastic constants, and $\boldsymbol{d}$ is the vector of viscosity constants. $\ddot{\boldsymbol{q}}$ that minimizes (20) and satisfies (19) was calculated by quadratic programming.

The initial posture at OTO that satisfies the COM and feet position constraints is first calculated by adjusting the position of the pelvis. The velocity at the initial posture is calculated by solving for the initial generalized velocity $\dot{q}_0$ that satisfies the constraints of the COM, angular momentum, and feet:

$$\boldsymbol{\lambda_0} = \boldsymbol{J^0_{all}}\dot{\boldsymbol{q}}_{\boldsymbol{0}} \tag{21}$$

where λ_0 is the initial velocity vector and $\boldsymbol{J^0_{all}}$ is the initial Jacobian matrix for the whole system. Using the calculated acceleration, the values of the generalized coordinates and their velocities were updated step by step, and, finally, the entire trajectories were obtained.