Bisimulation for stochastic hybrid models

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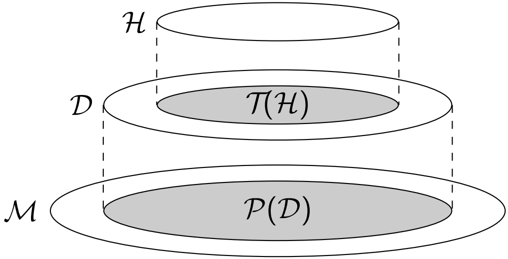
Three formalisms

stochastic HYPE models

transition-driven stochastic hybrid automata (TDSHA)

piecewise deterministic Markov processes (PDMPs)



Stochastic HYPE [Bortolussi, Galpin and Hillston, 2011]

- process algebra for modelling stochastic hybrid systems
- based on HYPE which models hybrid systems using fine-grained compositionality based on influences
- ODEs are constructed from influences affecting variables
- notion of event, each has name, guard and reset
- separate definitions for controller and uncontrolled system
- two semantics
  - compositional translation to TDHSA
  - labelled transition system
- bisimulation defined at abstract level over transition system

Transition-Driven Stochastic Hybrid Automata (TDSHA) [Bortolussi and Policriti, 2009]

- based on notion of hybrid automata
- set of modes, set of continuous variables
- addition of transitions with stochastic duration
- notion of named event with guard and reset
- flow at mode defined by continuous transitions
- continuous transitions define which variables they affect
- defined to be more amenable and compositional than PDMPs
- can be mapped to PDMPs

Piecewise Deterministic Markov Processes (PDMPs) [Davies, 1993]

- set of modes, each has locally Lipschitz continuous vector field
- hybrid state space defined over modes and open sets in $\mathbb{R}^n$
- continuous evolution in mode determined by vector field
- instantaneous jumps when boundary of current open set hit
- stochastic jumps determined by functional jump rate
- reset kernel that determines probabilistically mode and values of variables after jump
- no notion of named event
- bisimulation relation defined over hybrid state space

Bisimulation for stochastic HYPE

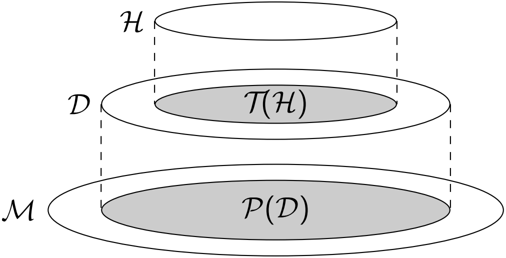
- each influence affects a continuous variable
- state: $\sigma : IN \to (\mathbb{R} \times IT)$, captures current influence details
- configuration: $\langle ConSys, \sigma \rangle \in \mathcal{F}$
- labelled transition system: $(\mathcal{F}, \mathcal{E}, \to \subseteq \mathcal{F} \times \mathcal{E} \times \mathcal{F})$
- assume $\equiv$, an equivalence relation over states
- B is a *system bisimulation with respect to* $\equiv$ if for all $(P, Q) \in B$, for all $a \in \mathcal{E}$, for all states σ, τ whenever
  1. $\langle P, \sigma \rangle \xrightarrow{a} \langle P', \sigma' \rangle$, $\langle Q, \tau \rangle \xrightarrow{a} \langle Q', \tau' \rangle$, $\sigma' \equiv \tau'$, $(P', Q') \in B$.
  2. $\langle Q, \tau \rangle \xrightarrow{a} \langle Q', \tau' \rangle$, $\langle P, \sigma \rangle \xrightarrow{a} \langle P', \sigma' \rangle$, $\sigma' \equiv \tau'$, $(P', Q') \in B$.
  3. $\sigma \equiv \tau$,
- notation: $P \sim^{\equiv} Q$

Bisimulation for PDMPs

- B, relation over modes and vectors of reals
- equality of some elements of vectors
- equality of guards and time points when guards become true
- equality of stochastic rates
- continuous flows must be related by B
- different length sequences of instantaneous transitions can match
- can apply to TDSHA
  - weak style that ignores event names
  - strong style that considers event names
- notation: $\sim$ for strong; $\approx$ for weak

What do we know so far?

- obvious relations: $D_1 \sim D_2 \Rightarrow D_1 \approx D_2$;
- $\doteq$, equivalence over states ensuring identical ODEs
- need to consider multiple transitions with the same event



$$H_1 \sim^{\doteq} H_2$$

$$\Downarrow ?$$

$$\mathcal{T}(H_1) \sim \mathcal{T}(H_2) \qquad D_1 \approx D_2$$

$$\Downarrow$$

$$\mathcal{P}(D_1) \approx \mathcal{P}(D_2)$$

Ongoing research

- consider all variants of strong, weak, labelled, unlabelled
- consider whether translations between formalisms surjective
- characterise $\mathcal{T}(\mathcal{H})$ and $\mathcal{P}(\mathcal{D})$ if not
- apply bisimulations to large examples
  - delay tolerant networks
  - systems biology
- through examples, understand expressive power