Bisimulation for stochastic hybrid models

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This report on work-in-progress considers semantic equivalences for formalisms describing stochastic hybrid systems, i.e. systems that show stochastic, instantaneous and continuous behaviour. The three formalisms investigated are stochastic HYPE (sHYPE) models [1] based on HYPE [6], [5], Transition-Driven Stochastic Hybrid Automata (TDSHA) [3], and Piecewise Deterministic Markov processes (PDMPs) [4]. The choice of these formalisms is not accidental. The semantics of sHYPE models are given in terms of TDHSA (as well as by means of a structured operational semantics) [1] and TDSHA were developed as a formalism to reason about stochastic hybrid systems without using the full complexity of PDMPs [2]. The left side of Figure 1 shows this relationship, where $\mathcal{T}(.)$ is the mapping from HYPE models to TDSHAs, and $\mathcal{P}(.)$ is the mapping from TDSHAs to PDMPs.

Stochastic HYPE is a process algebra and a bisimulation can be defined on the labelled transition system obtained from the operational semantics. The syntax for (well-defined) individual subcomponents is $S(\mathcal{W}) \stackrel{\mathit{def}}{=} \sum_{j=1}^{n} \mathrm{a}_j{:}(\iota, r_j, I_j(\mathcal{W})).S(\mathcal{W}) + \underline{\mathrm{init}}{:}(\iota, r, I(\mathcal{W})).S(\mathcal{W})$ where $\mathcal{W}$ is a subset of the continuous system variables. The prefix operator is $\mathrm{a}.(\iota, r, I(\mathcal{W})).S$ where either $\mathrm{a} = \underline{\mathrm{a}}$ is an instantaneous event or $\mathrm{a} = \overline{\mathrm{a}}$ is a stochastic event. Instantaneous events have event conditions of the form (G, R) where G is a guard or activation condition over variables, and R is a conjunction of resets that affect variables. Stochastic events have the form (f, R) where f defines the rate of the transition (giving an exponential distribution) and R is the same as above. The second part of the prefix, called an influence, consists of an influence name ι which is associated with exactly one variable of the system; an influence strength $r \in \mathbb{R}$ and an influence type $I(\mathcal{W})$ with an associated function over the variables $\mathcal{W}$. Subcomponents are composed in parallel by means of a multi-way synchronization operator. They are also synchronized with a controller, imposing causality on events.

The labelled transition system consists of configurations which are pairs of models and states, where elements of a state have the form $\iota \mapsto (r, I(\mathcal{W}))$ which get updated when an event fires. States define ODEs for each configuration. An ODE is constructed for each continuous variable of the model by summing over the product of influence strength and influence type function for each influence name that is associated with the variable, where the influence is defined in the state.

System bisimulation ($\sim_{\mathbf{sm}}$) is defined over sHYPE models, and matches over transitions [5]. It also requires that a pair of models in the relation have identical states. We have also relaxed this condition and allowed for an equivalence relation over states. We plan to use a relation $\doteq$ which ensures identical ODEs to define more interesting bisimulations (such as $\sim_{\mathbf{sm}}^{\doteq}$; note that $\sim_{\mathbf{sm}} = \sim_{\mathbf{sm}}^{=}$).

TDHSAs, by contrast, are a more direct formalism. A TDSHA has a set of modes, and each mode has associated continuous, instantaneous and stochastic transitions. Unlike sHYPE, guards and resets in instantaneous and stochastic transitions are not determined by the event of the transition. The continuous transitions are used to determine the flow at that mode. A synchronised product for TDSHA synchronising on event names has been defined [1]. The translation from HYPE models to TDSHAs is compositional. Modes are based on sets of influences, and influences can be mapped to continuous transitions. A TDSHA is defined for the uncontrolled system from products of TDSHAs for subcomponents, and the product of it and the TDHSA of the controller (with which event conditions are associated) gives the final TDSHA.

PDMPs, in the same fashion as TDSHAs, have a set of modes Q. The hybrid state space is defined by $D = \bigcup_{q \in Q} \{q\} \times D_q$ where each $D_q \subset \mathbb{R}^n$ is an open set. Each mode q has an associated locally Lipschitz continuous vector field which defines the evolution of the continuous variables while in q. A functional jump rate is defined over D and determines when stochastic

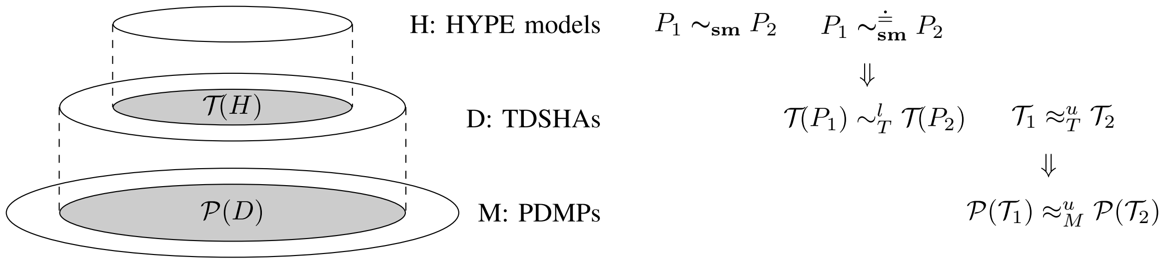

Figure 1. Relationships between formalisms and their bisimulations

jumps occur, using an exponential distribution. Instantaneous jumps occur when the boundary of the current mode is hit. Additionally a reset kernel is defined which determines probabilistically the change in mode and continuous variables, when an instantaneous or stochastic jump occurs.

Bisimulations for PDMPs [7] and TDSHAs, unlike those for HYPE models, are constructed as a relation over pairs of modes and vectors of reals. The relation B must have equality over output functions defined over the vectors of reals (this allows for some variables to be ignored, for example), equality of exit rates and equality of guards. Continuous flows must also be related by B and the time point at which guards become true must be the same. We require B to be measurable in order for the reset kernels to be equal over equivalence classes formed by B (in the case of TDHSAs) or to be equivalent probability measures with respect to the relation B (in the case of PDMPs). As long as the reset kernels have the appropriate properties, different length sequences of instantaneous transitions can be matched, hence this is a weak notion of bisimulation.

Since bisimulation for PDMPs does not take account of event names, we have defined two bisimulations for TDSHAs, $\sim_T^l$ which does consider event names and $\approx_P^u$ which does not consider these, and is weak in style, since it allows matching of sequences of transitions.

The right side of Figure 1 illustrates the expected relationships between the bisimulations mentioned above. We have proved the implication between TDSHAs and PDMPs and are now focussing on the implication from HYPE models to TDSHAs. This requires a characterisation of the TDSHAs that are obtained from HYPE models, but this is not straightforward because we need to consider multiple transitions with the same event that can be generated by certain controllers, for example $Con \stackrel{def}{=} \underline{\mathrm{a}}.Con + \underline{\mathrm{a}}.Con$. The importance of this equivalence over HYPE models is that HYPE models can be checked for equivalence without transformation to TDSHAs.

We do not limit ourselves to the equivalences in Figure 1. We can define combinations of weak and strong, and labelled and unlabelled bisimulation for each formalism, and compare these across formalisms and within formalisms. We already know certain obvious implications, for example $\mathcal{T}_1 \sim_T^l \mathcal{T}_2 \Rightarrow \mathcal{T}_1 \approx_T^u \mathcal{T}_2$.

We will also characterise the set $\mathcal{T}(H)$ (as mentioned above) and $\mathcal{P}(M)$. First, because we wish to understand what expressivity may have been lost in the two mappings, and second, because we want to know if it is possible to construct reverse implications from these sets. If it is not the case that these reverse implications hold, we can further characterise the subset for which they do hold, and hence develop further understanding of the two mappings.

Finally, we will illustrate the use of the bisimulations on sufficiently large examples, with delay tolerant networks and systems biology as possible application domains, and demonstrate the utility of working with higher level formalisms.

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