HYPE: A Process Algebra for Compositional Flows and Emergent Behaviours

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Introduction

- hybrid systems

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  - discrete behaviour

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  - well known
  - graphical rather than textual
  - not very compositional

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  - compositional language
  - semantic equivalences

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  - semantic equivalences
- running example: temperature control for an orbiter

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  - ACP^{srt}_{hs} – Bergstra and Middelburg
  - HyPA – Cuijpers and Reniers
  - hybrid χ – van Beek *et al*
  - ϕ-calculus – Rounds and Song

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- HYPE
  - more fine-grained approach, individual additive flows
  - influence of continuous semantics of PEPA

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- events — instantaneous, discrete changes

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ι: influence name; r: rate; $I(\vec{X})$: influence type with $[\![I(\vec{X})]\!] = f(\vec{X})$

where $\vec{X}$ is a formal parameter.

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Orbiter Temperature Control in HYPE

- orbiter, heater and shade to control temperature

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- environment

$$\mathrm{Time} \stackrel{def}{=} \underline{\mathrm{light}}:(t, 1, \mathit{const}).\mathrm{Time} + \underline{\mathrm{dark}}:(t, 1, \mathit{const}).\mathrm{Time} + \underline{\mathrm{init}}:(t, 1, \mathit{const}).\mathrm{Time}$$

$$\mathrm{Sun} \stackrel{def}{=} \underline{\mathrm{light}}:(s, r_s, \mathit{const}).\mathrm{Sun} + \underline{\mathrm{dark}}:(s, 0, \mathit{const}).\mathrm{Sun} + \underline{\mathrm{init}}:(s, 0, \mathit{const}).\mathrm{Sun}$$

$$\mathrm{Cool}(X) \stackrel{def}{=} \underline{\mathrm{init}}:(c, -1, \mathit{linear}(X)).\mathrm{Cool}(X)$$

Orbiter Temperature Control in HYPE (continued)

- orbiter

$$\text{Heat} \stackrel{\text{def}}{=} \underline{\text{on}}:(h, r_h, const).\text{Heat} + \underline{\text{off}}:(h, 0, const).\text{Heat} + \underline{\text{init}}:(h, 0, const).\text{Heat}$$

$$\text{Shade} \stackrel{\text{def}}{=} \underline{\text{up}}:(d, -r_d, const).\text{Shade} + \underline{\text{down}}:(h, 0, const).\text{Shade} + \underline{\text{init}}:(d, 0, const).\text{Shade}$$

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- uncontrolled system

$$\text{Sys} \stackrel{\mathit{def}}{=} (\text{Heat} \underset{\{\underline{\text{init}}\}}{\bowtie} \text{Shade}) \underset{\{\underline{\text{init}}\}}{\bowtie} (\text{Cool}(K) \underset{\{\underline{\text{init}}\}}{\bowtie} \text{Sun} \underset{\{\underline{\text{init}}, \underline{\text{light}}, \underline{\text{dark}}\}}{\bowtie} \text{Time})$$

Orbiter Temperature Control in HYPE (continued)

- controllers

$$
\begin{aligned}
Con_h &\stackrel{def}{=} \underline{on}.\underline{off}.Con_h \\
Con_d &\stackrel{def}{=} \underline{up}.\underline{down}.Con_d \\
Con_s &\stackrel{def}{=} \underline{light}.\underline{dark}.Con_s
\end{aligned}
$$

$$Con \stackrel{def}{=} Con_h \bowtie_{\emptyset} Con_d \bowtie_{\emptyset} Con_s$$

- controlled system

$$
\begin{aligned}
OTC &\stackrel{def}{=} Sys \bowtie_{M} \underline{init}.Con \\
&\text{with } M = \{\underline{init}, \underline{on}, \underline{off}, \underline{up}, \underline{down}, \underline{light}, \underline{dark}\}
\end{aligned}
$$

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  - *IN* influence names, *IT* influence types

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  - IN influence names, IT influence types
  - $ec : \mathcal{E} \rightarrow EC$, association of events with event conditions
  - EC, event conditions, (activation condition, reset)

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$$\mathcal{C}_s(\vec{X}) \stackrel{def}{=} \underline{\mathrm{a}}_1 : \alpha_1.\mathcal{C}_s(\vec{X}) + \ldots + \underline{\mathrm{a}}_n : \alpha_n.\mathcal{C}_s(\vec{X}) \quad \underline{\mathrm{a}}_i \neq \underline{\mathrm{a}}_j$$

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  - $\underline{\mathrm{a}} : (\iota, \_, \_)$ appears at most once

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  - synchronisation on shared events

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$$[\![const]\!] = 1 \qquad [\![linear(X)]\!] = X$$

Operational semantics

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- updating function: $\sigma[\iota \mapsto (r, I)]$

$$\sigma[\iota \mapsto (r, I)](x) = \begin{cases} (r, I) & \text{if } x = \iota \\ \sigma(x) & \text{otherwise} \end{cases}$$

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- change identifying function: $\Gamma : \mathcal{S} \times \mathcal{S} \times \mathcal{S} \rightarrow \mathcal{S}$

$$(\Gamma(\sigma, \tau, \tau'))(\iota) = \begin{cases} \tau(\iota) & \text{if } \sigma(\iota) = \tau'(\iota) \\ \tau'(\iota) & \text{if } \sigma(\iota) = \tau(\iota) \\ \text{undefined} & \text{otherwise} \end{cases}$$

Operational semantics (continued)

Prefix with influence:

$$\frac{}{\langle \underline{a}:(\iota, r, I).E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma[\iota \mapsto (r, I)]\rangle}$$

Prefix without influence:

$$\frac{}{\langle \underline{a}.E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma\rangle}$$

Choice:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle} \qquad \frac{\langle F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}$$

Constant:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle A, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}(A \stackrel{def}{=} E)$$

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Parallel without synchronisation:

$$\frac{\langle E, \sigma \rangle \xrightarrow{\underline{a}} \langle E', \sigma' \rangle}{\langle E \underset{M}{\bowtie} F, \sigma \rangle \xrightarrow{\underline{a}} \langle E' \underset{M}{\bowtie} F, \sigma' \rangle} \quad \underline{a} \notin M$$

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$\underline{a} \in M, \Gamma$ defined

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$\underline{a} \in M, \Gamma$ defined

Orbiter Temperature Control – transition derivation

$$\dfrac{\dfrac{\dfrac{\langle \underline{\text{dark}}.(s,0,c).\text{Sun},\, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Sun},\, \tau_1\rangle}{\vdots}}{\langle \text{Sun},\, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Sun},\, \tau_1\rangle} \qquad \dfrac{\dfrac{\langle \underline{\text{dark}}.(t,1,c).\text{Time},\, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Time},\, \tau_2\rangle}{\vdots}}{\langle \text{Time},\, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Time},\, \tau_2\rangle}}{\langle \text{Sun} \underset{M}{\bowtie} \text{Time},\, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Sun} \underset{M}{\bowtie} \text{Time},\, \tau_3\rangle}$$

$$\begin{array}{rcrcl}
\tau & = & & = & \{s \mapsto (r_s, const), t \mapsto (1, const), \ldots\} \\
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Orbiter Temperature Control – transition derivation

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Orbiter Temperature Control – transition derivation

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$$\dfrac{\langle \text{Sun}, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Sun}, \tau_1\rangle \qquad \langle \text{Time}, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Time}, \tau_2\rangle}{\langle \text{Sun} \bowtie_M \text{Time}, \tau\rangle \xrightarrow{\text{dark}} \langle \text{Sun} \bowtie_M \text{Time}, \tau_3\rangle}$$

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Orbiter Temperature Control – labelled transition system

8 configurations represent all possibilities with 8 distinct states

$$
\begin{aligned}
\sigma_0 &= \{h \mapsto (0, const),\ d \mapsto (0, const),\ s \mapsto (0, const)\} \cup D \\
\sigma_1 &= \{h \mapsto (0, const),\ d \mapsto (0, const),\ s \mapsto (r_s, const)\} \cup D \\
\sigma_2 &= \{h \mapsto (0, const),\ d \mapsto (-r_d, const),\ s \mapsto (0, const)\} \cup D \\
\sigma_3 &= \{h \mapsto (0, const),\ d \mapsto (-r_d, const),\ s \mapsto (r_s, const)\} \cup D \\
\sigma_4 &= \{h \mapsto (r_h, const),\ d \mapsto (0, const),\ s \mapsto (0, const)\} \cup D \\
\sigma_5 &= \{h \mapsto (r_h, const),\ d \mapsto (0, const),\ s \mapsto (r_s, const)\} \cup D \\
\sigma_6 &= \{h \mapsto (r_h, const),\ d \mapsto (-r_d, const),\ s \mapsto (0, const)\} \cup D \\
\sigma_7 &= \{h \mapsto (r_h, const),\ d \mapsto (-r_d, const),\ s \mapsto (r_s, const)\} \cup D
\end{aligned}
$$

where $D = \{c \mapsto (-1, linear(K)),\ t \mapsto (1, k)\}$

Hybrid semantics

- extract ODEs from each state σ in the lts of CS

$$CS_\sigma = \left\{ \text{ODE for variable } V \;\middle|\; V \in \mathcal{V} \right\} \text{ where}$$

$$\frac{dV}{dt} = \sum \left\{ r[\![I(\vec{W})]\!] \;\middle|\; iv(\iota) = V \text{ and } \sigma(\iota) = (r, I(\vec{W})) \right\}$$

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  - for any influence name associated with V
  - determine from σ its rate and influence type
  - multiply its rate and influence function together
  - sum these over all associated influence names

Orbiter Temperature Control – ODEs

Consider the state with the sun shining and the shade up

$$\begin{aligned}\sigma_3 \;=\; & \{h \mapsto (0, const),\ d \mapsto (-r_d, const),\\ & s \mapsto (r_s, const),\ c \mapsto (-1, linear(K)),\ t \mapsto (1, k)\}\end{aligned}$$

The ODEs for this state are

$$\frac{dT}{dt} = 1 \qquad \frac{dK}{dt} = r_s - r_d - K$$

Hybrid automata

- $(V, E, \mathbf{X}, \mathcal{E}, \mathit{flow}, \mathit{init}, \mathit{inv}, \mathit{event}, \mathit{jump}, \mathit{reset}, \mathit{urgent})$

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  - invariants: $\mathit{inv}(v)$
- (control) switches: $e \in E$
  - events: $\mathit{event}(e) \in \mathcal{E}$
  - predicate on $\mathbf{X}$: $\mathit{jump}(e)$
  - predicate on $\mathbf{X} \cup \mathbf{X}'$: $\mathit{reset}(e)$
  - boolean: $\mathit{urgent}(e)$

HYPE model to hybrid automaton

- modes V: set of reachable configurations

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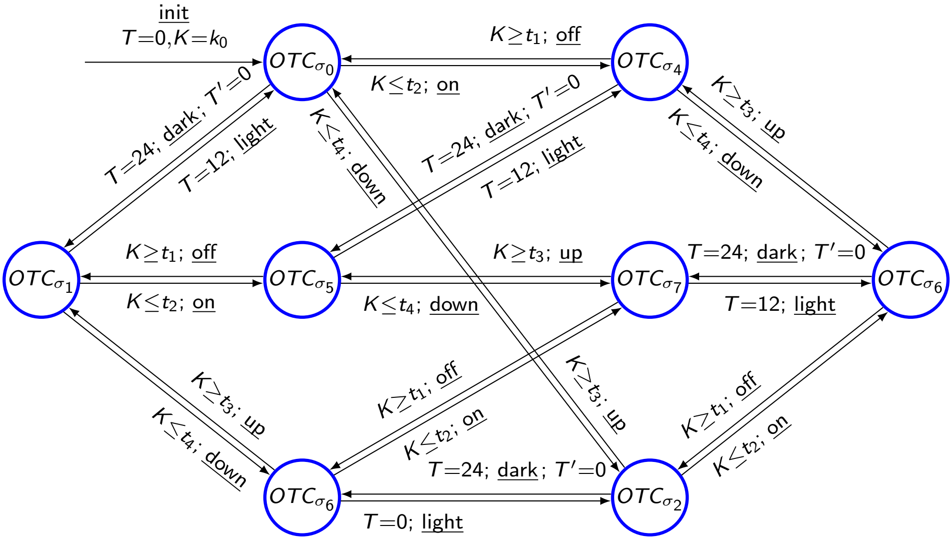
    else $jump(e) = true$ and $urgent(e) = false$
- $init(v) = \begin{cases} res_{\underline{\text{init}}} & \text{if } v = \langle P, \sigma \rangle \text{ with primes removed} \\ false & otherwise \end{cases}$

Orbiter Temperature Control – hybrid automaton



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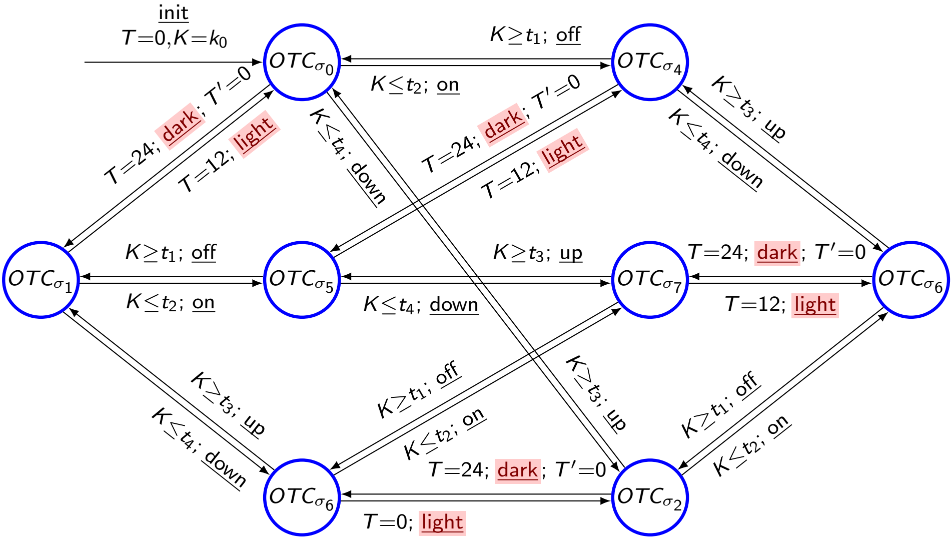

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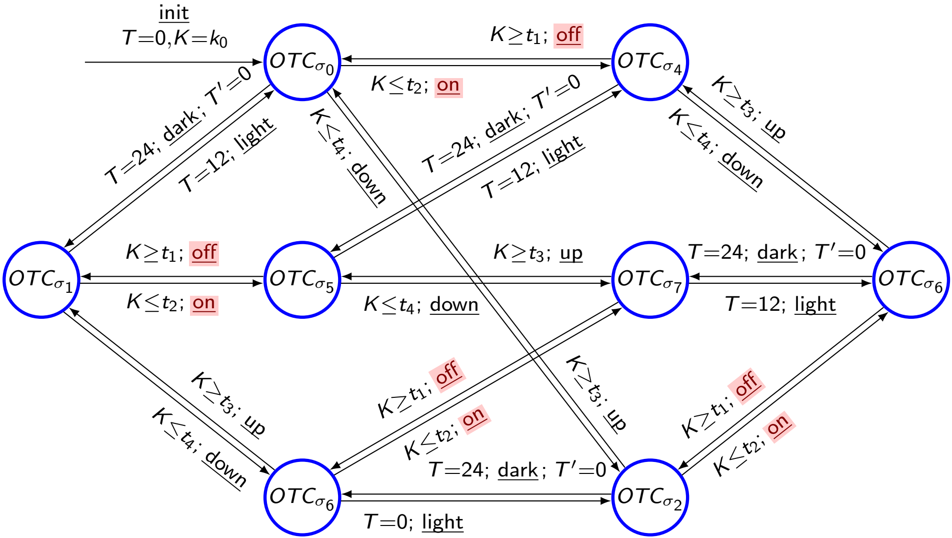

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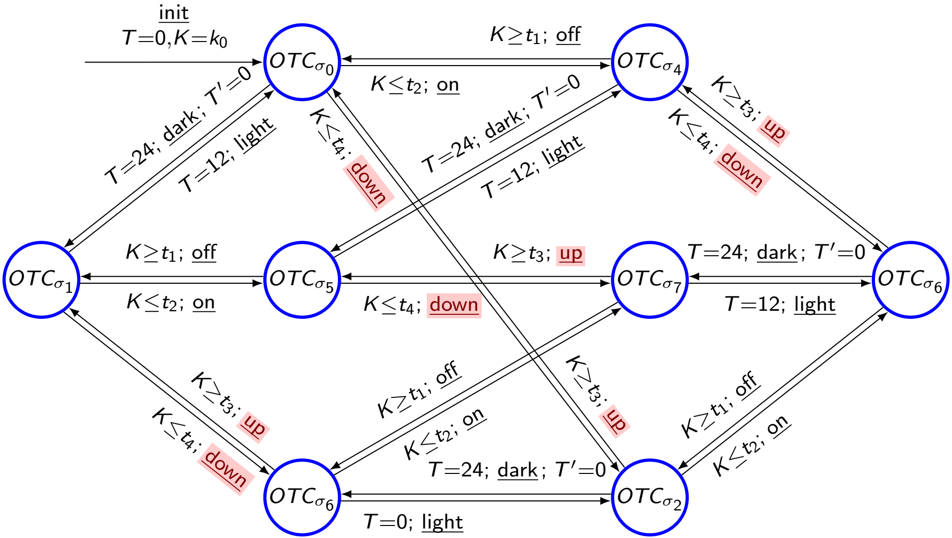
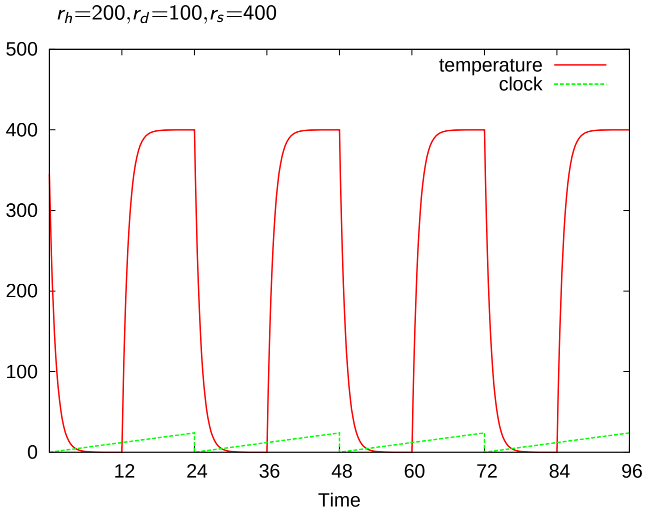


Orbiter Temperature Control – without control



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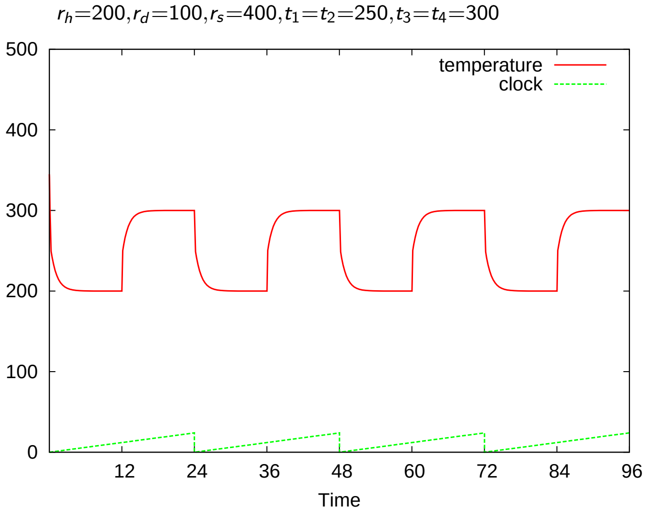

Equivalence semantics

- system bisimulation: relation B if for all $(P, Q) \in B$ whenever
  1. $\langle P, \sigma \rangle \xrightarrow{\underline{a}} \langle P', \sigma' \rangle$, there exists $\langle Q', \sigma' \rangle$ with $\langle Q, \sigma \rangle \xrightarrow{\underline{a}} \langle Q', \sigma' \rangle$ and $(P', Q') \in B$.
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- *Theorem 2*: if Σ_1 and Σ_2 have the same prefixes then $\Sigma_1 \underset{L}{\bowtie} \underline{\mathrm{init}}.Con \sim_s \Sigma_2 \underset{L}{\bowtie} \underline{\mathrm{init}}.Con$, assuming well-defined systems

Equivalence semantics

- system bisimulation: relation B if for all $(P, Q) \in B$ whenever
  1. $\langle P, \sigma\rangle \xrightarrow{\underline{a}} \langle P', \sigma'\rangle$, there exists $\langle Q', \sigma'\rangle$ with $\langle Q, \sigma\rangle \xrightarrow{\underline{a}} \langle Q', \sigma'\rangle$ and $(P', Q') \in B$.
  2. $\langle Q, \sigma\rangle \xrightarrow{\underline{a}} \langle Q', \sigma'\rangle$, there exists $\langle P', \sigma'\rangle$ with $\langle P, \sigma\rangle \xrightarrow{\underline{a}} \langle P', \sigma'\rangle$ and $(P', Q') \in B$.
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- *Theorem 3*: if $P \sim_s Q$ then $P_\sigma = Q_\sigma$ for all σ, assuming well-defined systems

Further work and conclusions

- further work
  - other equivalences
  - more modelling
  - addition of stochasticity

Further work and conclusions

- further work
  - other equivalences
  - more modelling
  - addition of stochasticity
- conclusions
  - HYPE for modelling hybrid systems
  - applied to orbiter, fan system, tanks system, bottling line, Repressilator
  - use of flows to obtain ODEs
  - separation of modelling concerns

Thank you

Orbiter in ACP_{hs}^{srt}

- different style of modelling, uses explicit ODEs

$$
\begin{aligned}
Time &\stackrel{def}{=} (T = 0) \wedge\!\!\blacktriangle\; TN \\
TN &\stackrel{def}{=} \big((dT/dt = 1) \curvearrowright\!\!\blacktriangledown\; \sigma^{*}_{\mathrm{rel}}\big(\big(T = 12\big) :\!\longrightarrow \\
&\qquad\qquad\qquad\qquad \big((T^{\bullet} = {}^{\bullet}T) \ulcorner\!\!\blacktriangledown\; light \cdot TD\big)\big) \\
TD &\stackrel{def}{=} \big((dT/dt = 1) \curvearrowright\!\!\blacktriangledown\; \sigma^{*}_{\mathrm{rel}}\big(\big(T = 24\big) :\!\longrightarrow \\
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\text{Start} &\stackrel{\text{def}}{=} (K = k_0) \curlywedge DOD \\
DOD &\stackrel{\text{def}}{=} (dK/dt = -K) \curvearrowright \sigma^{*}_{\text{rel}}\Big(\big((K^{\bullet} = {}^{\bullet}K) \ulcorner \; light \cdot LOD\big) \\
&\qquad\qquad + \big((K \le t_2) :\longrightarrow \big((K^{\bullet} = {}^{\bullet}K) \ulcorner \; on \cdot DND\big)\big)\Big) \\
LOD &\stackrel{\text{def}}{=} (dK/dt = r_s - K) \curvearrowright \sigma^{*}_{\text{rel}}\Big(\big((K^{\bullet} = {}^{\bullet}K) \ulcorner \; dark \cdot DOD\big) \\
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