A hybrid view of PEPA

Vashti Galpin*, Luca Bortolussi†, Jane Hillston*, Mirco Tribastone‡
*LFCS, School of Informatics, University of Edinburgh;
†DMI, Università degli Studi di Trieste; ‡PST, Ludwig-Maximilians-Universität München

From PEPA to Transition-Driven Stochastic Hybrid Automata (TDSHA)

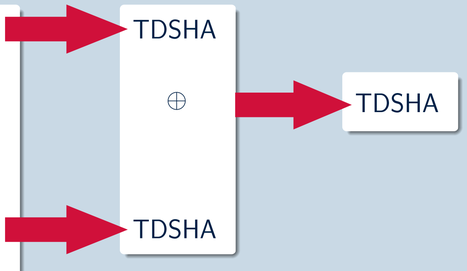
- Reduce state space by using fluid approximation while retaining some discrete aspects
- Actions classified as *discrete (and stochastic)* or continuous (and deterministic)
- Derivatives classified as *discrete* if only affected by *discrete* actions otherwise continuous
- Map each sequential component to a TDSHA with multiple modes
- Compose TDSHAs using synchronisation product combining transitions with same action
- Assume multiple independent copies of components, hence vector notation for composition

PEPA client-server model

$\mathrm{Cr} \stackrel{def}{=} (\mathrm{request}, r_{rq}).\mathrm{Ct} \qquad \mathrm{Ct} \stackrel{def}{=} (\mathrm{think}, r_{th}).\mathrm{Cr}$

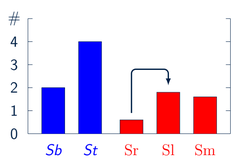
$(\#\mathrm{Cr}, \#\mathrm{Ct}) \underset{\{\mathrm{request}\}}{\bowtie} (\#\mathrm{Sr}, \#\mathrm{Sl}, \#Sb, \#St)$

$\mathrm{Sr} \stackrel{def}{=} (\mathrm{request}, r_{rp}).\mathrm{Sl} + (break, r_{bk}).Sb$
$\mathrm{Sl} \stackrel{def}{=} (\log, r_{lg}).\mathrm{Sr} + (remove, r_{rm}).\mathrm{Sm}$
$\mathrm{Sm} \stackrel{def}{=} (maint, r_{mn}).\mathrm{Sr} \qquad Sb \stackrel{def}{=} (fix, r_{fx}).St$
$St \stackrel{def}{=} (\mathrm{test}, r_{ts}).St + (compl, r_{cm}).\mathrm{Sr}$

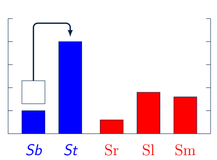


How transitions affect quantities of each derivative in TDSHA (assuming 10 servers)

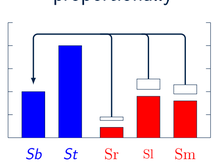
continuous to continuous with continuous action
$\mathrm{Sr} \xrightarrow{(\mathrm{request}, \#\mathrm{Sr}.r_{rp})}_{\star} \mathrm{Sl}$
flow defined by ODE



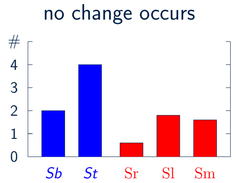
discrete to *discrete* with *discrete* action
$Sb \xrightarrow{(fix, \#Sb.r_{mn})}_{\star} St$
unit quantity is shifted



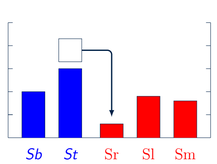
continuous to *discrete* with *discrete* action
$\mathrm{Sr} \xrightarrow{(break, \#\mathrm{Sr}.r_{bk})}_{\star} Sb$
unit quantity is shifted proportionally



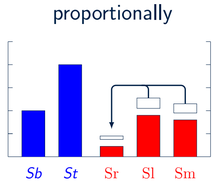
discrete to *discrete* with continuous action
$St \xrightarrow{(\mathrm{test}, \#St.r_{ts})}_{\star} St$
between one derivative no change occurs



discrete to continuous with *discrete* action
$St \xrightarrow{(compl, \#St.r_{cm})}_{\star} \mathrm{Sr}$
unit quantity is shifted



continuous to continuous with *discrete* action
$\mathrm{Sm} \xrightarrow{(maint, \#\mathrm{Sm}.r_{mn})}_{\star} \mathrm{Sr}$
unit quantity is shifted proportionally



Transition Driven Stochastic Hybrid Automata (TDSHA)

- set of modes, set of continuous variables
- set of continuous transitions at each mode which define ODEs at that mode
- set of *stochastic* transitions between modes
- simulate behaviour and obtain traces

Results: theoretical

Theorem

If all actions are *discrete*, then the TDSHA obtained is a *Markov chain* identical to that obtained from the original PEPA model.

Theorem

If all actions are continuous, then the TDSHA obtained is a set of ODEs identical to those obtained from the original PEPA model.

Discussion

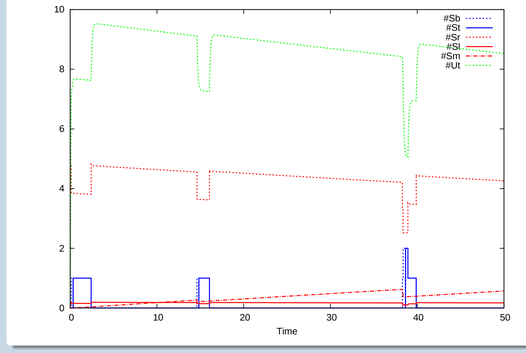
- Generally, more continuous actions means smaller state space and faster computation
- Question: what accuracy is lost with introduction of continuous actions?
- Question: can sensible partitioning of actions into continuous and discrete be done automatically?
- Question: does the rate of the reaction affect whether it should be continuous or discrete?
- Unit quantity shift from a continuous variable may require additional quantity shifts from other continuous variables
- Question: what is the best way to shift a unit quantity from continuous variables?

Reference

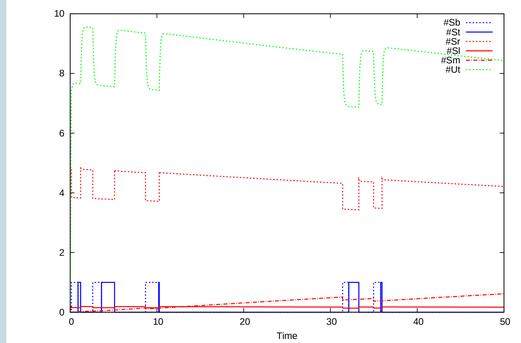
L. Bortolussi, V. Galpin, J. Hillston, and M. Tribastone, PEPA with hybrid semantics, *QEST 2010*, 181-190.

Results: simulation traces for 100 clients and 5 servers

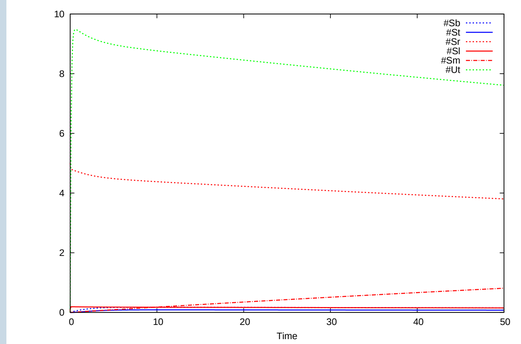
Simulation: client-server model as given



Simulation: maint is continuous



Simulation: all actions continuous



Simulation: parameters for all traces

$r_{rp} = 20$	$r_{lg} = 500$	$r_{rm} = 0.1$	$r_{mn} = 0.001$
$r_{br} = 0.02$	$r_{fix} = 0.5$	$r_{ts} = 1$	$r_{cm} = 1$
$r_{rq} = 50$	$r_{th} = 10$		