Discretization and Equivalences in Bio-PEPA

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Outline

Bio-PEPA

Syntax and semantics

Equivalence choice

Equivalence definition

Results

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- stochastic process algebra for modelling biological systems [Ciocchetta and Hillston 2008]

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- Bio-PEPA systems with finite number of levels, CTMCs

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  - P is a well-defined model component

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Bio-PEPA semantics (continued)

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$$\frac{S(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')}{C(\ell) \xrightarrow{(\alpha,w)}_c S'(\ell')} \quad C \stackrel{\mathit{def}}{=} S$$

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- Cooperation for $\alpha \notin L$

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- qualitative, only consider α

Example: reaction with enzyme, max level 3

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$$
\begin{tikzcd}
(3,3,0,0) \arrow[r, shift left, "\alpha"] \arrow[r, leftarrow, shift right, "\beta"'] & (2,2,1,0) \arrow[r, shift left, "\alpha"] \arrow[r, leftarrow, shift right, "\beta"'] \arrow[d, "\gamma"] & (1,1,2,0) \arrow[r, shift left, "\alpha"] \arrow[r, leftarrow, shift right, "\beta"'] \arrow[d, "\gamma"] & (0,0,3,0) \arrow[d, "\gamma"] \\
 & (2,3,0,1) \arrow[r, shift left, "\alpha"] \arrow[r, leftarrow, shift right, "\beta"'] & (1,2,1,1) \arrow[r, shift left, "\alpha"] \arrow[r, leftarrow, shift right, "\beta"'] \arrow[d, "\gamma"] & (0,1,2,1) \arrow[d, "\gamma"] \\
 & & (1,3,0,2) \arrow[r, shift left, "\alpha"] \arrow[r, leftarrow, shift right, "\beta"'] & (0,2,1,2) \arrow[d, "\gamma"] \\
 & & & (0,3,0,3)
\end{tikzcd}
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Example: reaction with enzyme, max level 7

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\begin{tikzcd}
7700 \arrow[r, shift left, "\alpha"] & 6610 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 5520 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 4430 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 3340 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 2250 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 1160 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 0070 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & 6701 \arrow[r, shift left, "\alpha"] & 5611 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 4521 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 3431 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 2341 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 1251 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 0161 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & 5702 \arrow[r, shift left, "\alpha"] & 4612 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 3522 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 2432 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 1342 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 0252 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & & 4703 \arrow[r, shift left, "\alpha"] & 3613 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 2523 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 1433 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 0342 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & & & 3704 \arrow[r, shift left, "\alpha"] & 2614 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 1524 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 0614 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & & & & 2705 \arrow[r, shift left, "\alpha"] & 1615 \arrow[l, shift left, "\beta"] \arrow[r, shift left, "\alpha"] \arrow[d, "\gamma"] & 0525 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & & & & & 1706 \arrow[r, shift left, "\alpha"] & 0616 \arrow[l, shift left, "\beta"] \arrow[d, "\gamma"] \\
 & & & & & & & 0707
\end{tikzcd}
$$

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- prove it is a congruence for relevant operators

$$P_1 \equiv P_2 \quad \text{and} \quad Q_1 \equiv Q_2 \quad \textit{implies} \quad P_1 \mathbin{[\!]} Q_1 \equiv P_2 \mathbin{[\!]} Q_2$$

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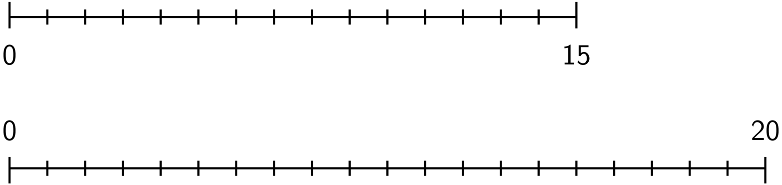
Equivalence illustrated

- $B \stackrel{def}{=} (\alpha, 3) \downarrow B + (\beta, 4) \uparrow B + (\gamma, 1) \uparrow B$



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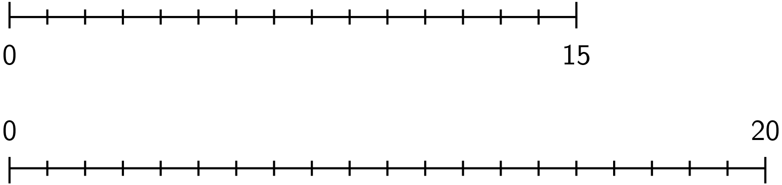

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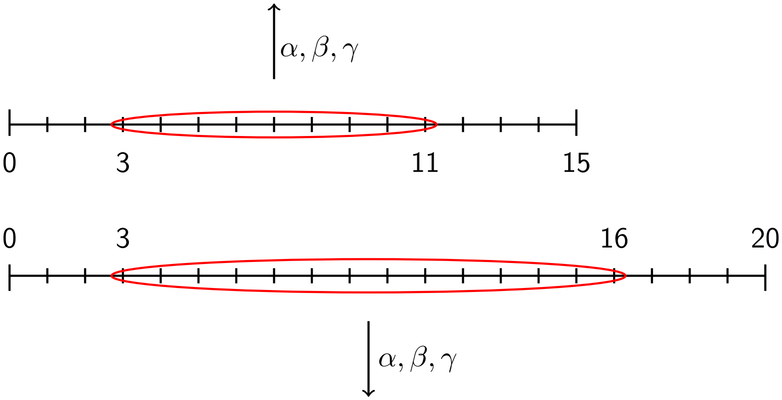

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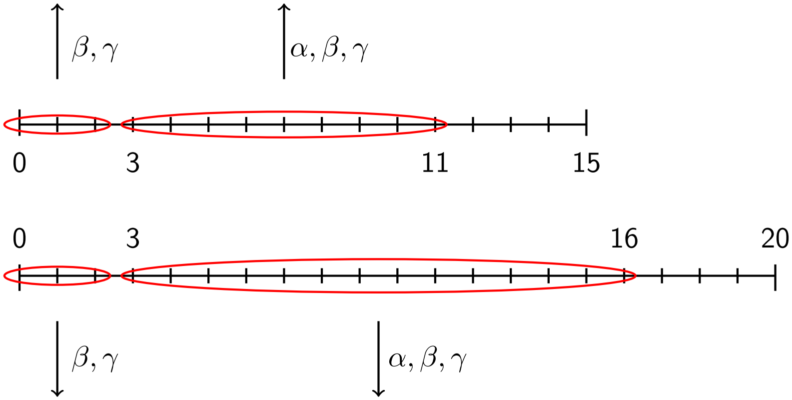

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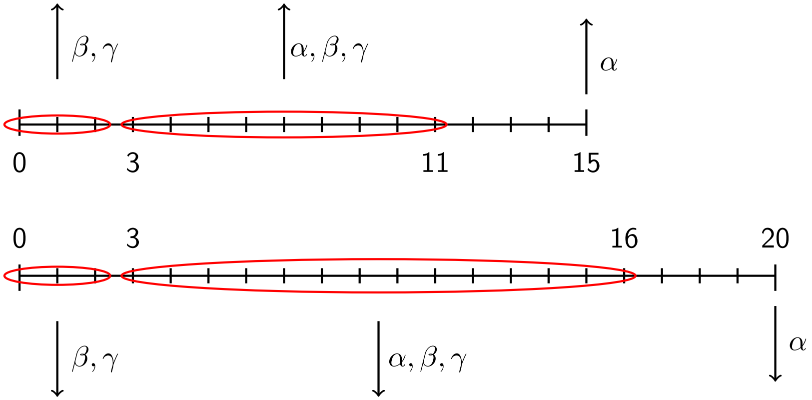

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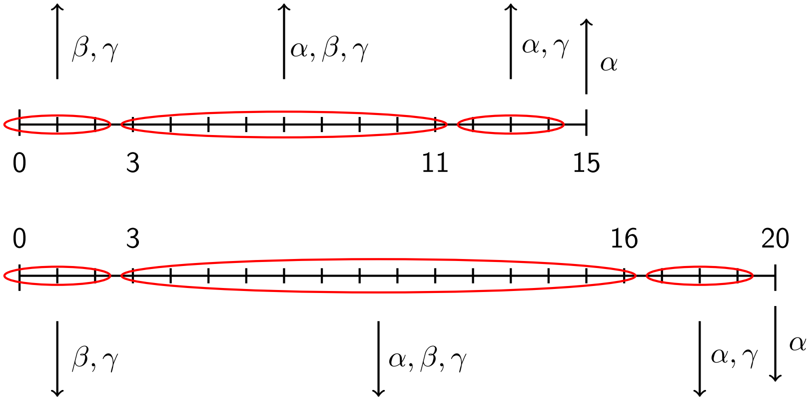

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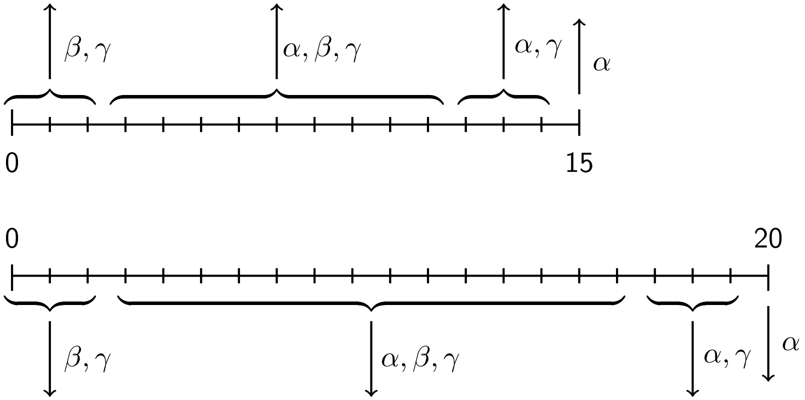

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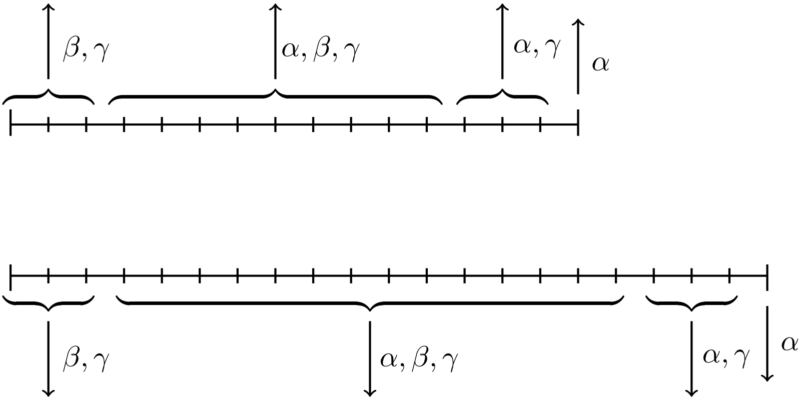

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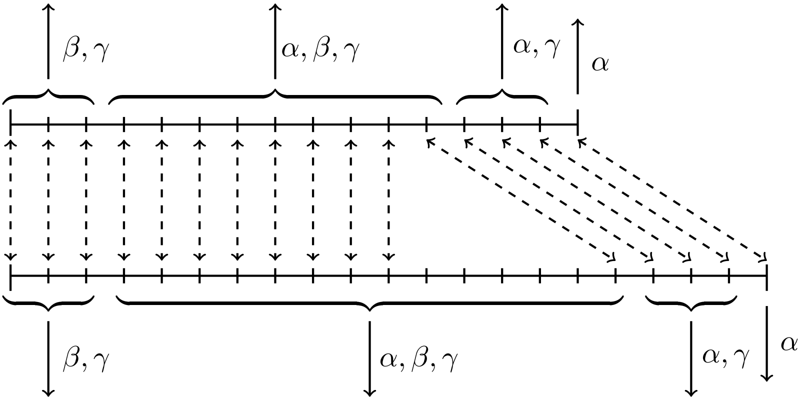

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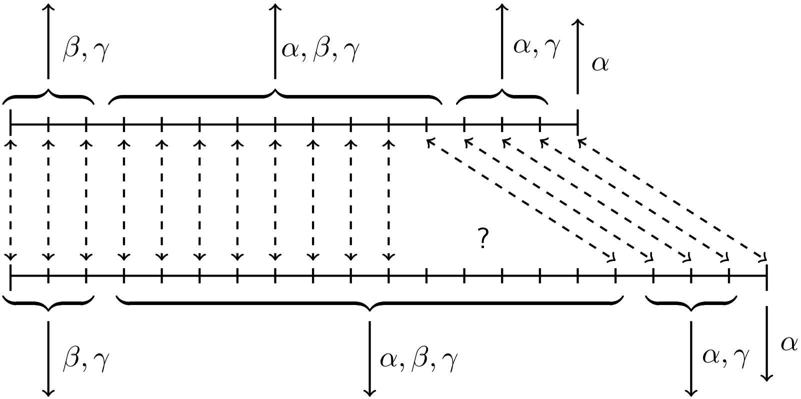

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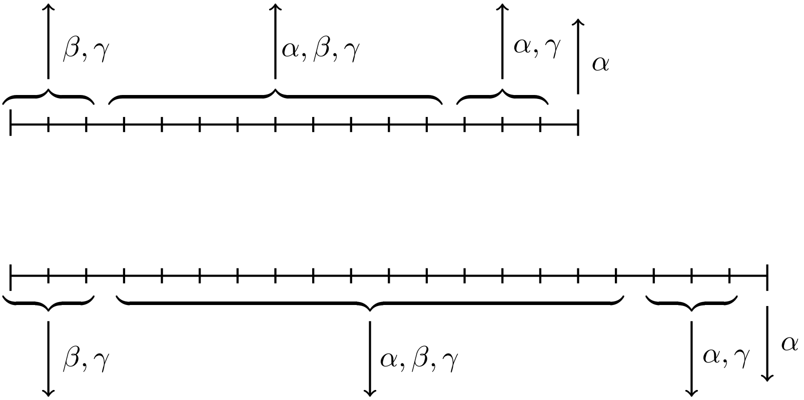

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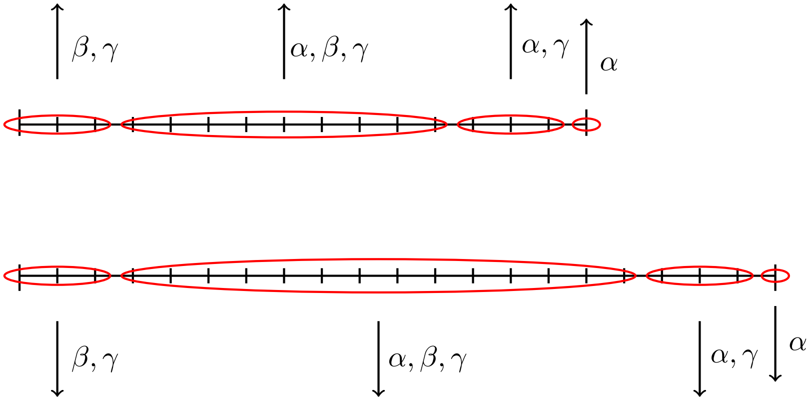

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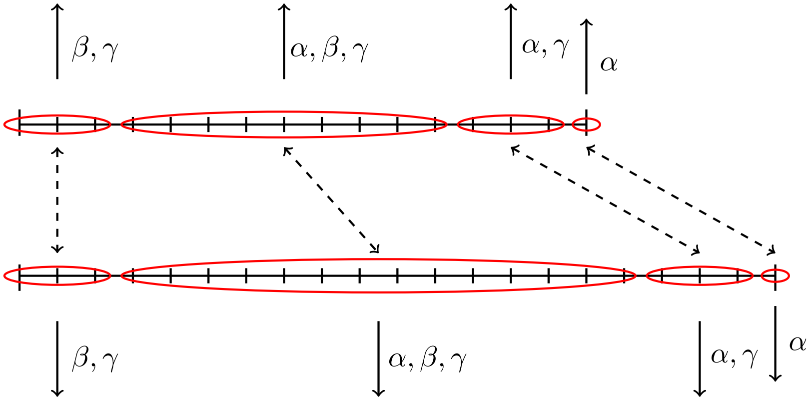

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- $(P, Q) \in \mathcal{H}$ if they can perform the same actions
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Results (continued)

- CADP: current action decomposition property
  - $(P_1 \underset{L}{\bowtie} Q_1, P_2 \underset{L}{\bowtie} Q_2) \in \mathcal{H}$ and derivatives of same model then $(P_1, P_2) \in \mathcal{H}$, $(Q_1, Q_2) \in \mathcal{H}$

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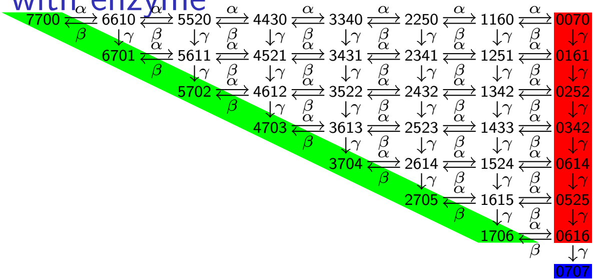
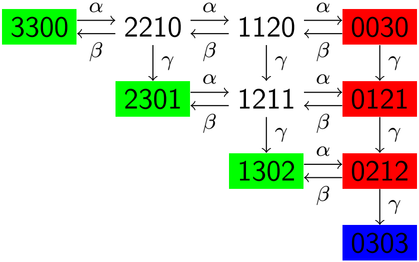
- for a well-defined Bio-PEPA system, $P^n \doteq P^m$ if they both have CADP and $n, m \geq k_\downarrow + \max\{k_\downarrow, k_\uparrow\} + k_\uparrow$

Example: reaction with enzyme





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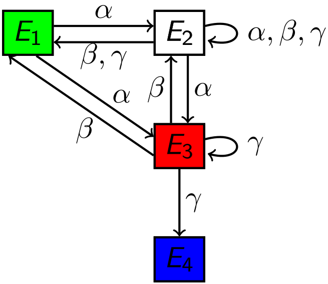

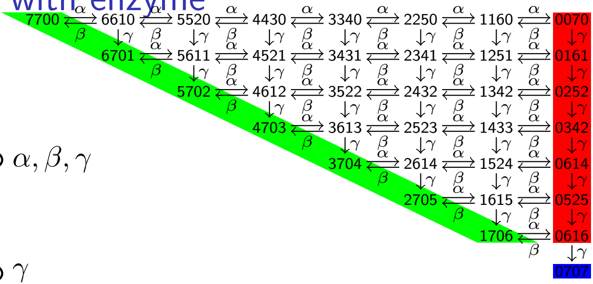

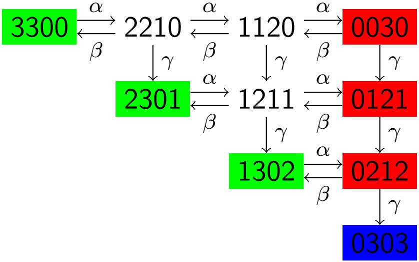

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Thank you