A comparison of bisimulation-based semantic equivalences for noninterleaving behaviour over CCS processes

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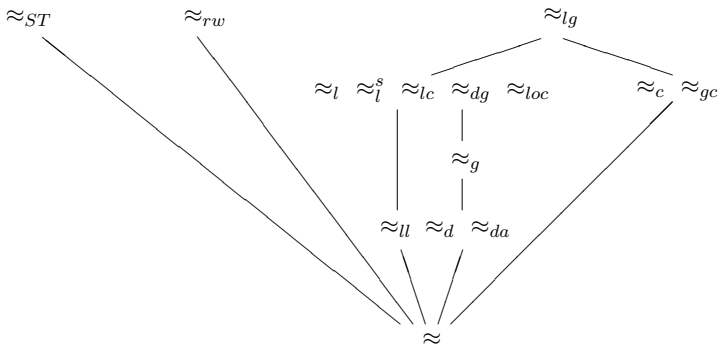
Introduction

- process algebras
  - mathematical models of concurrency
  - CCS (Calculus of Communicating Systems) – Milner
  - three components
    - syntax
    - operational semantics
    - semantic equivalences – bisimulation
- extensions to CCS – noninterleaving behaviour based on
  - location
  - causality
  - others – time, probability, priority, ...
- what is the relationship between these extensions?
- comparison – hierarchy of semantic equivalences

Hierarchy of semantic equivalences



Outline

- CCS
- examples of extensions to CCS
- bisimulation-based semantic equivalences
- hierarchy
  - basis for comparison
  - construction
  - example of incomparability
- semantic equivalences that cannot be included in the hierarchy
- conclusions

CCS

- syntax for processes: $P ::= 0 \mid \alpha.P \mid P + P \mid P|P \mid P\backslash L \mid P[f]$
- operational semantics

$$\frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \qquad \frac{P \xrightarrow{\alpha} P'}{Q + P \xrightarrow{\alpha} P'} \qquad \frac{P \xrightarrow{\alpha} P'}{P\backslash L \xrightarrow{\alpha} P'\backslash L} \quad \alpha, \overline{\alpha} \notin L$$

$$\frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]} \qquad \frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \qquad \frac{P \xrightarrow{\alpha} P'}{Q \mid P \xrightarrow{\alpha} Q \mid P'} \qquad \frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q'}$$

- proofs of transitions

$$\frac{\dfrac{a.b.0 \xrightarrow{a} b.0}{a.b.0 + c.0 \xrightarrow{a} b.0} \qquad \overline{a}.0 \xrightarrow{\overline{a}} 0}{(a.b.0 + c.0) \mid \overline{a}.0 \xrightarrow{\tau} b.0 \mid 0}$$

- labelled transition system to describe behaviour of processes

Extensions to CCS

- different labelled transition systems
- model different behaviour
- CCS with locations
  - syntax: as CCS plus $l :: P$
  - operational semantics

$$\frac{}{\alpha.P \xrightarrow[l]{\alpha} l :: P} \qquad \frac{P \xrightarrow[u]{\alpha} P'}{l :: P \xrightarrow[lu]{\alpha} l :: P'}$$

- CCS with split actions (ST)
  - different action set: $s(a)$, $f(a)$

$$\frac{}{a.P \xrightarrow{s(a)} f(a).P} \qquad \frac{}{f(a).P \xrightarrow{f(a)} P}$$

Extensions to CCS (cont.)

- distributed CCS
  - same syntax
  - operational semantics: $P \xrightarrow{a} \langle P', P'' \rangle$
- decomposing CCS into sets of processes
- transition systems labelled with proofs of transitions

Semantic equivalences

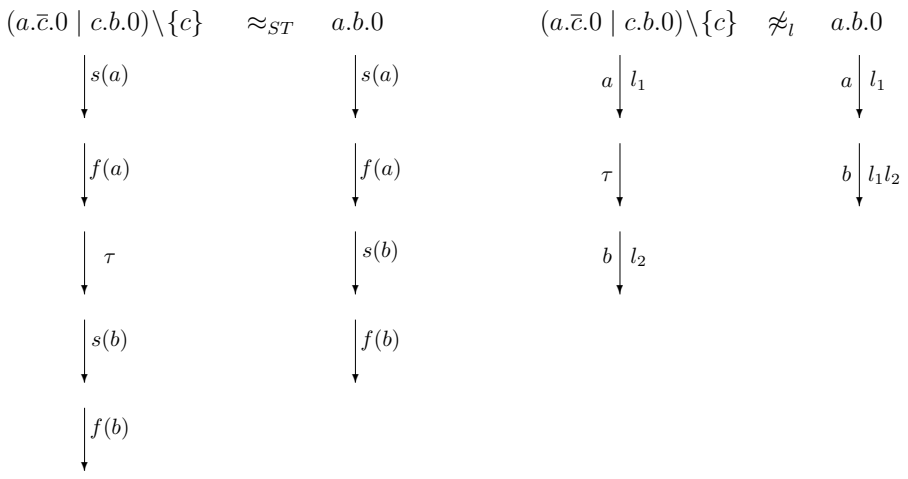
- bisimulation – equates processes with similar behaviour
- informally, two processes are bisimilar ($P \sim Q$) if
  1. whenever $P \xrightarrow{a} P'$ there exists Q' such that $Q \xrightarrow{a} Q'$ and $P' \sim Q'$
  2. whenever $Q \xrightarrow{a} Q'$ there exists P' such that $P \xrightarrow{a} P'$ and $P' \sim Q'$
- distinguishes branching behaviour: $a(b.0 + c.0) \not\sim a.b.0 + a.c.0$
- interleaving over CCS: $a.b.0 + b.a.0 \sim a.0 \mid b.0$
- can abstract from τ actions – weak bisimulation ($P \approx Q$)
- variants for extensions to CCS
  - match on all elements of label – location bisimulation matches on action and location
  - requirements on process resulting from transitions – distributed bisimulation requires local processes bisimilar, global processes bisimilar
  - noninterleaving

Construction of hierarchy

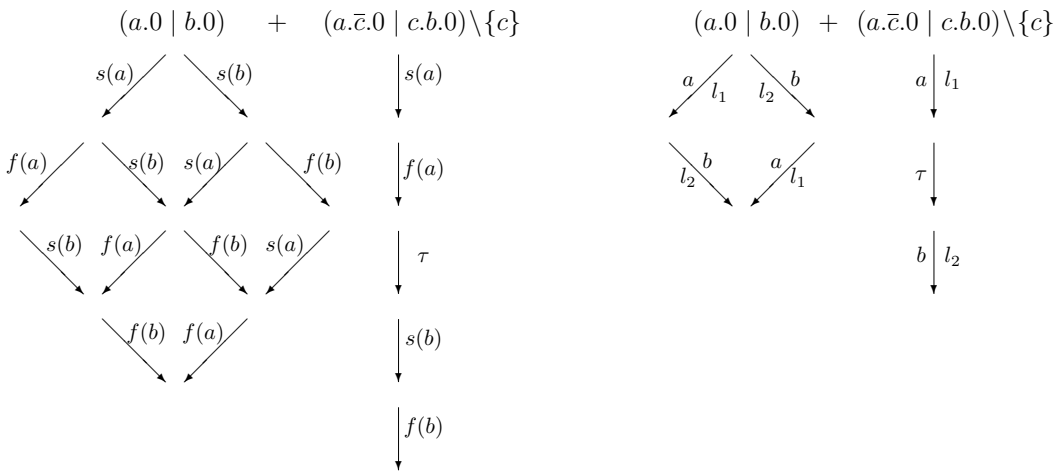
- basis for comparison
  - extensions where CCS terms without additional operators have interesting behaviour
  - *ad hoc* – based on existing results, new examples to show incomparability, logical inference
- semantic equivalences are (equivalence) relations – compare as relations
- ways in which two equivalences can be related
  - equal – equate same CCS processes
  - proper subset – one equivalence (finer) equates few processes than the other (coarser)
  - incomparable – equate different processes

Incomparability of semantic equivalences

- Example: location bisimulation and ST bisimulation (informally)
  - two pairs of CCS processes



$$(a.0 \mid b.0) + (a.\overline{c}.0 \mid c.b.0)\backslash\{c\} \quad \begin{matrix} \not\approx_{ST} \\ \approx_{l} \end{matrix} \quad a.0 \mid b.0$$



Hierarchy

- interpretation
  - directed graph – Hasse diagram
  - higher in graph – finer semantic equivalence
  - lower in graph – coarser semantic equivalence
  - no (directed) path between two equivalences – incomparable
- observations
  - all equivalence finer than standard bisimulation
  - many location-based equivalences grouped together
  - relationship between location and causality
  - $\approx_{ST}$ and $\approx_{rw}$ incomparable

Exclusions from hierarchy

- some semantic equivalences can not be included
  - CCS processes do not show noninterleaving behaviour
  - CCS processes only show subset of noninterleaving behaviour
  - conservative extension
  - semantic equivalence does not abstract from internal actions
  - not based on CCS operators
  - subset of CCS operators
  - subset of CCS processes
  - not bisimulation-based semantic equivalence

Conclusions

- comparison of 14 semantic equivalences
- comparison can be done by *ad hoc* approach
- specific requirements for comparison
- there are other approaches to comparison