A comparison of the ODE semantics of PEPA with timed continuous Petri nets

Vashti Galpin
LFCS
University of Edinburgh

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Outline

PEPA

- Performance Evaluation Process Algebra [Hillston 1996]
  - syntax, structured operational semantics
  - equivalence semantics
  - analysis of dynamic behaviour
  - stochastic, action durations from exponential distribution

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  - stochastic, action durations from exponential distribution
- syntax
  - $S ::= (\alpha, r).S \mid S + S \mid C_s$, sequential component
  - $P ::= P \underset{L}{\bowtie} P \mid P/L \mid C$, model component
  - C_s and C constants
  - cooperations of sequential components
  - ergodic continuous time Markov chain (CTMC)

Structured operational semantics

- Prefix and Constant

$$\frac{}{(\alpha, r).E \xrightarrow{(\alpha,r)} E} \qquad \frac{E \xrightarrow{(\alpha,r)} E'}{A \xrightarrow{(\alpha,r)} E'}(A \stackrel{def}{=} E)$$

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- Hiding

$$\frac{E \xrightarrow{(\alpha,r)} E'}{E/L \xrightarrow{(\alpha,r)} E'/L}(\alpha \notin L) \qquad \frac{E \xrightarrow{(\alpha,r)} E'}{E/L \xrightarrow{(\tau,r)} E'/L}(\alpha \in L)$$

Structured operational semantics (continued)

- Cooperation

$$\frac{E \xrightarrow{(\alpha,r)} E'}{E \underset{L}{\bowtie} F \xrightarrow{(\alpha,r)} E' \underset{L}{\bowtie} F}(\alpha \notin L) \qquad \frac{F \xrightarrow{(\alpha,r)} F'}{E \underset{L}{\bowtie} F \xrightarrow{(\alpha,r)} E \underset{L}{\bowtie} F'}(\alpha \notin L)$$

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$$\frac{E \xrightarrow{(\alpha,r_1)} E' \quad F \xrightarrow{(\alpha,r_2)} F'}{E \underset{L}{\bowtie} F \xrightarrow{(\alpha,R)} E' \underset{L}{\bowtie} F'}(\alpha \in L)$$

$$R = \frac{r_1}{r_\alpha(E)} \frac{r_2}{r_\alpha(F)} \min(r_\alpha(E), r_\alpha(F))$$

Modelling

- operational semantics generate a labelled multi-transition system

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  - same behaviour
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- analysis of dynamic behaviour
  - state transition diagram $\longrightarrow$ continuous time Markov Chain
  - syntax $\longrightarrow$ activity matrix $\longrightarrow$ ODEs
  - syntax $\longrightarrow$ rate equations $\longrightarrow$ stochastic simulation

Motivation and background

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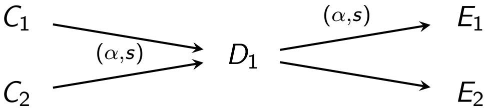
- infinite or finite server semantics?

ODE semantics of PEPA

- numerical vector form $(n_1, \ldots n_m)$
- how many copies of each derivative is present in a given state
- continuous approximation of changes in numbers



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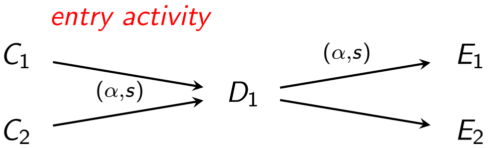

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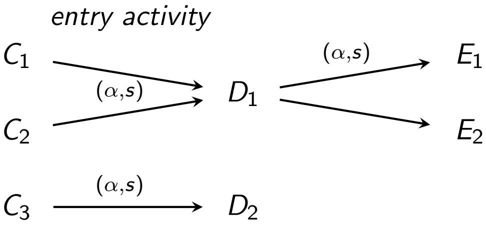

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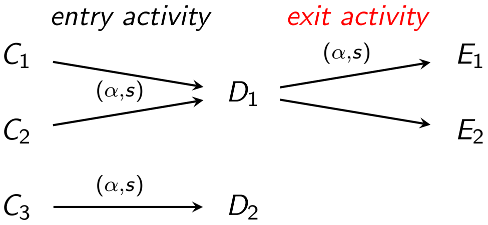

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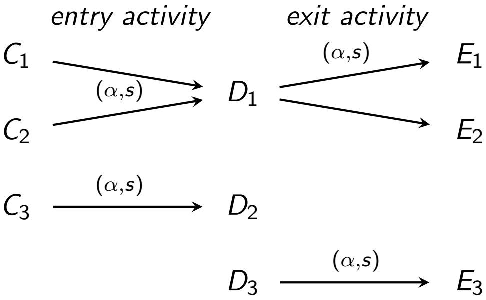

Change in number of copies of component D

$$\frac{dN(D,\tau)}{d\tau} = \sum_{\substack{(\alpha,r)\\ \text{entry}\\ \text{activity}}} r \times \min\{N(C,\tau) \mid C \xrightarrow{(\alpha,r)}\}$$

$$- \sum_{\substack{(\alpha,r)\\ \text{exit}\\ \text{activity}}} r \times \min\{N(C,\tau) \mid C \xrightarrow{(\alpha,r)}\}$$

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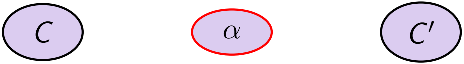
- create activity graph and matrix from syntax

Activity graph and activity matrix

- graph nodes are activities

- $C \xrightarrow{(\alpha,r)} C'$



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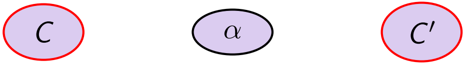

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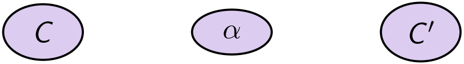

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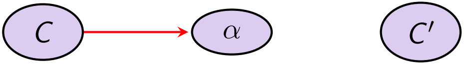

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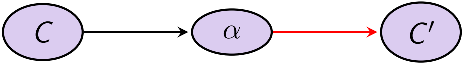

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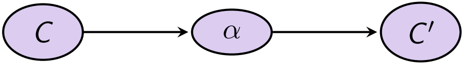
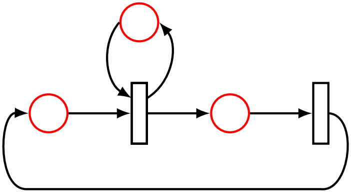
- edges are added
  - from a derivative to an exit activity for that derivative
  - from an entry activity for a derivative to that derivative
- activity matrix, derivatives $\times$ activities
  - $(d, a) = -1$ if a exit activity for d
  - $(d, a) = +1$ if a entry activity for d
- $C \xrightarrow{(\alpha,r)} C'$ then $(C, \alpha) = -1$ and $(C', \alpha) = +1$



Timed continuous Petri nets

- places P,



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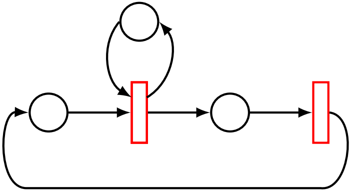

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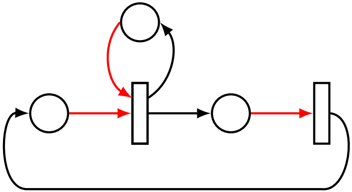
- arcs from places to transitions $Pre : P \times T \to \{0, 1\}$



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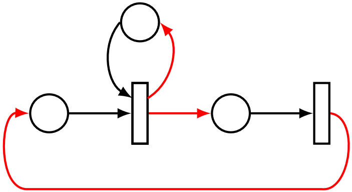

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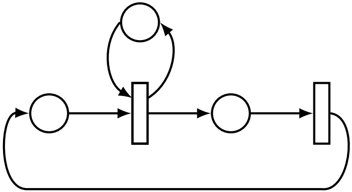
- cost matrix $C = Post - Pre$



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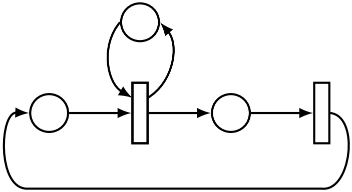
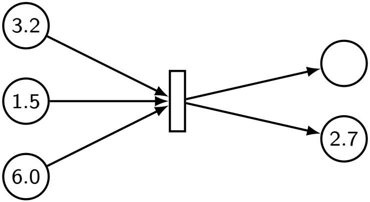
- standard definitions of ${}^{\bullet}p$, ${}^{\bullet}t$, $p^{\bullet}$, $t^{\bullet}$



Dynamic behaviour

- firing rates $\lambda : T \to (0, \infty)$



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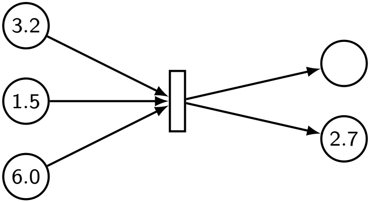

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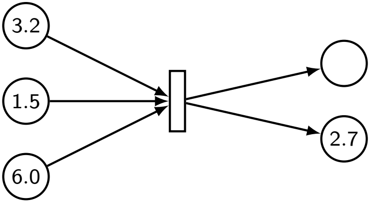

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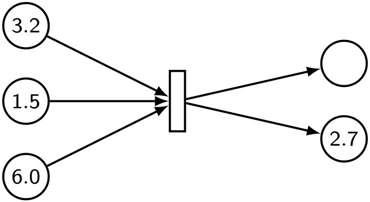

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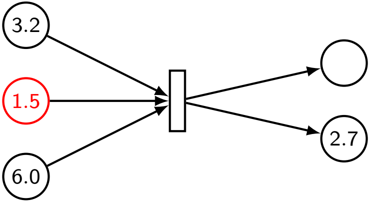

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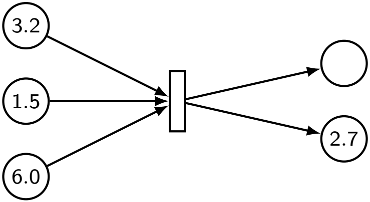

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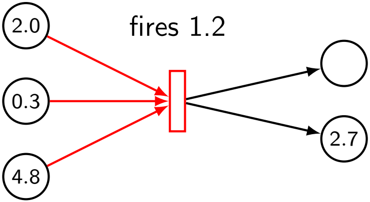

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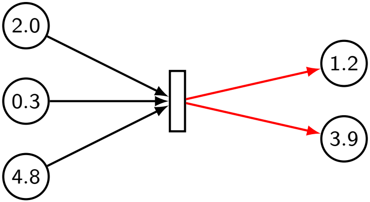

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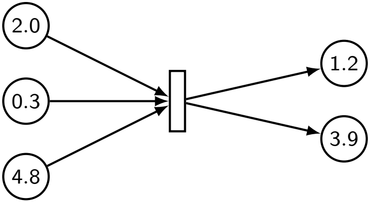


Change in marking at place p

- infinite server semantics: many servers, many clients

Change in marking at place p

- infinite server semantics: many servers, many clients
- fundamental equation for Petri nets

$$m(\cdot, \tau + \delta\tau) = m(\cdot, \tau) + C(\cdot, t) \cdot \sigma(\tau)$$

- change in marking of place p

$$\begin{aligned}
\frac{dm(p,\tau)}{d\tau} &= \sum_{j=1}^{n} C(p, t_j).\lambda(t).\min_{p' \in {}^\bullet t}\{m(p', \tau)\} \\
&= \sum_{t \in {}^\bullet p} \lambda(t).\min_{p' \in {}^\bullet t}\{m(p', \tau)\} \\
&\quad - \sum_{t \in p^\bullet} \lambda(t).\min_{p' \in {}^\bullet t}\{m(p', \tau)\}
\end{aligned}$$

Comparison

- translate a PEPA model into a timed continuous Petri net
- example – clients and servers

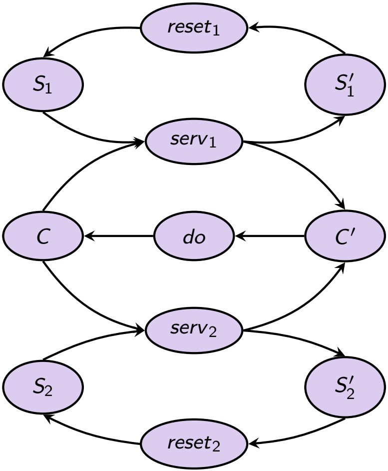
$$
\begin{aligned}
C &\stackrel{def}{=} (serv_1, s_1).C' + (serv_2, s_2).C' \\
C' &\stackrel{def}{=} (do, d).C
\end{aligned}
$$

$$
\begin{aligned}
S_1 &\stackrel{def}{=} (serv_1, s_1).S_1' & \qquad S_2 &\stackrel{def}{=} (serv_2, s_2).S_2' \\
S_1' &\stackrel{def}{=} (reset_1, r_1).S_1 & \qquad S_2' &\stackrel{def}{=} (reset_2, r_2).S_2
\end{aligned}
$$

$$
Sys \stackrel{def}{=} \left(C(100) \underset{\{serv_1, serv_2\}}{\bowtie} \left(S_1(50) \parallel S_2(50)\right)\right)
$$

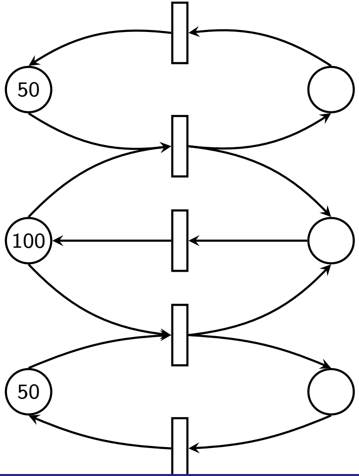
Activity graph

- activities and derivatives



Petri net

- activities become transitions and derivatives become places



Petri net (continued)

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- a marking value of x at p is the same as x copies of p

$$m(p, \tau) = N(p, \tau)$$

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$$m(p,\tau) = N(p,\tau)$$

- initial marking $m(p,0) = N(p,0)$ for each p

Comparison of ODEs

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Further work

- finite server semantics: many clients, few servers
  - special case of infinite in discrete Petri nets
  - can apply to PEPA
  - two definitions for continuous Petri nets

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Thank you