Towards a spatial stochastic process algebra

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Introduction

- spatial concepts in a stochastic process algebra
- location can affect time taken
- aims
  - generality and general results hence wide application
  - CTMCs and steady state
  - separation of concerns
  - performance evaluation
  - no unnecessary increase in state space
  - single discovery of state space
  - efficient experiments

Introduction (cont.)

- examples of locations
  - nodes in networks
  - points in *n*-dimensional space
- applications – networks, epidemiology
- related research
  - PEPA nets (Gilmore *et al*)
  - StoKlaim (de Nicola *et al*)
  - biological models – BioAmbients, attributed π-calculus
- work in progress
- general process algebra then applications and example

Syntax

- $\mathcal{L}$ set of locations
- $\mathcal{P}_{\mathcal{L}} = 2^{\mathcal{L}}$ powerset
- let $L \in \mathcal{P}_{\mathcal{L}}$
- sequential components

$$S ::= (\alpha @ L, r).S \mid S + S \mid C_s @ L$$

- model components

$$P ::= P \underset{M}{\bowtie} P \mid P/M \mid C$$

Location functions

- process location function
  - $ploc(C_s@L) = L \cup ploc(S)$ where $C_s@L \stackrel{def}{=} S$
  - $ploc(P) = \ldots$
- action location function
  - $aloc((\alpha@L, r).S) = L \cup aloc(S)$
  - $aloc(P) = \ldots$
- location function
  - $loc(P) = ploc(P) \cup aloc(P)$
- all static definitions
- current location function
  - needs a dynamic definition

Operational semantics

- relation for labels from $\mathcal{A} \times \mathcal{P}_{\mathcal{L}} \times \mathbb{R}^+$
- Prefix $$\frac{}{(\alpha@L, r).S \xrightarrow{(\alpha@L', r)} S} \quad L' = apref((\alpha@L, r).S)$$
- Cooperation $$\frac{P_1 \xrightarrow{(\alpha@L_1, r_1)} P_1' \quad P_2 \xrightarrow{(\alpha@L_2, r_2)} P_2'}{P_1 \underset{M}{\bowtie} P_2 \xrightarrow{(\alpha@L, R)} P_1' \underset{M}{\bowtie} P_2'} \quad \alpha \in M$$

$$L = async(P_1, P_2, L_1, L_2) \quad R = rsync(P_1, P_2, L_1, L_2, r_1, r_2)$$

- other rules defined in the obvious manner
  Constant, Choice, Hiding, other two Cooperation rules

Operational semantics (cont.)

- parameterised by three functions
  - *apref* – determines location in Prefix rule
  - *async* – determines location in Cooperation rule
  - *rsync* – determines rate in Cooperation rule
- PEPA rate function

$$RPEPA(P_1, P_2, L_1, L_2, r_1, r_2) = \frac{r_1}{r_\alpha(P_1)} \frac{r_2}{r_\alpha(P_2)} \min(r_\alpha(P_1), r_\alpha(P_2))$$

- can define *apref* and *async* for PEPA as well
- use of different functions to obtain different process algebras

Possible interpretations

$\lvert ploc(C_s)\rvert = 1$	process at one location
$\lvert ploc(C_s)\rvert > 1$	process is mobile *or*
	process at one of some possible locations
$\lvert aloc(C_s)\rvert = 1$	all actions occur at one location
$\lvert aloc(C_s)\rvert > 1$	interaction is mobile *or*
	actions occur at one of some possible locations
$P \xrightarrow{(\alpha @ L, r)} P',\ \lvert L\rvert = 1$	synchronisation happens at one location
$P \xrightarrow{(\alpha @ L, r)} P',\ \lvert L\rvert > 1$	synchronisation involves actions or
	processes at different locations
...	...

And that's not all

- beyond just adding locations, want to use them for performance evaluation
- aims
  - relate locations to each other
  - want interesting theoretical results
  - want to minimise time in analysis
- graph structure over locations
- weighted hypergraph: $G = (\mathcal{L}, E, w)$ with $E \subseteq \mathcal{P}_{\mathcal{L}}$ and $w : E \to \mathbb{R}$ *or* weighted directed graph
- weights modify rates on actions between locations
- weights assigned or calculated

Theoretical results – aim

- combine graph and known structures to obtain new results quickly

Analysis	**PEPA model**	**Modified by G**	**Form of result**
PEPA model	P	P_G	
$\downarrow$			
LMTS	$\mathcal{M}$	$\mathcal{M}_G$	$\mathcal{M} \circledcirc G = \mathcal{M}_G$
$\downarrow$			
CTMC	$\mathcal{Q}$	$\mathcal{Q}_G$	$\mathcal{Q} \boxdot G = \mathcal{Q}_G$
$\downarrow$			
steady state	Π	Π_G	$\Pi \boxdot G = \Pi_G$

Application – plant epidemiology

- epidemiology with spatial aspects
- grapevine diseases in vineyards
- mealy bug insect – vector for leaf roll disease
- two separate issues
  - delay before vector arrives at uninfected plant – dependent on distance between plants, hence dependent on vineyard layout
  - delay before newly arrived vector infects plant
- three distinct patterns of infection
- build models to explain this

Application – networking

- networking performance
- scenario
  - arbitrary topology
  - single packet traversal through network
  - processes can be colocated
- want to model different traffic situations
- choose functions to create process algebra
  - each sequential component must have single fixed location
  - communication must be pairwise and directional

Application – networking (cont.)

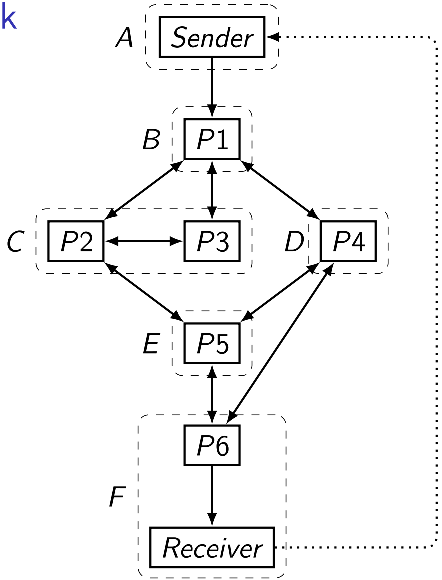
- functions

$$apref(S) = \begin{cases} l & \text{if } ploc(S) = \{l\} \\ \perp & \text{otherwise} \end{cases}$$

$$async(P_1, P_2, L_1, L_2) = \begin{cases} (l_1, l_2) & \text{if } L_1 = \{l_1\}, L_2 = \{l_2\} \\ \perp & \text{otherwise} \end{cases}$$

$$rsync(P_1, P_2, L_1, L_2, r_1, r_2) \\ = \begin{cases} \frac{r_1}{r_\alpha(P_1)} \frac{r_2}{r_\alpha(P_2)} \min(r_\alpha(P_1), r_\alpha(P_2)) \cdot w((l_1, l_2)) & \\ \qquad \text{if } L_1 = \{l_1\}, L_2 = \{l_2\}, (l_1, l_2) \in E & \\ \perp & \text{otherwise} \end{cases}$$

Example network



PEPA model

$$
\begin{aligned}
Sender@A &\stackrel{\mathit{def}}{=} (prepare, \rho).Sending@A \\
Sending@A &\stackrel{\mathit{def}}{=} (cSto1, r_1).(ack, r_{ack}).Sender@A
\end{aligned}
$$

$$
\begin{aligned}
Receiver@F &\stackrel{\mathit{def}}{=} (c6toR, r_6).Receiving@F \\
Receiving@F &\stackrel{\mathit{def}}{=} (consume, \gamma).(ack, r_{ack}).Receiver@F
\end{aligned}
$$

$$
\begin{aligned}
&\ldots \\
P2@C &\stackrel{\mathit{def}}{=} (c1to2, r).P2'@C + (c3to2, r).P2'@C + (c5to2, r).P2'@C \\
P2'@C &\stackrel{\mathit{def}}{=} (c2to1, r).P2@C + (c2to3, r).P2@C + (c2to5, r).P2@C \\
&\ldots
\end{aligned}
$$

$$
\begin{aligned}
Network \stackrel{\mathit{def}}{=} \; & (Sender@A \underset{L_S}{\bowtie} (P1@B \underset{L_1}{\bowtie} (P2@C \underset{L_2}{\bowtie} (P3@C \underset{L_3}{\bowtie} \\
& (P4@D \underset{L_4}{\bowtie} (P5@E \underset{L_5}{\bowtie} (P6@F \underset{L_R}{\bowtie} Receiver@F)))))))
\end{aligned}
$$

Graphs

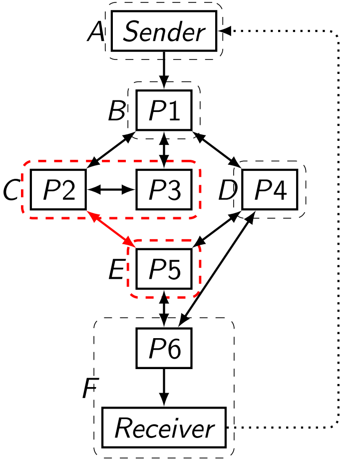
- rates: $r = r_1 = r_6 = 10$
- the weighted graph G has no effect on the rates in the model

	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		1	
D		1			1	1
E			1	1		1
F	1			1	1	1

Graphs

- G_1 represents heavy traffic between C and E

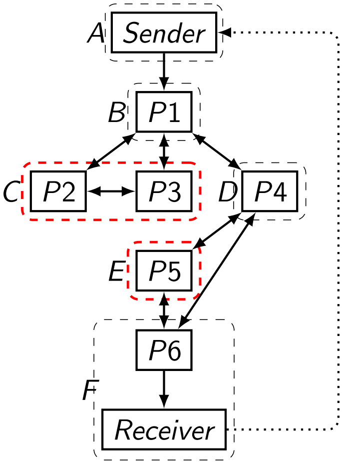
	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		0.1	
D		1			1	1
E			0.1	1		1
F	1			1	1	1



Graphs

- G_2 represents no connectivity between C and E

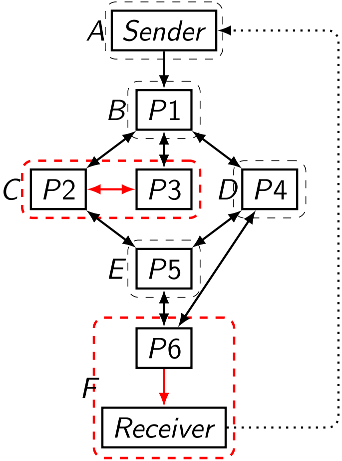
	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		0	
D		1			1	1
E			0	1		1
F	1			1	1	1



Graphs

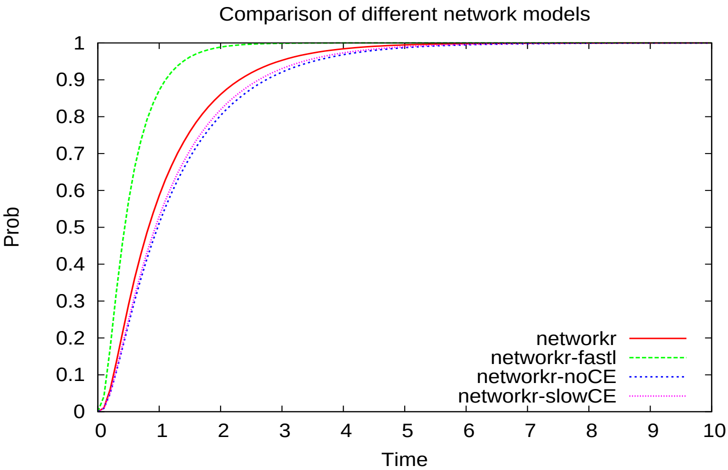
- G_3 represents high connectivity between colocated processes

	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	10		1	
D		1			1	1
E			1	1		1
F	1			1	1	10



Analysis

- cumulative density function of passage time



Further work and conclusions

- further work
  - finalise general process algebra
  - more specific applications
  - theoretical results
  - semantic equivalences
- conclusions
  - first steps towards a very general stochastic process algebra with locations
  - most important aspect is that locations affect rates

Thank you

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