Continuous approximation of PEPA models and Petri nets

Vashti Galpin
Laboratory for Foundations of Computer Science
School of Informatics
University of Edinburgh

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Outline

Motivation

- large systems and state space explosion

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- how do these compare?
- what are the server semantics of PEPA?

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  - stochastic, action durations from exponential distribution
  - syntax, structured operational semantics
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  - parallel cooperation with multiway synchronisation

$$C_1[n_1] \underset{L_1}{\bowtie} C_2[n_2] \underset{L_2}{\bowtie} \dots \underset{L_{m-1}}{\bowtie} C_m[n_m]$$

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  - identical rates for shared activities

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- each sequential component may have a number of derivatives

$$A \stackrel{\text{def}}{=} (a_1, r_1).B + (a_2, r_2).C \quad B \stackrel{\text{def}}{=} (b, s).A \quad C \stackrel{\text{def}}{=} (c, t).A$$

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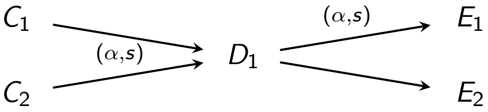
- express states in numerical vector form $(n_1, \ldots n_m)$
- number of copies of each component/derivative
- transitions update the vector

$$\begin{array}{ccc} A & \xrightarrow{(a_1,r_1)} & B \\ (N(A), N(B), N(C)) & \xrightarrow{(a_1,r_1)} & (N(A)-1, N(B)+1, N(C)) \end{array}$$

ODE semantics of PEPA (continued)

- continuous approximation of changes in numbers
- consider what actions lead to change in numbers



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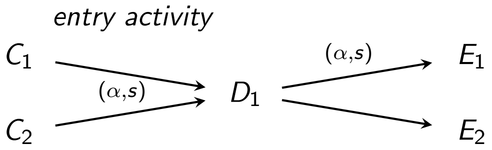

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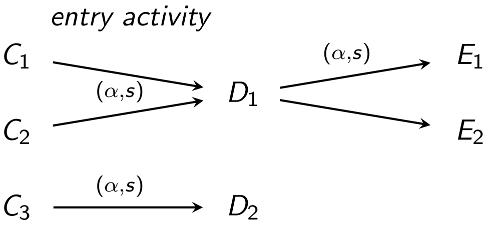

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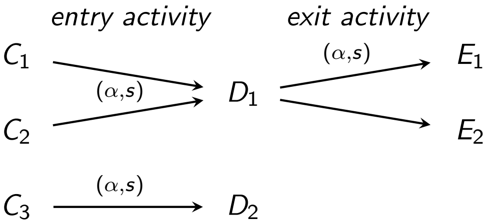

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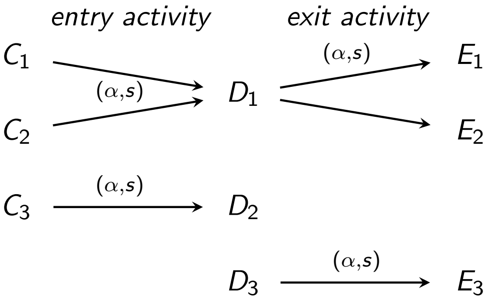

Change in number of copies of component D

$$\frac{dN(D,\tau)}{d\tau} = \sum_{\substack{(\alpha,r)\\ \text{entry}\\ \text{activity}}} r \times \min\{N(C,\tau) \mid C \xrightarrow{(\alpha,r)}\} - \sum_{\substack{(\alpha,r)\\ \text{exit}\\ \text{activity}}} r \times \min\{N(C,\tau) \mid C \xrightarrow{(\alpha,r)}\}$$

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- can express ODEs as activity graph and activity matrix

Activity graph

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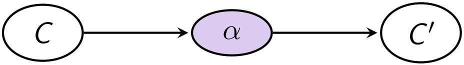
- $C \xrightarrow{(\alpha,r)} C'$



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- presets and postsets

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- cost matrix $C = Post - Pre$

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- cost matrix $C = Post - Pre$
- firing rates $\lambda : T \rightarrow (0, \infty)$

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- infinite server semantics
  - assume many clients and many servers
  - t fires $\mathit{enab}(t, \tau)$

Change in marking at place p

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- fundamental equation for Petri nets

$$m(\cdot,\tau+\delta\tau) = m(\cdot,\tau) + C\cdot\sigma(\tau)$$

- change in marking of place p

$$\begin{aligned}
\frac{dm(p,\tau)}{d\tau} &= \sum_{j=1}^{n} C(p,t_j).\lambda(t_j).\min_{p'\in{}^\bullet t_j}\{m(p',\tau)\} \\
&= \sum_{t\in{}^\bullet p} \lambda(t).\min_{p'\in{}^\bullet t}\{m(p',\tau)\} \\
&\quad -\sum_{t\in p^\bullet} \lambda(t).\min_{p'\in{}^\bullet t}\{m(p',\tau)\}
\end{aligned}$$

PEPA model to continuous Petri net

- translate a PEPA model into a timed continuous Petri net
- example – clients and servers

$$\begin{array}{lcl} C & \stackrel{def}{=} & (serv_1, s_1).C' + (serv_2, s_2).C' \\ C' & \stackrel{def}{=} & (do, d).C \end{array}$$

$$\begin{array}{lcllcl} S_1 & \stackrel{def}{=} & (serv_1, s_1).S_1' & \quad S_2 & \stackrel{def}{=} & (serv_2, s_2).S_2' \\ S_1' & \stackrel{def}{=} & (reset_1, r_1).S_1 & \quad S_2' & \stackrel{def}{=} & (reset_2, r_2).S_2 \end{array}$$

$$Sys \stackrel{def}{=} \big(C[100] \underset{\{serv_1, serv_2\}}{\bowtie} \big(S_1[50] \parallel S_2[50]\big)\big)$$

ODEs

$$\begin{aligned}
\frac{dN(C,\tau)}{d\tau} &= +d.N(C',\tau) - s_1.\min(N(C,\tau),N(S_1,\tau)) \\
&\qquad - s_2.\min(N(C,\tau),N(S_2,\tau)) \\
\frac{dN(C',\tau)}{d\tau} &= -d.N(C',\tau) + s_1.\min(N(C,\tau),N(S_1,\tau)) \\
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\frac{dN(S_i,\tau)}{d\tau} &= r_i.N(S_i') - s_i.\min(N(C,\tau),N(S_i,\tau)) \quad i = 1,2 \\
\frac{dN(S_i',\tau)}{d\tau} &= s_i.\min(N(C,\tau),N(S_i,\tau)) - r_i.N(S_i') \quad i = 1,2
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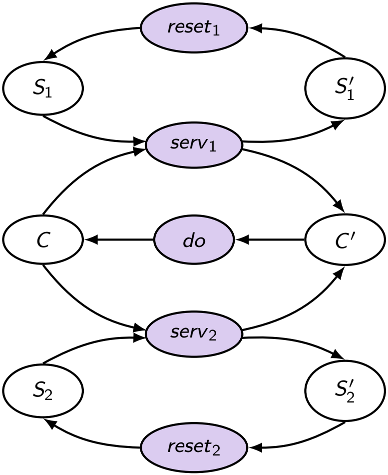
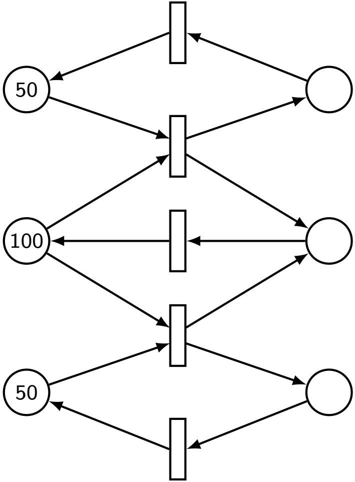
Activity graph

- activities and components



Petri net

- activities become transitions and components become places



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$$m(p,\tau) = N(p,\tau)$$

- initial marking $m(p,0) = N(p,0)$ for each p

Comparison of ODEs

$$\frac{dm(p,\tau)}{d\tau} = \sum_{t \in {}^\bullet p} \lambda(t). \min_{p' \in {}^\bullet t} \{m(p',\tau)\} \quad - \sum_{t \in p^\bullet} \lambda(t). \min_{p' \in {}^\bullet t} \{m(p',\tau)\}$$

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- both approaches give the same equations

Continuous Petri net to PEPA model

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- concept of implicit/complementary place: marking of p does not change enabling degree of any t

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- ODEs remain unchanged by addition of complementary places

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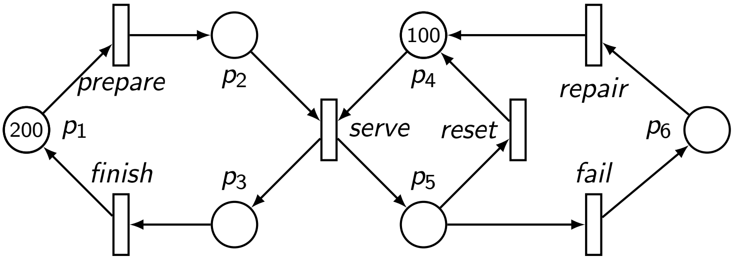
Complementation

- example



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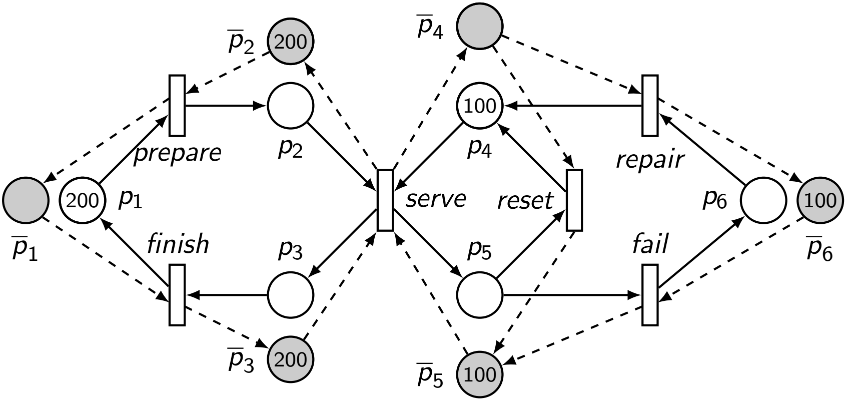

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  - for each p define model component

$$M_p \stackrel{def}{=} C_p || \ldots || C_p || C_{\overline{p}} || \ldots || C_{\overline{p}}$$

with $m(p, 0)$ copies of C_p and $m(\overline{p}, 0)$ copies of $C_{\overline{p}}$.

Construction of PEPA model

- use algorithm by Hillston, Recalde, Ribaudo and Silva [2001]
- converts stochastic Petri net to PEPA model
  - convert each pair p and $\overline{p}$ into a sequential component

$$C_p \stackrel{def}{=} \sum_{t \in p^\bullet} (t, \lambda(t)).C_{\overline{p}} \qquad C_{\overline{p}} \stackrel{def}{=} \sum_{t \in {}^\bullet p} (t, \lambda(t)).C_p$$

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    with $m(p, 0)$ copies of C_p and $m(\overline{p}, 0)$ copies of $C_{\overline{p}}$.
  - recursively build up synchronisation sets and system equation

Construction of PEPA model (cont.)

- example

$$C_{p_1} \stackrel{def}{=} (prepare, p).C_{\overline{p}_1} \qquad C_{\overline{p}_1} \stackrel{def}{=} (finish, f).C_{p_1}$$

Construction of PEPA model (cont.)

- example

$$
\begin{array}{lll}
C_{p_1} \stackrel{def}{=} (prepare, p).C_{\overline{p}_1} & & C_{\overline{p}_1} \stackrel{def}{=} (finish, f).C_{p_1} \\
C_{p_2} \stackrel{def}{=} (serve, s).C_{\overline{p}_2} & & C_{\overline{p}_2} \stackrel{def}{=} (prepare, p).C_{p_2}
\end{array}
$$

Construction of PEPA model (cont.)

- example

$$
\begin{array}{lllllll}
C_{p_1} & \stackrel{def}{=} & (prepare, p).C_{\overline{p}_1} & C_{\overline{p}_1} & \stackrel{def}{=} & (finish, f).C_{p_1} \\
C_{p_2} & \stackrel{def}{=} & (serve, s).C_{\overline{p}_2} & C_{\overline{p}_2} & \stackrel{def}{=} & (prepare, p).C_{p_2} \\
C_{p_3} & \stackrel{def}{=} & (finish, f).C_{\overline{p}_3} & C_{\overline{p}_3} & \stackrel{def}{=} & (serve, s).C_{p_3} \\
C_{p_4} & \stackrel{def}{=} & (serve, s).C_{\overline{p}_4} & C_{\overline{p}_4} & \stackrel{def}{=} & (reset, r).C_{p_4} + (repair, e).C_{p_4} \\
C_{p_5} & \stackrel{def}{=} & (reset, r).C_{\overline{p}_5} + (fail, a).C_{\overline{p}_5} & C_{\overline{p}_5} & \stackrel{def}{=} & (serve, s).C_{p_5} \\
C_{p_6} & \stackrel{def}{=} & (repair, e).C_{\overline{p}_6} & C_{\overline{p}_6} & \stackrel{def}{=} & (fail, a).C_{p_6}
\end{array}
$$

Construction of PEPA model (cont.)

- example

$$
\begin{array}{lllllll}
C_{p_1} & \stackrel{def}{=} & (prepare, p).C_{\overline{p}_1} & & C_{\overline{p}_1} & \stackrel{def}{=} & (finish, f).C_{p_1} \\
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C_{p_6} & \stackrel{def}{=} & (repair, e).C_{\overline{p}_6} & & C_{\overline{p}_6} & \stackrel{def}{=} & (fail, a).C_{p_6}
\end{array}
$$

$$
\begin{aligned}
\mathcal{M} \stackrel{def}{=} \; & C_{p_1}[200] \underset{\{prepare, finish\}}{\bowtie} C_{\overline{p}_2}[200] \underset{\{serve\}}{\bowtie} C_{\overline{p}_3}[200] \underset{\{serve\}}{\bowtie} \\
& C_{p_4}[100] \underset{\{serve, reset, repair\}}{\bowtie} C_{\overline{p}_5}[100] \underset{\{fail\}}{\bowtie} C_{\overline{p}_6}[100]
\end{aligned}
$$

ODE extraction

- view PEPA model as high/low concentration, C_p and $C_{\overline{p}}$

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- extract ODEs

Comparison of ODEs

$$\frac{dN(C_p,\tau)}{d\tau} = \sum_{\substack{(t,\lambda(t))\\ \text{entry}\\ \text{activity}}} \lambda(t).\min\{N(C_{p'},\tau) \mid C_{p'} \xrightarrow{(t,\lambda(t))}\} - \sum_{\substack{(t,\lambda(t))\\ \text{exit}\\ \text{activity}}} \lambda(t).\min\{N(C_{p'},\tau) \mid C_{p'} \xrightarrow{(t,\lambda(t))}\}$$

$$\frac{dm(p,\tau)}{d\tau} = \sum_{t \in {}^\bullet p} \lambda(t).\min_{p' \in {}^\bullet t \,\cap\, P}\{m(p',\tau)\} - \sum_{t \in p^\bullet} \lambda(t).\min_{p' \in {}^\bullet t \,\cap\, P}\{m(p',\tau)\}$$

Comparison of ODEs

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- both approaches give the same equations

Further work and conclusions

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  - different approaches to finite server semantics
  - robustness of ODEs

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